# LDDR. Niveau 2: TE 37 Solutions.

## Problème 1

$\int \ln^2(2x)\,dx = \int 1\cdot \ln^2(2x)\,dx = I$

$u' = 1 \Rightarrow u = x$

$v = \ln^2(2x) \Rightarrow v' = \frac{2}{2}\ln(2x) = \ln(2x)$

$I = x\cdot \ln^2(2x) - \int x\ln(2x)\,dx$

$u' = x \Rightarrow u = \frac{1}{2}x^2$

$v = \ln(2x) \Rightarrow v' = \frac{2}{2x} = \frac{1}{x}$

Donc $\int x\ln(2x)\,dx = \frac{1}{2}x^2\ln(2x) - \frac{1}{2}\int x\,dx$

$= \frac{1}{2}x^2\ln(2x) - \frac{1}{4}x^2$

Alors $I = x\ln^2(2x) - \frac{1}{2}x^2\ln(2x) + \frac{1}{4}x^2 + c$

## Problème 2.

$f(x) = \frac{2x^2-1}{2x-4}$

a)

$$\begin{array}{r|l} 2x^2-1 & 2x-4 \\ \cline{2-2} -2x^2+4x & x+2 \\ \cline{1-1} 4x-1 & \\ -4x+8 & \\ \cline{1-1} 7 & \end{array}$$

AO: $d(x) = y = x+2$

b) $A = \int_b^0 (d(x) - f(x))\,dx = \int_b^0 \left(\cancel{x}\cancel{+2}\cancel{-x}\cancel{-2} - \frac{7}{2x-4}\right)dx$

$= -\frac{7}{2}\int_b^0 \frac{1}{x-2}\,dx = -\frac{7}{2}\left[\ln|x-2|\right]_b^0 =$

$= -\frac{7}{2}\left(\ln 2 - \ln(b-2)\right) = 10 \Rightarrow$

$\Rightarrow \ln 2 - \ln(b-2) = -\frac{20}{7} \Rightarrow$

$\ln 2 + \frac{20}{7} = \ln(b-2) \Rightarrow$

$b-2 = \exp(\ln 2 + \frac{20}{7}) \Rightarrow$

$b = \exp(\ln 2 + \frac{20}{7}) + 2$

<u>Problème 3</u> $f(x) = \frac{5e^x}{e^{2x}+4}$ $\quad g(x) = 1$

$f(x) = 1 \Rightarrow 5e^x = e^{2x}+4 \Rightarrow e^{2x} - 5e^x + 4 = 0$

$\Delta = 25 - 16 = 9$ $\quad e^x = \frac{5 \pm 3}{2}$ $\begin{cases} e^x = 4 \\ e^x = 1 \end{cases}$

$e^x = 4 \Rightarrow x = \ln 4$

$e^x = 1 \Rightarrow x = 0$

$V = \pi \int_0^{\ln 4} (f^2(x) - g^2(x))\,dx =$

$f^2(x) = \frac{25e^{2x}}{(e^{2x}+4)^2}$ On cherche une primitive

$e^{2x} = t \Rightarrow 2e^{2x}dx = dt \Rightarrow dx = \frac{dt}{2e^{2x}}$

$25\int \frac{e^{2x}}{(e^{2x}+4)^2}\,dx = 25\int \frac{t}{(t+4)^2} \cdot \frac{dt}{2 \cdot t} = \frac{25}{2}\int \frac{dt}{(t+4)^2} =$

$= \frac{25}{2}\int (t+4)^{-2}dt = \frac{25}{2}(-1)(t+4)^{-1} = -\frac{25}{2(t+4)} =$

$= \frac{-25}{2(e^{2x}+4)}$

$g^2(x) = 1 \Rightarrow G(x) = x$

Donc

$$V = \pi \left( \left( F(\ln 4) - G(\ln 4) \right) - \left( F(0) - G(0) \right) \right)$$

$$F(\ln(4)) = \frac{-25}{2(e^{2\ln 4}+4)} = -\frac{25}{2(e^{\ln 4^2}+4)} =$$

$$= -\frac{\overset{5}{\cancel{25}}}{2 \cdot \underset{4}{\cancel{20}}} = -\frac{5}{8}$$

$$F(0) = -\frac{25}{2(e^0+4)} = -\frac{\overset{5}{\cancel{25}}}{2 \cdot \underset{1}{\cancel{5}}} = -\frac{5}{2}$$

$$G(\ln 4) = \ln 4 \qquad G(0) = 0$$

$$V = \pi \left( \left( -\frac{5}{8} - \ln 4 \right) - \left( -\frac{5}{2} - 0 \right) \right) =$$

$$= \pi \left( -\frac{5}{8} - \ln 4 + \frac{20}{8} \right) = \pi \left( \frac{15}{8} - \ln 4 \right)$$