CALCUL INTEGRAL

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c,\quad n \neq -1 \qquad \int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\quad n \neq -1$$

- *Exemples :*

i. $\int x^2 dx = \frac{1}{3} x^3 + c$ ii. $\int \frac{1}{x^3} dx = \int x^{-3} dx = \frac{1}{-3+1} x^{-3+1} = \frac{1}{-2} x^{-2} = -\frac{1}{2x^2} + c$

iii. $\int \sqrt[3]{x} dx = \int x^{1/3} dx = \frac{1}{\frac{1}{3}+1} x^{\frac{1}{3}+1} = \frac{1}{\frac{4}{3}} x^{\frac{4}{3}} = \frac{3}{4}\sqrt[3]{x^4} + c$

iv. $\int \frac{1}{\sqrt[4]{x}} dx = \int x^{-1/4} dx = \frac{1}{-\frac{1}{4}+1} x^{-\frac{1}{4}+1} = \frac{1}{\frac{3}{4}} x^{\frac{3}{4}} = \frac{4}{3}\sqrt[4]{x^3} + c$

v. $\int (2x+1)^2 dx = \frac{1}{3\cdot 2}(2x+1)^3 + c = \frac{1}{6}(2x+1)^3 + c$

vi. $\int (-x+2)^2 dx = \frac{1}{(-1)\cdot 3}(-x+2)^3 + c = -\frac{1}{3}(-x+2)^3 + c$

vii. $\int x\sqrt[3]{x} dx = \int x \cdot x^{1/3} dx = \int x^{4/3} dx = \frac{1}{\frac{4}{3}+1} x^{\frac{4}{3}+1} = \frac{1}{\frac{7}{3}} x^{\frac{7}{3}} = \frac{3}{7}\sqrt[3]{x^7} + c$

$$\int \frac{1}{x} dx = \ln|x| + c \qquad \int \frac{1}{ax+b} dx = \frac{1}{a}\ln|ax+b| + c$$

- *Exemples:*

i. $\int \frac{1}{3x-1} dx = \frac{1}{3}\ln|3x-1| + c$ ii. $\int \frac{1}{-x+3} dx = \frac{1}{(-1)}\ln|-x+3| + c = -\ln|-x+3| + c$

$$\int e^x dx = e^x + c \qquad \int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$$

- *Exemples:*

i. $\int e^{3x+2} dx = \frac{1}{3} e^{3x+2} + c$ ii. $\int e^{-x} dx = -e^{-x} + c$ iii. $\int e^{x/2} dx = \frac{1}{1/2} e^{e^{x/2}} = 2e^{e^{x/2}} + c$

$$\int a^x dx = \frac{1}{\ln a} a^x + c$$

- *Exemples*:

i. $\int 3^x dx = \frac{1}{\ln 3} 3^x + c$ ii. $\int 3^{2x} dx = \frac{1}{\ln 3} \cdot \frac{1}{2} 3^{2x} + c$

$$\boxed{\int \sin(x)dx = -\cos(x)+c \qquad \int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b)+c}$$

$$\boxed{\int \cos(x)dx = \sin(x)+c \qquad \int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b)+c}$$

- *Exemples* :

i. $\int \sin(-3x+1)dx = -\frac{1}{-3}\cos(-3x+1)+c = \frac{1}{3}\cos(-3x+1)+c$

ii. $\int \cos\left(\frac{x}{3}+2\right)dx = \frac{1}{1/3}\sin\left(\frac{x}{3}+2\right) = 3\sin\left(\frac{x}{3}+2\right)+c$

$$\boxed{\int \frac{1}{\cos^2(x)}dx = \tan(x)+c \qquad \int \frac{1}{\sin^2(x)}dx = -\cot an(x)+c \qquad \int \frac{1}{x^2+1}dx = \arctan(x)+c}$$

Intégration par parties:

$$\boxed{\int u'(x)v(x)dx = u(x)v(x) - \int u(x)v'(x)dx}$$

	p(x)e^{ax+b}	**p(x)sin(ax+b)**	**p(x)cos(ax+b)**	**e^{ax+b}sin(cx+d)**	**p(x)ln(ax+b)**
u´	e^{ax+b}	sin(ax+b)	cos(ax+b)	e^{ax+b}	p(x)
u	$\frac{1}{a}e^{ax+b}$	$-\frac{1}{a}\cos(ax+b)$	$\frac{1}{a}\sin(ax+b)$	$\frac{1}{a}e^{ax+b}$	$\int p(x)dx$
v	p(x)	p(x)	p(x)	sin(cx+d)	ln(ax+b)
v´	p´(x)	p´(x)	p´(x)	c·cos(cx+d)	$\frac{a}{ax+b}$

