TEST#4	LINEAR ALGEBRA	3MG Level 2
2023/02/22		3MG02

With "formulaire" / 34 Name: _ 90'

Give justifications when needed

Exercise 1. / 2 (10')

In V_3 we consider the subspace F generated by $\overrightarrow{v_1}, \overrightarrow{v_2}$ and $\overrightarrow{v_3}$ three non-zero vectors.

a) What's the condition, and how to verify it, these three vectors to form a basis of V_3 ?

Answer:

b) Under what condition(s) $\dim(F) = 1$?

Answer:

Exercise 2. / 8 (15') 1-1-2-1.5-2.5

1) Give the expression of the linear application that corresponds to a rotation by $-90°$ in V_2.

Answer: $f: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} \quad \end{pmatrix}$

2) Give an application from V_3 to V_2 that is not a linear application and justify why.

Answer and justification: $f: \begin{matrix} V_3 \to V_2 \\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} \quad \end{pmatrix} \end{matrix}$

3) Give the matrix of...

a) ...the linear application $g: \begin{matrix} V_3 \to V_2 \\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} 3x - z \\ 2y + 5z \end{pmatrix} \end{matrix}$ in the standard basis. **Answer:**

b) ...the rotation by $+90°$ about the vector $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ in V_3. **Answer:**

c) ...the symmetry about the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ in V_2.

Computations and answer:

Exercise 3. / 6 (15') 3-3

a) What is an eigenvector of a linear application f ?
Answer:

b) The vector $\vec{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ is an eigenvector of $M = \begin{pmatrix} -6 & 4 \\ 2 & k \end{pmatrix}$. Determine the eigenvalue of $\vec{a}$ and the value of k.

c) Determine the eigenvalues of the transformation whose matrix is $F = \begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix}$.

Exercise 4. / 4 (15') 1.5-2.5

In the orthonormal basis $\mathcal{B} = (\vec{a}, \vec{b}, \vec{c})$ the matrix of the transformation f is $F = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

a) Write $\overrightarrow{v'}$ the image of $\vec{v} = 3\vec{a} - \vec{b} + 5\vec{c}$ as a linear combination of $\vec{a}, \vec{b}$ and $\vec{c}$.

Answer: $\overrightarrow{v'} = \_\_\vec{a} + \_\_\vec{b} + \_\_\vec{c}$

b) Give the geometrical description of f, a composition. Illustration welcome.
Answer:

Exercise 5. / 7 (15') 1-3-3

We consider the orthogonal projection on the plane $\pi: 2x - y + 2z = 0$.

In the standard basis we name F the matrix associated.

a) What is the value of $\det(F)$ and why?

Answer:

b) Determine the second column of F.

c) Give an eigenbasis $\mathcal{B}'$ and the eigenmatrix F' associated.

Answer:

Exercise 6. / 7 (10') 1-1.5-2.5-1-1

We consider an orthogonal matrix M with determinant 1.

a) How to find the axis $\vec{a}$ of the transformation f associated?

Answer:

b) What would make you conclude that f is a symmetry about $\vec{a}$? What computations...?

c) If f is not a symmetry it is a rotation with angle $\alpha \neq 180°$. ($f = R(\vec{a}, \alpha)$)

Explain how to get the angle α , with its sign. Illustration welcome.

d) Give the determinant of M^3 with a justification.

Answer:

e) Vector $\vec{a}$ (see question a)) is an eigenvector of M^3. What is its eigenvalue?

Answer: