Domain :

- Dénominateur $\neq 0$

f(x) = $\frac{x^2-1}{(x+2)(x-3)}$. $D_f = \mathbb{R}\setminus\{-2, 3\}$

f(x) = $\frac{x^2}{\ln(x)}$. $\ln(x) \neq 0 \Leftrightarrow x \neq 1$ $D_f = \mathbb{R}\setminus\{1\}$

- $\sqrt[paire]{g(x)}$, $g(x) \geq 0$

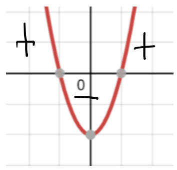
f(x) = $\sqrt{x-1}$, $x-1 \geq 0 \Leftrightarrow x \geq 1$ $D_f = [1, +\infty[$

f(x) = $\sqrt{x^2-4}$, $x^2-4 \geq 0$,

JAMAIS $x \geq \pm 2$, pas de sens,

$x^2 - 4$ parabole :

$D_f =]-\infty ;-2] \cup [2 ;+\infty[$



- $\log_a(g(x))$, $0<a$, $a \neq 1$ et $g(x) > 0$

f(x) = ln(x-2), $x-2 > 0 \Leftrightarrow x > 2$, $D_f =]2, +\infty[$

f(x) = $\ln(9-x^2)$, $9-x^2 > 0$, $D_f =]-3 ;3[$

$$\boxed{e^{g(x)} = a \Leftrightarrow g(x) = \ln(a)}$$

$e^x = 1 \Leftrightarrow x = \ln(1) = 0$, $e^{x-2} = 4 \Leftrightarrow x-2 = \ln(4)$

$$\boxed{\ln(g(x)) = a \Leftrightarrow g(x) = e^a}$$

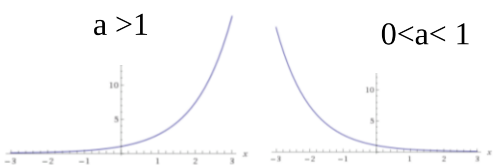
$\ln x = 0 \Leftrightarrow x = e^0 = 1$, $\ln(2-x) = 1 \Leftrightarrow 2-x = e^1$

$\ln(x^2-1) = 4 \Leftrightarrow x^2-1 = e^4$

Limites

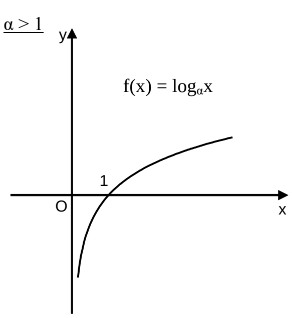
$\frac{\text{nbre}}{\infty} = 0$	$\frac{\text{nbre}}{0} = \infty$

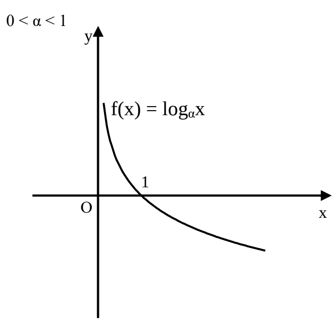
a >1 0<a< 1



$e^x, 10^x$

$e^{-x}, 10^{-x}$

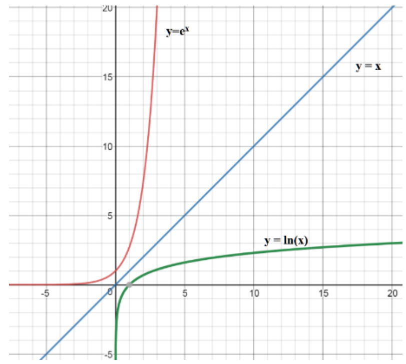




- formes indéterminées $\frac{\infty}{\infty}$, $\frac{0}{0}$, $0 \cdot \infty$

Attention: $\frac{\infty}{0} = \infty$, $\frac{0}{\infty} = 0$

1. e^x
2. P(x)
3. lnx



$\lim_{x\to+\infty} x^2e^{-x} = \lim_{x\to+\infty} \frac{x^2}{e^x} = 0.$

$\lim_{x\to-\infty} (x^2+2)e^x = +\infty \cdot 0 = 0$

$\lim_{x\to0^+} \frac{\ln(x)}{x} = \frac{-\infty}{0^+} = -\infty$

$\lim_{x\to0+} x^2\ln(x) = 0^+ \cdot (-\infty) = 0^-$

Asymptotes

AV (exclus) : $\lim_{x\to x_o} f(x) =$ ∞ , AV x = xo ; $y_0 \in \mathbb{R}$, Trou (xo, yo)

f(x) = $e^x \cdot x$, pas AV car $D_f = \mathbb{R}$

f(x) = xln(x) , $D_f =]0 , +\infty[$,

$\lim_{x\to0} f(x) = 0 \cdot (-\infty) = 0$ Trou (0,0)

f(x) = $\frac{e^x}{x+1}$, $D_f = \mathbb{R}\setminus\{-1\}$, $\lim_{x\to-1} f(x) = \frac{e^{-1}}{0}\left[\frac{nb}{0}\right] = \infty$,

AV x = -1

AH : $\pm\infty$ $\lim_{x\to\infty} f(x) =$ ∞ , pas AH AO? ; $y_0 \in \mathbb{R}$, AH : y = yo

f(x) = $e^x \cdot x$ $\lim_{x\to+\infty} f(x) = +\infty$, pas AH , $\lim_{x\to-\infty} f(x) = 0 \cdot (-\infty) = 0$, AH y = 0

$f(x) = \frac{2x}{1-e^x}$, $\lim_{x\to+\infty} f(x) = \frac{+\infty}{-\infty} = 0^-$, AH y=0,

$\lim_{x\to-\infty} f(x) = \frac{-\infty}{1-0} = -\infty$, pas AH $\to$ AO ?

AO : $\pm\infty$ y = mx +h

m = $\lim\limits_{x\to\infty}\frac{f(x)}{x} \in \mathbb{R}$, sinon on arrête

h = $\lim\limits_{x\to\infty}[f(x) - mx] \in \mathbb{R}$ sinon on arrête

$f(x) = \frac{2x}{1-e^x}$, $m = \lim\limits_{x\to-\infty}\frac{f(x)}{x} = \lim\limits_{x\to-\infty}\frac{2x}{x(1-e^x)} = \lim\limits_{x\to-\infty}\frac{2}{1-e}x = \frac{2}{1-0} = 2 \in \mathbb{R}$,

$h = \lim\limits_{x\to-\infty}(f(x) - 2x) = \lim\limits_{x\to-\infty}\left(\frac{2x}{1-e^x} - 2x\right) = \lim\limits_{x\to-\infty}\left(\frac{2x-2x(1-e^x)}{1-e^x}\right) = \lim\limits_{x\to-\infty}\frac{2x(1-1+e^x)}{1-e^x} = \lim\limits_{x\to-\infty}\frac{2x\cdot e^x}{1-e^x} = \frac{-\infty\cdot 0}{1} = 0 \in \mathbb{R}$, AO : y = 2x

Fonctions rationnelles : f(x) = $\frac{P(x)}{Q(x)}$

- Deg(P) < deg(Q) : AH y = 0 (axe des x)

f(x) = $\frac{x+4}{-x^3+1}$, AH : y = 0

- Deg(P) = deg(Q) : AH y = quotient des coefficients des plus grandes puissances.

f(x) = $\frac{-3x+4}{2x+3}$. AH : y = $\frac{-3}{2}$

- Deg(P) = deg(Q) +1 , on effectue la division euclidienne P :Q. Le quotient de cette division est l'asymptote oblique.

f(x) = $\frac{x^2+2x-1}{x-3}$ = x+5 + $\frac{14}{x-3}$. AO : y = x+5

Dérivées

$(u\cdot v)' = u'\cdot v + u\cdot v'$ $\left(\frac{u}{v}\right)' = \frac{u'\cdot v - u\cdot v'}{v^2}$