- *Exemples*:

i. $I = \int 2xe^x dx$ $u' = e^x \Leftrightarrow u = e^x$ $v = 2x \Leftrightarrow v' = 2$

$I = 2xe^x - 2\int e^x dx = 2xe^x - 2e^x + c$

ii. $I = \int (x^2+1)e^{3x}dx$ $u' = e^{3x} \Leftrightarrow u = \frac{1}{3}e^{3x}$ $v = x^2 + 1 \Leftrightarrow v' = 2x$

$I = \frac{1}{3}(x^2+1)e^{3x} - \frac{2}{3}\int xe^{3x}dx$

On applique encore une fois la méthode pour l'intégrale $\int xe^{3x}dx$: $u' = e^{3x} \Leftrightarrow u = \frac{1}{3}e^{3x}$ $\quad v = x \Leftrightarrow v' = 1$

Alors $\int xe^{3x}dx = \frac{1}{3}x e^{3x} - \int e^{3x}dx = \frac{1}{3}x e^{3x} - \frac{1}{3}e^{3x}$ et $I = \frac{1}{3}(x^2+1)e^{3x} - \frac{2}{3}\left(xe^{3x} - e^{3x}\right) + c$

iii. $I = \int \ln(x)dx$ $\quad u' = 1 \Leftrightarrow u = x \quad v = \ln(x) \Leftrightarrow v' = \frac{1}{x}$

$I = x\ln(x) - \int x\frac{1}{x}dx = x\ln(x) - \int dx = x\ln(x) - x + c$

iv. $I = \int \frac{\ln(x)}{x}dx$ $\quad u' = \frac{1}{x} \Leftrightarrow u = \ln(x) \quad v = \ln(x) \Leftrightarrow v' = \frac{1}{x}$

$I = \ln^2(x) - \int \frac{\ln(x)}{x}dx \Leftrightarrow I = \ln^2(x) - I \Leftrightarrow 2I = \ln^2(x) \Leftrightarrow I = \frac{\ln^2(x)}{2} + c$

v. $I = \int 3x\sin(2x+3)dx$ $\quad u' = \sin(2x+3) \Leftrightarrow u = -\frac{1}{2}\cos(2x+3) \quad v = 3x \Leftrightarrow v' = 3$

$I = -\frac{1}{2}\cos(2x+3)\cdot 3x + \frac{3}{2}\int \cos(2x+3)dx = -\frac{1}{2}\cos(2x+3)\cdot 3x + \frac{3}{2}\cdot\frac{1}{2}\sin(2x+3) =$

$= -\frac{1}{2}\cos(2x+3)\cdot 3x + \frac{3}{4}\sin(2x+3) + c$

vi. $I = \int e^x\sin(x)dx$ $\quad u' = e^x \Leftrightarrow u = e^x \quad v = \sin(x) \Leftrightarrow v' = \cos(x)$

$I = e^x\sin(x) - \int e^x\cos(x)dx$

On applique la méthode encore une fois pour trouver l'intégrale $\int e^x\cos(x)dx$

$u' = e^x \Leftrightarrow u = e^x \quad v = \cos(x) \Leftrightarrow v' = -\sin(x)$

Alors $\int e^x\cos(x)dx = e^x\cos(x) + \int e^x\sin(x)dx$, donc

$I = e^x\sin(x) - (e^x\cos(x) + \int e^x\sin(x)dx) \Leftrightarrow I = e^x\sin(x) - e^x\cos(x) - I \Leftrightarrow 2I = e^x(\sin(x)-\cos(x)) \Leftrightarrow$

