

MATHEMATICS

Series A

The answers must be justified.

Problem 1 2.5 points

- Sketch the graph of $\mathbb{R} \rightarrow \mathbb{R}, x \mapsto y = x^2 - 4x$.
- Find the number m such that $f:]-\infty; 1] \rightarrow [m; +\infty[, x \mapsto y = f(x) = x^2 - 4x$ is bijective.
- Find the formula $f^{-1}(x)$ of the inverse function of f .

Problem 2 3.0 points

Consider the function f defined by $f(x) = 3 - 2 \cdot |x - 4|$.

- Complete the following expression without using the absolute value.

$$f(x) = \begin{cases} \dots & \text{if } x \dots \\ \dots & \text{if } x \dots \end{cases}$$

- Sketch the graph of f .
- Solve the inequality $f(x) \leq -\frac{1}{3}x$.

Problem 3 2.0 points

Let $x(t) = 1 - 2 \cdot 3^{-k \cdot t}$. It is given that $x(5) = -0.9$.

Calculate the smallest integer number t such that $x(t) \geq 0.99$. Use at least 4 decimal places.

Problem 4 2.0 points

Consider the functions f and g defined by $f(x) = |4 - x|$ and $g(x) = 50 + 4 \cdot \log(x)$.

Calculate the exact values of x such that $(g \circ f)(x) = (f \circ g)(100^{100})$.

Problem 5 0.5 point

Consider the functions $f:]-\infty; 0] \rightarrow [0; +\infty[, x \mapsto y = f(x) = |2x + \sqrt{x^2}|$.

Find m such that $f(x) = m \cdot x$.