LDDR - Niveau 2 : TE 12 (solutions)

Problème 1 $f(x) = \frac{1}{2}x^2 e^{-x/3}$ $\quad g(x) = (x^2-3x)e^{-x/3}$

$f(x) = g(x) \Rightarrow \frac{1}{2}x^2 e^{-x/3} = (x^2-3x)e^{-x/3} \Leftrightarrow x^2 = 2x^2 - 6x$

$\Rightarrow x^2 - 6x = 0 \Rightarrow x(x-6) = 0 \Rightarrow x = 0 \quad x = 6$

$A = \int_0^6 (f(x) - g(x))\,dx = \int_0^6 \left(\frac{1}{2}x^2 - x^2 + 3x\right)e^{-x/3}\,dx$

$= \int_0^6 \left(-\frac{1}{2}x^2 + 3x\right)e^{-x/3}\,dx$

On trouve une primitive de $h(x) = \left(-\frac{1}{2}x^2 + 3x\right)e^{-x/3}$ par identification des coefficients

$H(x) = (ax^2 + bx + c)e^{-x/3} \Rightarrow$

$H'(x) = (2ax + b)e^{-x/3} - \frac{1}{3}(ax^2 + bx + c)e^{-x/3}$

$= e^{-x/3}\left(-\frac{1}{3}ax^2 + (2a - \frac{1}{3}b)x + b - \frac{1}{3}c\right)$

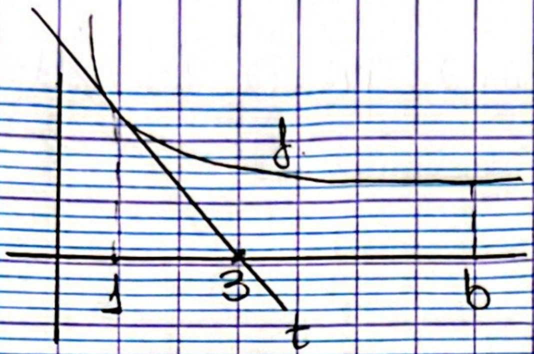
$H'(x) = h(x) = e^{-x/3}\left(-\frac{1}{2}x^2 + 3x + 0\right)$

$$\Rightarrow \begin{cases} -\frac{1}{3}a = -\frac{1}{2} \Rightarrow a = \frac{3}{2} \\ 2a - \frac{1}{3}b = 3 \Leftrightarrow 3 - \frac{1}{3}b = 3 \Rightarrow b = 0 \\ b - \frac{1}{3}c = 0 \Rightarrow c = 0 \end{cases}$$

$H(x) = \frac{3}{2}x^2 e^{-x/3}$

Donc $A = H(6) - H(0) = \left(\frac{3}{2} \cdot 36e^{-2} - 0\right) = 54e^{-2}$

oblème 2.



$f(x) = \frac{4}{\sqrt{x}} = 4x^{-1/2} \quad x=1 \Rightarrow f(1) = 4$

$f'(x) = 4\cdot\left(-\frac{1}{2}\right)x^{-3/2} = -\frac{2}{\sqrt{x^3}} \quad f'(1) = -2 = m$

$y = mx + h$

$f(1) = f'(1)\cdot 1 + h \Rightarrow 4 = -2\cdot 1 + h \Rightarrow h = 6$

$t(x) = -2x+6$

$t(x) = 0 \Leftrightarrow -2x+6 = 0 \Leftrightarrow x = 3$

$V = \pi\int_1^b f^2(x)dx - \pi\int_1^3 t^2(x)dx$

$f^2(x) = \frac{16}{x} \Rightarrow F(x) = 16\ln x$

$t^2(x) = (-2x+6)^2 = 4x^2 - 24x + 36$

$\int_1^b f^2(x)dx = F(b) - F(1) = 16\ln b - \cancel{16\ln 1}^{\,0}$

$= 16\ln b$

$T(x) = \frac{4}{3}x^3 - 12x + 36x = 4\left(\frac{1}{3}x^3 - 3x + 9x\right)$

$\int_1^3 t^2(x)dx = T(3) - T(1) = 108 - \left(24 + \frac{4}{3}\right) = \frac{248}{3}$

$V = \pi\left(16\ln b - \frac{248}{3}\right) = 100 \Rightarrow 16\ln b = \frac{100}{\pi} + \frac{248}{3}$

$\Rightarrow 16\ln b = \frac{100}{\pi} + \frac{248}{3} \Rightarrow \ln b = \frac{100}{16\pi} + \frac{31}{6}$

$\Rightarrow b = \exp\left(\frac{25}{4\pi} + \frac{31}{6}\right)$

Problème 3 $f(x) = \dfrac{x^2-x}{x-2}$

$$\begin{array}{r|l} x^2 - x & x-2 \\ -x^2+2x & x+1 \\ \hline x & \\ -x+2 & \\ \hline 2 & \end{array}$$

AO $y = x+1 = d$

$f(x) = 0 \Leftrightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0 \quad x = 1$

$A = \int_{-2}^{1} (d(x) - f(x))dx = \int_{-2}^{1} (\cancel{x} + \cancel{1} - \cancel{x} - \cancel{1} - \frac{2}{x-2})dx =$

$= -2\int_{-2}^{1} \frac{1}{x-2}dx = -2\ln|x-2|\Big|_{-2}^{1} = -2\ln 1 + 2\ln 4 =$

$= 2\ln 4$

Problème 4. $\int x\sqrt{x-1}\,dx$

$t = \sqrt{x-1} \Rightarrow t^2 = x-1 \Rightarrow x = t^2+1$, $2t\,dt = dx$

$I = \int x \cdot t \cdot 2t\,dt = 2\int (t^2+1)t^2\,dt =$

$= 2\int (t^4 + t^2)\,dt = 2 \cdot \frac{1}{5}t^5 + \frac{2}{3}t^3 =$

$= \frac{2}{5}(\sqrt{x-1})^5 + \frac{2}{3}(\sqrt{x-1})^3 + C.$