Without "formulaire"   / 40 pts

CALCULUS   20pts

Exercise 1   / 4 pts

Determine the derivative of $f(x) = 4 - \frac{3x+5}{x^2-1}$ and of $g(x) = \sqrt{x^2 \cdot \sin(x)}$

Write the answer in the simplest form.

Exercise 2   / 7 pts

The total area of a cylinder is 2000π.

Determine its dimensions (radius and height) such that its volume is optimized.

Is it a maximum or a minimum? Justify and determine that volume.

Exercise 3   / 4 pts

The graph of the function $f(x) = ax^3 + bx^2 + cx$ has an inflexion point at $(2; -3)$ and the tangent at its inflexion point is $t: y = -x - 1$.

To determine the coefficients $a, b, c \in \mathbb{R}$ a system of equations must be solved.
Determine that system, without solving it.

Exercise 4   / 5 pts

The curves $f(x) = 5(x-3)^2 + a$ and $g(x) = \frac{b}{x}$ intersect at $(4; 1)$.

Determine the values of $a, b \in \mathbb{R}$.

Determine the measure of the acute angle between these curves at that intersection point.

Exercise 5   / 20 pts

a) (2.5pts)   Determine $\frac{1+2i}{3-4i}$ in the algebraic form

b) (3pts)   Determine the argument of $z = z_1 - z_2$, with $z_1 = 2cis(80°)$ and $z_2 = cis(-10°)$

c) (3pts)   Express the complex number $z = \frac{-\overline{z_1}}{(2z_2)^3}$ in the trigonometric form, with $z_1 = 4cis(20°)$ and $z_2 = 2cis(40°)$.

d) (2.5pts)   Solve the equation $2z - 3\bar{z} = 5 - 2i$

e) (4pts)   Precisely describe the geometric shapes defined by   a) $z \cdot \bar{z} = 4$ b) $Re(i\,\bar{z}) = 2$

f) (5pts)   Among the five solutions of the equation $z^5 = -7 + 24i$, determine the real part of the one that is the closest to the imaginary axis.   *Give your answer rounded to 3 digits.*