$I = \frac{e^x(\sin(x)-\cos(x))}{2} + c$

Changement de variables

$$\left.\begin{array}{l}\int f'(x)\sin\left(f(x)\right)dx\\ \int f'(x)\,e^{f(x)}dx\\ \int f'(x)\ln(f(x))dx\\ \int \frac{f'(x)}{f(x)}dx\end{array}\right\}\ f(x)=t \Rightarrow f'(x)dx = dt \Rightarrow dx = \frac{1}{f'(x)}dt$$. En remplaçant on obtient :

$$\left.\begin{array}{l}\int f'(x)\sin\left(f(x)\right)dx\\ \int f'(x)\,e^{f(x)}dx\\ \int f'(x)\ln(f(x))dx\\ \int \frac{f'(x)}{f(x)}dx\end{array}\right\} = \left\{\begin{array}{l}\int \sin\left(t\right)dt=-\cos(t)= -\cos(f(x))+c\\ \int e^{t}dt = e^{t} = e^{f(x)}+c\\ \int \ln(t)dt= t\ln(t)-t=f(x)\ln(f(x))-f(x)+c\\ \int \frac{1}{t}dt = \ln(t) = \ln(f(x)) + c\end{array}\right.$$

$$\int f'(x)\sqrt[n]{f(x)}dx\ ,\ \sqrt[n]{f(x)} = t \Rightarrow f(x) = t^{n} \Rightarrow f'(x)dx = nt^{n-1}\,dt \Rightarrow dx = \frac{n}{f'\left(x\right)}t^{n-1}dt$$

On obtient: $\int f'(x)\sqrt[n]{f(x)}dx = n\int t\cdot t^{n-1}dt = n\int t^{n}dt = \frac{n}{n+1}t^{n+1} = \frac{n}{n+1}(\sqrt[n]{f(x)})^{n+1} + c$

- *Exemples :*

i. $I = \int (2x+1)\sin(x^{2}+x+2)dx$, $x^2 + x + 2 = t \Rightarrow (2x + 1)dx = dt \Rightarrow dx = \frac{1}{2x+1}dt$

$I = \int (2x+1)\sin(t)\frac{1}{2x+1}dt = \int \sin(t)dt = -\cos(t) = -\cos(x^2 + x + 2) + c$

ii. $I = \int (6x^{2}+4x)e^{x^{3}+x^{2}}dx$ $x^3 + x^2 = t \Rightarrow (3x^2 + 2x)dx = dt \Rightarrow dx = \frac{1}{3x^{2}+2x}dt$

$I = \int 2(3x^{2}+2x)e^{t}\frac{1}{3x^{2}+2x}dt = \int 2e^{t}dt = 2e^{t} = 2\,e^{x^{3}+x^{2}} + c$

iii. $I = \int (2x-1)\ln(x^{2}-x)dx$ $x^2 - x = t \Rightarrow (2x - 1)dx = dt \Rightarrow dx = \frac{1}{2x-1}dt$

$I = \int (2x-1)\ln(t)\frac{1}{2x-1}dt = \int \ln(t)dt = t\ln(t)-t = (x^{2}-x)\ln(x^{2}-x)-(x^{2}-x) = + c$

iv. $I = \int \frac{6x+3}{x^{2}+x}dx$, $x^2 + x = t \Rightarrow (2x + 1)dx = dt \Rightarrow dx = \frac{1}{2x+1}dt$

$$I = \int \frac{3(2x+1)}{t}\frac{1}{2x+1}dt = 3\int \frac{1}{t}dt \;= 3\ \ln(t) = 3\ln(x^2+x) + c$$

v. $I = \int x\sqrt{x^2-1}dx$, $\sqrt{x^2-1} = t \Rightarrow x^2 - 1 = t^2 \Rightarrow 2xdx = 2t\,dt \Rightarrow dx = \frac{1}{x}tdt$

$$I = \int x \cdot t \cdot \frac{1}{x} \cdot tdt = \int t^2dt = \frac{1}{3}t^3 = \frac{1}{3}\left(\sqrt{x^2-1}\right)^3 + c$$

vi. $I = \int \frac{1}{x\ln(x)}dx$, $\ln(x) = t \Rightarrow \frac{1}{x}dx = dt \Rightarrow dx = xdt$, donc $I = \int \frac{1}{x \cdot t}xdt = \int \frac{1}{t}dt = \ln(t) = \ln(\ln(x)) + c$

$$\boxed{I = \int \frac{\sin^{2n+1}(x)}{\cos^{2n}(x) \text{ ou } \cos^{2n+1}(x)}dx \quad \cos(x) = t \Rightarrow -\sin(x)\ dx = dt \Rightarrow dx = \frac{1}{-\sin(x)}dt}$$

$$\boxed{I = \int \frac{\cos^{2n+1}(x)}{\sin^{2n}(x) \text{ ou } \sin^{2n+1}(x)}dx \quad \sin(x) = t \Rightarrow \cos(x)\ dx = dt \Rightarrow dx = \frac{1}{\cos(x)}dt}$$