$\left(u(v(x))\right)' = u'(v(x))\cdot v'(x)$

$(x^n)' = nx^{n-1}$ $(g^n(x))' = n\, g^{n-1}(x)\, g'(x)$

f(x) = $(2x^3+4x)^5 \Rightarrow$ f ′(x) = $5(2x^3+4x)^4(6x^2+4)$

f(x) = $\sin^3 x \Rightarrow$ f ′(x) = $3\sin^2 x\cos x$

f(x) = $\ln^2 x \Rightarrow$ f ′(x) = $2\ln x\,\frac{1}{x} = \frac{2\ln x}{x}$

f(x) = $(\sin x + 2x)^5 \Rightarrow$ f ′(x) = $5(\sin x + 2x)^4(\cos x+2)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$ $\left(\sqrt{g(x)}\right)' = \frac{g'(x)}{2\sqrt{g(x)}}$

f(x) = $\sqrt{4x^3+2} \Rightarrow$ f ′(x) = $\frac{12x^2}{2\sqrt{4x^3+2}} = \frac{6x^2}{\sqrt{4x^3+2}}$

$(\sin(x))' = \cos x$ $\left(\sin(g(x))\right)' = \cos(g(x))\cdot g'(x)$

$(\cos x)' = \sin x$ $\left(\cos(g(x))\right)' = -\sin(g(x))\cdot g'(x)$

f(x) = $\sin(5x^3 - 4x) \Rightarrow$ f ′(x) = $\cos(5x^3-4x)(15x^2-4)$

f(x) = $\cos\left(\sqrt{x} + \frac{1}{x}\right) \Rightarrow$ f ′(x) = $-\sin\left(\frac{1}{2\sqrt{x}} - \frac{1}{x^2}\right)$

$(\ln x)' = \frac{1}{x}$ $\left(\ln(g(x))\right)' = \frac{g'(x)}{g(x)}$

$(\log_a x)' = \frac{1}{x\cdot \ln a}$

f(x) = $\ln(-2x^3+4x) \Rightarrow$ f ′(x) = $\frac{-6x^2+4}{-2x^3+4x}$

f(x) = $\ln\left(\frac{2x+3}{4x-1}\right) \Rightarrow$ f ′(x) = $\frac{4x-1}{2x+3}\cdot\frac{2(4x-1)-(2x+3)\cdot 4}{(4x-1)^2} = \frac{8x-2-8x-12}{(2x+3)(4x-1)} = \frac{-14}{(2x+3)(4x-1)}$

$(e^x)' = e^x$ $\left(e^{g(x)}\right)' = e^{g(x)}\cdot g'(x)$, $(a^x)' = a^x \ln a$

f(x) = $e^{x^2-4x} \Rightarrow$ f ′(x) = $e^{x^2-4x}\cdot(2x-4)$

f(x) = $e^{\frac{x}{2}} \Rightarrow$ f ′(x) = $\frac{1}{2}e^{\frac{x}{2}}$ f(x) = $e^{-x} \Rightarrow$ f ′(x) = $-e^{-x}$

Tangente

La valeur f ′(x) est la pente de la tangente à la courbe f(x) en x = x_0.

Point à tangente horizontale PTH $\Leftrightarrow$ f′(x) = 0

Point d'inflexion PI $\Leftrightarrow$ f ′′(x) = 0

$y = mx + h$,   y = f(x_o), m = f′(x_o)

Donner l'équation de la tangente au graphe de f, f(x) = ln(x^2 -3x + 8), au point d'abscisse x_o = 1.

- f(1) = ln(1 – 3 +8) = ln6
- f′(x) = $\frac{2x-3}{x2\ \ -3x+8}$, f′(1) = $-\frac{1}{6}$
- ln6 = $-\frac{1}{6}\cdot 1$ + h $\Rightarrow$ h = ln6 +$\frac{1}{6}$, Donc la tangente : y = $-\frac{1}{6}$ x + ln6 +$\frac{1}{6}$