

With "formulaire"

44 pts

Name: _____

Answers must be justified with computations

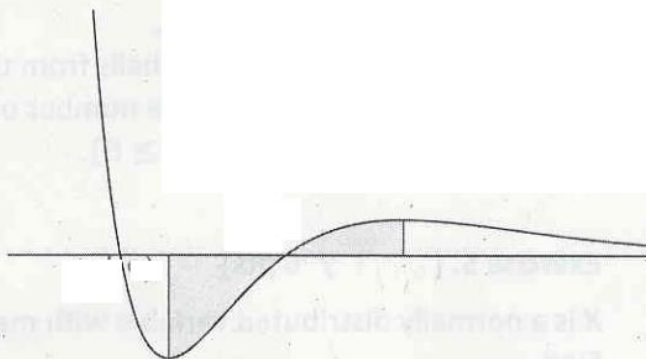
Exercise 1. [/ 16 pts]

- 1) Determine the derivative and an antiderivative of the function $f(x) = (x^2 - 1)e^{-x}$ whose graph is represented below.

- 2) The graph intersects the x-axis at $(-1; 0)$ and $(1; 0)$.

Determine the abscissas of the stationary points.

- 3) Determine the measure of the colored surface.



Answer **one** of the following two questions :

- 4) Determine $\int \frac{\ln(x)}{x} dx$ using the method of your choice.

or

- 4) Find an antiderivative of $f(x) = \frac{3x^2 - 2}{x + 4}$

Exercise 2. [/ 6 pts]**Remember :**

A function f is a density function of a continuous random variable X if $\int_{-\infty}^{+\infty} f(x) dx = 1$ (integral on the domain of the function).

- 1) Determine $k \in \mathbb{R}$ such that $f(x) = \begin{cases} kx^2, & \text{if } x \in [1; 2] \\ 0, & \text{else} \end{cases}$ is a density function.

- 2) The random variable X has density function $f(x) = \begin{cases} 0.5e^x, & \text{if } x < 0 \\ 0.5e^{-x}, & \text{if } x \geq 0 \end{cases}$

Determine $P(-1 \leq X \leq 2)$.

Exercise 3. [/ 8 pts]

A box contains one red ball and three blue balls. You draw balls one by one *without replacement* and place them in a bag until you have two blue balls.

We call X the number of balls in your bag.

- 1) Draw a tree diagram of the situation and determine the probability distribution of X .
- 2) Compute the average and the standard deviation of that distribution.

Exercise 4. [/ 3 pts]

You draw with replacement 10 balls from the same box containing one red ball and three blue balls. We name X the number of blue balls drawn.

Determine the probability $P(X \geq 8)$.

Exercise 5. [/ 6 pts]

X is a normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$. Find

- 1) $P(X = 30)$
- 2) $P(X \leq 40)$
- 3) $P(X \geq 22)$
- 4) k such that $P(X \geq k) = 0.5$
- 5) $P(30 \leq X \leq 36)$

Exercise 6. [/ 5 pts]

We assume that the weight of the potatoes produced in a farm follows a normal distribution with mean $\mu = 150\text{g}$ and standard deviation $\sigma = 30\text{g}$.

The production is separated in three :

the heaviest 15% of the production is sold at the Saturday market,

the 20% less heaviest part of the production is used to feed animals

and the other middle ones are sold in a shop.

Determine the possible sizes of potatoes that can be found in the shop.