- *<u>Exemples :</u>*

i. $I = \int \tan(x)dx = \int \frac{\sin(x)}{\cos(x)}dx$, $\cos(x) = t \Rightarrow -\sin(x)\ dx = dt \Rightarrow dx = \frac{1}{-\sin(x)}dt$

$$I = -\int \frac{\sin(x)}{t}\frac{1}{\sin(x)}dt = -\int \frac{1}{t}dt = -\ln(t) = -\ln(\sin(x)) + c$$

ii. $I = \int \sin^2(x)\cos^3(x)dx$ $\sin(x) = t \Rightarrow \cos(x)\ dx = dt \Rightarrow dx = \frac{1}{\cos(x)}dt$

$$I = \int t^2\cos^3(x)\frac{dt}{\cos(x)} = \int t^2\cos^2(x)dt = \int t^2(1-t^2)dt = \int (t^2-t^3)dt = \frac{1}{3}t^3 - \frac{1}{4}t^4 = \frac{1}{3}\sin^3(x) - \frac{1}{4}\sin^4(x) + c$$

Primitive d'une fonction rationnelle $f(x) = \frac{p(x)}{d(x)}$

❖ **deg(p) ≥ deg(d)** : On effectue la division euclidienne p(x): d(x) et on écrit $f(x) = q(x) + \frac{r(x)}{d(x)}$ où q(x) est le quotient et r(x) le reste de la division.

- *<u>Exemples :</u>*

i. $I = \int \frac{x^2+3x-1}{x-1}dx$,

$$\begin{array}{r|l} x^2 + 3x - 1 & x-1 \quad d(x) \\ \hline -x^2+x & x+4 \quad q(x) \\ \hline 4x-1 & \\ -4x+4 & \\ \hline 3 & r(x) \end{array}$$

Alors I = $\int \frac{x^2+3x-1}{x-1}dx = \int\left(x+4+\frac{3}{x-1}\right)dx = \frac{1}{2}x^2+4x+3\ln|x-1|+c$

ii. I = $\int \frac{2x^2+2x+2}{x^2+1}dx$,

$$\begin{array}{l|l} 2x^2+2x+2 & x^2+1 \quad d(x) \\ \hline -2x^2 \qquad -2 & 2 \quad q(x) \\ \hline 2x & \\ \quad r(x) & \end{array}$$

Alors I = $\int\left(2+\frac{2x}{x^2+1}\right)dx = \int 2dx+\int\frac{2x}{x^2+1}dx = 2x + \int\frac{2x}{x^2+1}dx$

$\int\frac{2x}{x^2+1}dx$, $x^2 + 1 = t \Rightarrow 2xdx = dt \Rightarrow dx = \frac{1}{2x}dt$

$\int\frac{2x}{x^2+1}dx = \int\frac{2x}{t}\frac{1}{2x}dt = \int\frac{1}{t}dt = \ln(t) = \ln(x^2+1) + c$

Donc I = $2x + \ln(x^2+1) + c$

iii. I = $\int\frac{3x^2-4x+1}{x}dx = \int\left(\frac{3x^2}{x}+\frac{-4x}{x}+\frac{1}{x}\right)dx = \int\left(3x-4+\frac{1}{x}\right)dx = \frac{3}{2}x^2-4x+\ln|x|+c$

- **deg(p) < deg(d)**

a) $\int\frac{f'(x)}{f(x)}dx$ déjà vu

b) *si le dénominateur a de zéros réels* . On décompose la fonction en fractions équivalentes.

- *Exemples :*

i. I = $\int\frac{x}{x^2-7x+10}dx$

$\frac{x}{x^2-7x+10} = \frac{x}{(x-5)(x-2)} = \frac{A}{x-5}+\frac{B}{x-2} \Leftrightarrow x = A(x-2)+B(x-5) \Leftrightarrow x = (A+B)x-2A-5B \Leftrightarrow$

$\begin{cases} A+B=1 \\ -2A-5B=0 \end{cases}$, $A = \frac{5}{3}$ et $B = -\frac{2}{3}$

Alors I = $\int\frac{5}{3(x-5)}+\frac{2}{3(x-2)}dx = \frac{5}{3}\int\frac{1}{x-5}dx+\frac{2}{3}\int\frac{1}{x-2}dx = \frac{5}{3}\ln(x-5)+\frac{2}{3}\ln(x-2)+c$

ii. I = $\int\frac{1}{x^2(x+1)}dx$

$\frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} \Leftrightarrow 1 = Ax(x+1) + b(x+1) + Cx^2 \Leftrightarrow (A+C)x^2 + (A+B)x + B = 1$

