Exercise 1

a) They must be linearly independent. It's when $\det(v_1\, v_2\, v_3) \neq 0$.

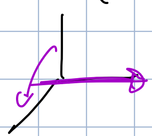
b) $v_1 \parallel v_2 \parallel v_3$

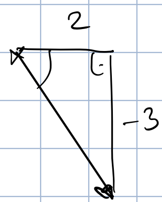
Exercise 2

1) $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y \\ -x \end{pmatrix}$

2) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x+1 \\ y+z \end{pmatrix}$ Not linear as $f(\vec{0}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \neq \vec{0}$

3) a) $G = \begin{pmatrix} 3 & 0 & -1 \\ 0 & 2 & 5 \end{pmatrix}$

b)  axis $M = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

c) 

$\alpha = \tan^{-1}\left(-\frac{3}{2}\right) \cong -56{,}31$ $\quad 2\alpha \cong -112{,}6°$

$S_{2\alpha} = \begin{pmatrix} \cos(2\alpha) & \sin(\alpha) \\ \sin(2\alpha) & -\cos(\alpha) \end{pmatrix} = \begin{pmatrix} -5/13 & -12/13 \\ -12/13 & 5/13 \end{pmatrix}$

$\left(\cong \begin{pmatrix} -0{,}38 & -0{,}92 \\ -0{,}92 & 0{,}38 \end{pmatrix} \right)$

Exercise 3

a) A vector $\vec{v} \neq \vec{0}$ whose image $\vec{v}'$ is $\parallel \vec{v}$, so a multiple of $\vec{v}$.

b) $M\vec{a} = \lambda \vec{a}$ $\quad \begin{pmatrix} -6 & 4 \\ 2 & k \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \lambda \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\quad \begin{cases} -24+12 = 4\lambda \\ 8+3k = 3\lambda = -9 \end{cases}$ $\quad -12 = 4\lambda \quad \underline{\underline{\lambda = -3}}$

$3k = -17 \quad \underline{\underline{k = -17/3}}$

c) $p(\lambda) = \begin{vmatrix} 6-\lambda & -1 \\ 2 & 3-\lambda \end{vmatrix} = \lambda^2 - 9\lambda + 20$

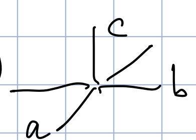
$p(\lambda) = 0$ $\quad$ ~~SDF~~ ... $p(\lambda) = (\lambda-5)(\lambda-4)$

$\underline{\lambda = 4 \text{ and } \lambda = 5}$

Exercise 4

a) $\vec{a}' = -2\vec{a}$ $\quad \vec{b}' = -2\vec{b}$ $\quad \vec{c}' = \vec{0}$ $\quad$ so $\vec{v}' = 3\vec{a}' - \vec{b}' + 5\vec{c}'$

$\underline{\vec{v}' = -6\vec{a} + 2\vec{b}}$ $(+0 \cdot \vec{c})$

b) 

proj on "plane $\vec{a}, \vec{b}$" $\parallel$

composed with a homothety with factor -2

(there are other possibilities)

Exercise 5

$\pi: 2x - y + 2z = 0 \qquad \vec{n} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \qquad \|\vec{n}\| = 3$

a) $\det(F) = 0$ as it is a projection.

b) $\vec{e_2}' = \vec{e_2} - \dfrac{\vec{e_2} \cdot \vec{n}}{\|\vec{n}\|^2} \cdot \vec{n} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \dfrac{-1}{9} \cdot \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2/9 \\ 8/9 \\ 2/9 \end{pmatrix}$

c) $B' = \left(\vec{n}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \right) \qquad F' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

must be $\perp$ to $\vec{n}$

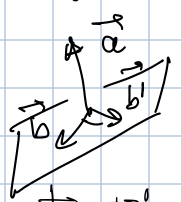
Exercise 6

In V_3, M orthogonal with $\det = 1$

a) Solve $M\vec{v} = \vec{v}$

b) Choose $\vec{b} \perp \vec{a}$, compute $\vec{b}' = M\vec{b}$ and obtain $\vec{b}' = -\vec{b}$

c)



$\cos(\gamma) = \dfrac{\vec{b} \cdot \vec{b}'}{\|\vec{b}\| \cdot \|\vec{b}'\|} \qquad \gamma = \pm \ldots$

$\vec{b} \times \vec{b}' = k \cdot \vec{a}$

It's a positive angle if $k > 0$
negative angle if $k < 0$

d) $\det(M^3) = \underline{\underline{1}}$ as $\det(M^3) = \det(M) \cdot \det(M) \cdot \det(M)$

e) $\vec{a}$ is 1-eigen under f so it is also 1-eigen under $f \circ f \circ f$

So it's eigenvalue is $\underline{\underline{1}}$.