LDDR: Nível 2: TE 6 Trigo

Exercício 1

1) $a=12,\ b=9,\ c=5$

$c^2 = a^2+b^2-2ab\cos\gamma \Rightarrow 2ab\cos\gamma = a^2+b^2-c^2$

$\Rightarrow \cos\gamma = \frac{a^2+b^2-c^2}{2ab} \Rightarrow \cos\gamma = \frac{12^2+9^2-5^2}{2\cdot 12\cdot 9} = 0.926$

$\Rightarrow \gamma = 22.2^\circ$

$b^2 = a^2+c^2-2ac\cos\beta \Rightarrow \cos\beta = \frac{a^2+c^2-b^2}{2\cdot a\cdot c} \Rightarrow$

$\cos\beta = 0.733 \Rightarrow \beta = 42.83^\circ$

$\alpha = 180-\beta-\gamma \approx 115$

Exercício 2

1) $\sin(3x)+4\cos(3x)=0 \Rightarrow$

$\Rightarrow \sin(3x) = -4\cos(3x) \Rightarrow \frac{\sin(3x)}{\cos(3x)} = -4$

$\Rightarrow \tan(3x) = -4 \Rightarrow 3x = -76 + k180 \Rightarrow$

$x = -25,32 + 60k \quad k\in\mathbb{Z}$

2) $\cos^2(x)+2\sin(x) = -1 \Rightarrow 1-\sin^2 x+2\sin x = -1$

$\Rightarrow \sin^2 x - 2\sin x - 2 = 0$

$\Delta = 4+8 = 12$

$\sin x = \frac{2\pm\sqrt{12}}{2}$ $\begin{cases} \sin x = 2.73 \text{ imp.} \\ \sin x = -0.73 \end{cases}$

$\sin x = -0.73 \Rightarrow \begin{cases} x = 137 + k360 \\ x = -137 + k360 \end{cases}$

3) $\sin(4x+20^\circ) = -\sin(2x) \Rightarrow$

$\Rightarrow \sin(4x+20^\circ) = \sin(-2x) \Rightarrow$

$\Rightarrow \begin{cases} 4x+20 = -2x + k360 \\ 4x+20 = 180+2x+k360 \end{cases} \Rightarrow$

$\begin{cases} 6x = -20 + k360 \Rightarrow x = -\frac{10}{3} + k60 \\ 2x = 160 + k360 \Rightarrow x = 80 + 180k \end{cases}$

4) $2\arccos(x) = \arctan 1 \Rightarrow$

$2\arccos(x) = \frac{\pi}{4} \Rightarrow \arccos(x) = \frac{\pi}{8}$

$\Rightarrow x = \cos\frac{\pi}{8}$

Exercice 3. $\sin x = -0.8 \quad x \in Q_{III} \quad \cos y = 0.8 \quad y \in Q_{IV}$

$\cos(x-y) = \cos x \cos y + \sin x \sin y \quad (1)$

$\cos x = ? \qquad \sin y = ?$

$\sin^2 x + \cos^2 x = 1 \Rightarrow 0.8^2 + \cos^2 x = 1 \Rightarrow$

$\cos^2 x = 0.36 \Rightarrow \cos x = -0.6$ car $x \in Q_{III}$

$\sin^2 y + \cos^2 y = 1 \Rightarrow \sin^2 y = 1 - 0.8^2 = 0.36$

$\Rightarrow \sin y = -0.6$ car $y \in Q_{IV}$

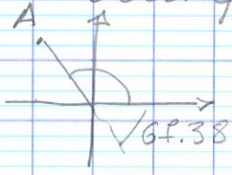
Donc (1) donne:

$\cos(x-y) = -0.6 \cdot 0.8 + (-0.8) \cdot (-0.6) = 0$

Exercice 4. 1. $A(-5,12)$ $r = \sqrt{5^2+12^2} = \sqrt{169} = 13$

$\tan\varphi = -\frac{12}{5} \Rightarrow \varphi = -67.38° + 180$

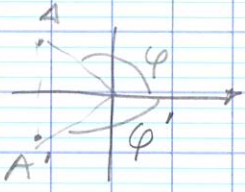
$\Rightarrow \varphi = 112.62°$

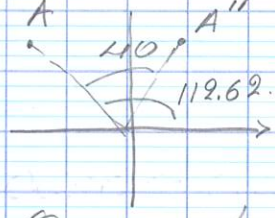


2) $A'(-5,-12)$ $r = 13$

$\varphi' = -112.62°$

$= 247.38°$



3) 

$\varphi'' = 112.62 - 40 = 72.62°$

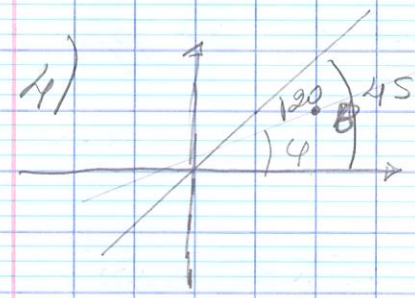
$r'' = 13.$

On va trouver les cordonnées cartésiennes de A''.

$x = r\cdot\cos\varphi'' = 3.88.$

$y = r\cdot\sin\varphi'' = 12.41$

Alors la distance par l'axe y est $x = 3.88$

4) 

$\varphi = 45 - 20 = 25° = r$

$m = \tan 25° = 0.47.$

$y = mx + h \Rightarrow y = 0.47x + h$

B: $1 = 0.47\cdot 2 + h \Rightarrow$

$\Rightarrow h = 0.07.$

$y = 0.47x + 0.07.$

Exercice 3. 1) $f(x) = \tan(2x) = \dfrac{\sin(2x)}{\cos(2x)}$

$\cos(2x) = 0 \Rightarrow 2x = \dfrac{\pi}{2} + k\pi \Rightarrow x = \dfrac{\pi}{4} + \dfrac{k\pi}{2}$.

$D_f = \mathbb{R} \setminus \left\{ \dfrac{\pi}{4} + \dfrac{k\pi}{2} \right\}$.

2) $M_{ax} = 2$ $\qquad T = \dfrac{2\pi}{\alpha} = \dfrac{3\pi}{4} \Rightarrow \alpha = \dfrac{8}{3}$.

$T = \dfrac{3\pi}{4}$

Donc $f(x) = 2\cos\left(\dfrac{8}{3}x\right)$

3) $f(x) = \sin(\pi x)$.

4) La période de $\sin\left(\dfrac{x}{5}\right)$ est $\dfrac{2\pi}{\frac{1}{5}} = 10\pi$.

La période de $\tan\left(\dfrac{x}{4}\right)$ est $\dfrac{\pi}{\frac{1}{4}} = 4\pi$.

La période de f est le ppmc de $(10\pi, 4\pi)$

$\Rightarrow T = 20\pi$.

5) $f(x) = 5\sin(4x+3) - 2$.

$-1 \leq \sin(4x+3) \leq 1 \Rightarrow -5 \leq 5\sin(4x+3) \leq 5$

$\Rightarrow -7 \leq f(x) \leq 3 \Rightarrow D_f = [-7, 3]$.