$\begin{cases} A+C=0 \\ A+B=0, \; A=-1, \; B=1, \; C=1 \\ B=1 \end{cases}$

$I = -\int \frac{1}{x}dx + \int \frac{1}{x^2}dx + \int \frac{1}{x+1}dx = -\ln|x| - \frac{1}{x} + \ln|x+1| + c$

c) *si le dénominateur n'a pas de zéros réels*

On a : $\frac{1}{ax^2+bx+c}$, avec $\Delta < 0$, on écrit

$ax^2 + bx + c = a\left[\left(x+\frac{b}{2a}\right)^2 + \left(\frac{\sqrt{b^2-4ac}}{2a}\right)^2\right]$ et on utilise la méthode de changement de variables.

- *Exemples :*

i. $I = \int \frac{5}{2x^2+x+5}dx = 5\int \frac{1}{2\left[\left(x+\frac{1}{4}\right)^2 + \frac{39}{16}\right]}dx = \frac{5}{2}\int \frac{1}{\left(x+\frac{1}{4}\right)^2 + \frac{39}{16}}dx$, $x + \frac{1}{4} = \sqrt{\frac{39}{16}}t \Rightarrow dx = \frac{\sqrt{39}}{4}dt$

$I = \frac{5}{2}\int \frac{\frac{\sqrt{39}}{4}}{\frac{39}{16}t^2 + \frac{39}{16}}dt = \frac{5}{2}\cdot\frac{\frac{\sqrt{39}}{4}}{\frac{39}{16}}\int \frac{1}{t^2+1}dt = \frac{10\sqrt{39}}{39}\arctan(t) = \frac{10\sqrt{39}}{39}\arctan\left(\frac{4x+1}{\sqrt{39}}\right) + c$

d) *Si le dénominateur est du deuxième dégrée avec $\Delta < 0$ et le numérateur du premier dégrée.* On essaie a « construire » au dominateur la dérivée du dénominateur.

- *Exemples :*

i. $\int \frac{x}{x^2-3x+5}dx$. $(x^2 - 3x + 5)' = 2x - 3$

$I = \frac{1}{2}\int \frac{2x}{x^2-3x+5}dx = \frac{1}{2}\int \frac{2x-3+3}{x^2-3x+5}dx = \frac{1}{2}\int \frac{2x-3}{x^2-3x+5}dx + \frac{1}{2}\int \frac{3}{x^2-3x+5}dx$

$I_1 = \frac{1}{2}\int \frac{2x-3}{x^2-3x+5}dx$, $t = x^2 - 3x + 5 \Rightarrow dt = (2x-3)dx \Rightarrow dx = \frac{dt}{2x-3}$

$I_1 = \frac{1}{2}\int \frac{2x-3}{t}\frac{dt}{2x-3} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln(t) = \frac{1}{2}\ln(x^2-3x+5)$

$I_2 = \frac{1}{2}\int \frac{3}{x^2-3x+5}dx = \frac{3}{2}\int \frac{1}{\left(x-\frac{3}{2}\right)^2+\frac{11}{4}}dx$, $x - \frac{3}{2} = \frac{\sqrt{11}}{2}t \Rightarrow dx = \frac{\sqrt{11}}{2}dt$

$I_2 = \frac{3}{2}\cdot\frac{\sqrt{11}}{2}\int \frac{1}{\frac{11}{4}t^2+\frac{11}{4}}dt = \frac{3}{2}\cdot\frac{\sqrt{11}}{2}\cdot\frac{4}{11}\int\frac{1}{t^2+1}dt = \frac{3\sqrt{11}}{11}\int\frac{1}{t^2+1}dt = \frac{3\sqrt{11}}{11}\text{Arctan}(t) =$

$= \frac{3\sqrt{11}}{11}\text{Arctan}\left(\frac{(2x-3)\sqrt{11}}{11}\right)$ .Donc $I = I_1 + I_2 = \frac{1}{2}\ln(x^2-3x+5) + \frac{3\sqrt{11}}{11}\arctan\left(\frac{(2x-3)\sqrt{11}}{11}\right) + c$

ii. $I = \int \frac{x^2+3x+4}{x(x^2+1)}dx$

$\frac{x^2+3x+4}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \Leftrightarrow x^2 + 3x + 4 = A(x^2+1) + x(Bx+C) \Leftrightarrow x^2 + 3x + 4 = (A+B)x^2 + Cx + A$

