Name:

MATHEMATICS

Series A

Problem 1 1.5 points

Find the sum of the following series and give the answer in the form of a fraction.

$$\sum_{k=2}^{+\infty} (-1)^k \frac{3^k}{4^k}$$

Problem 2 2.0 points

Let $f^{(n)}$ be the n-th derivative of the function f defined by $f(x) = e^{-k \cdot x} = \exp(-k \cdot x)$.

Show by induction that $f^{(n)}(x) = (-1)^n \cdot k^n \cdot e^{-k \cdot x}$ for $n \geq 1$.

Write a detailed proof.

Problem 3   2.5 points

Find an antiderivative of each of the following functions.

a) $f(x) = \sqrt[n]{m \cdot x + h}$ with $n \geq 2$ and $m \neq 0$ b) $f(x) = (x - x^2) \cdot e^{-2x}$

Problem 4   4.0 points

Consider the function f defined by $f(x) = \sin\left(\frac{\pi}{3} \cdot x\right)$.

Let t be the line tangent to the graph of f at $x_0 = 2$.

Calculate the area of the shaded surface.