$\begin{cases} A+B=1 \\ C=3 \\ A=4 \end{cases}$, $A = 4$, $C = 3$, $B = -3$

$I = \int \frac{4}{x} + \frac{-3x+3}{x^2+1}dx = 4\int\frac{1}{x}dx + \int\frac{-3x+3}{x^2+1}dx = 4\ln|x| + \int\frac{-3x+3}{x^2+1}dx$

$\int\frac{-3x+3}{x^2+1}dx = -3\int\frac{x}{x^2+1}dx + 3\int\frac{1}{x^2+1}dx = -\frac{3}{2}\int\frac{2x}{x^2+1}dx + 3\int\frac{1}{x^2+1}dx = -\frac{3}{2}\ln(x^2+1) + 3\arctan(x) + c$

Donc $I = 4\ln|x| - \frac{3}{2}\ln(x^2+1) + 3\arctan(x) + c$

e) *Cas spécial* $f(x) = \frac{p(x)}{q(x)}$ *ou p(x) et q(x) sont polynômes avec* $\deg(p) < \deg(q) = 2$, $q(x) = ax^2 + bx + c$

1^er cas : q(x) a deux zéros distincts x_1 et x_2

$\int\frac{r(x)}{q(x)}dx = \frac{\alpha}{a}\ln|x-x_1| + \frac{\beta}{a}\ln|x-x_2| + c$ avec α et β tels que $r(x) = \alpha(x-x_2) + \beta(x-x_1)$

2^e cas : q(x) a un zero unique x_0

$\int\frac{r(x)}{q(x)}dx = \frac{\alpha}{a}\ln|x-x_0| - \frac{\beta}{a(x-x_0)} + c$ avec α et β tels que $r(x) = \alpha(x-x_0) + \beta$

3^e cas : q(x) n'a aucun zéro réel

$$\int \frac{r(x)}{q(x)}dx = \alpha \ln|ax^2 + bx + c| + \frac{2\beta}{\sqrt{4ac-b^2}} \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) + c \text{ avec } \alpha \text{ et } \beta \text{ tels que } r(x) = \alpha(2ax+b) + \beta$$

- *Exemples :*

i. $\int \frac{1}{x^2-1}dx$ (1er cas). $x_1 = 1$, $x_2 = -1$, a = 1 , b = 0, c = -1. r(x) = 1 = α(x+1) + β(x-1) ⇒ 1 = (α+ β)x + (α- β) ⇒

$\begin{cases} \alpha+\beta = 0 \\ \alpha - \beta = 1 \end{cases} \Rightarrow \alpha = \frac{1}{2}, \beta = -\frac{1}{2}$. Donc $\int \frac{1}{x^2-1}dx = \frac{\frac{1}{2}}{1}\ln|x-1| + \frac{-\frac{1}{2}}{1}\ln|x+1| = \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1|$

ii. $\int \frac{2x}{(x-1)^2}dx$ (2e cas) ,$x_0 = 1$, $(x-1)^2 = x^2 - 2x + 1$, a = 1, b = -2, c= 1 ,

$r(x) = 2x = \alpha(x-1) + \beta \Rightarrow 2x = \alpha x - \alpha + \beta \Rightarrow \begin{cases} \alpha = 2 \\ -\alpha + \beta = 0 \end{cases} \Rightarrow \alpha = 2, \beta = 2$. Donc

$$\int \frac{2x}{(x-1)^2}dx = \frac{2}{1}\ln|x-1| - \frac{2}{1(x-1)} + c = 2\ln|x-1| - \frac{2}{(x-1)} + c$$

iii. $\int \frac{x}{x^2-x+1}dx$, (3e cas) , a = 1, b = -1, c = 1, r(x) = x= α(2x+1) + β ⇒ x = 2αx + α + β ⇒

$\begin{cases} 2\alpha = 1 \\ \alpha + \beta = 0 \end{cases}$, α = ½ , β = -½.

Donc $\int \frac{x}{x^2-x+1}dx = \frac{1}{2}\ln|x^2 - x + 1| + \frac{-2\cdot\frac{1}{2}}{\sqrt{4-1^2}}\arctan\left(\frac{2x-1}{\sqrt{4-1^2}}\right) + c = \frac{1}{2}\ln|x^2 - x + 1| + \frac{-1}{\sqrt{3}}\arctan\left(\frac{2x-1}{\sqrt{3}}\right) + c$