LJP: TE 2 EQUATIONS

Exercice 1

$$\begin{array}{r|l} x^3+3x^2-14x+8 & x^2+5x-4 = D \\ -x^3-5x^2+4x & x-2 = Q \\ \hline -2x^2-10x+8 & \\ +2x^2+10x-8 & \\ \hline 0 = R & \end{array}$$

Exercice 2

$P(x) = 4kx^5+13x^3-2k+7$

$P(2)=0 \Rightarrow 4\cdot k\cdot 32+13\cdot 8-2k+7=0 \Leftrightarrow$

$\Leftrightarrow 128k+104-2k+7=0 \Leftrightarrow$

$\Leftrightarrow 126k=-111 \Leftrightarrow k=-\frac{111}{126}$

Exercice 3

A. $\sqrt[3]{2x-x^3+7} = 1-x \Leftrightarrow 2x-x^3+7=(1-x)^3 \Leftrightarrow$

$\Leftrightarrow 2x-x^3+7=1-3x+3x^2-x^3 \Leftrightarrow$

$\Leftrightarrow 3x^2-5x-6=0 \quad \Delta = 25+4\cdot 6\cdot 3 = 97$

$x_{1,2} = \frac{5\pm\sqrt{97}}{6}$.

B. $2x+\sqrt{6+6x}=4 \Leftrightarrow \sqrt{6+6x}=4-2x \Leftrightarrow$

$\Leftrightarrow 6+6x=(4-2x)^2 \Leftrightarrow 6+6x=16-16x+4x^2 \Leftrightarrow$

$\Leftrightarrow 4x^2-22x+10=0 \Leftrightarrow 2x^2-11x+5=0$

$\Delta = 11^2-4\cdot 2\cdot 5 = 81 \quad x_{1,2} = \frac{11\pm 9}{4} \begin{cases} x_1=5 \\ x_2=\frac{1}{2}. \end{cases}$

$x=5 \quad 2\cdot 5+\sqrt{6+6\cdot 5}=4 \Rightarrow$

$10+\sqrt{36}=4 \Rightarrow 10+6=4$ imp.

$\boxed{x=\frac{1}{2}} \quad 2\cdot\frac{1}{2}+\sqrt{6+6\frac{1}{2}}=4 \Rightarrow 1+\sqrt{9}=4$ ✓

C. $10x^4+9x^2-4=0 \qquad x^2=t\geq 0$

$10t^2+9t-4=0 \qquad \Delta = 81+4\cdot 4\cdot 10 = 241$

$t_{1,2} = \frac{-9\pm\sqrt{241}}{20}$

$t_1 = \frac{-9+\sqrt{241}}{20} = x^2 \Rightarrow x = \pm\sqrt{\frac{-9+\sqrt{241}}{20}}$

$t_2 = \frac{-9-\sqrt{241}}{20} = x^2$ impossible

Exercice 4

$P(x) = 10x^4 - 19x^3 - 58x^2 + 61x + 42 = 0$

Rp. $\pm 1, \pm 2, \pm 3, \pm 14 \ldots$

$x = 1 \quad P(1) \neq 0.$

$x = -1 \quad P(-1) \neq 0.$

$x = 2 \quad P(2) \neq 0.$

$x = -2 \quad P(-2) = 0.$

					-2
10	-19	-58	61	42	
↓	-20	78	-40	-42	
10	-39	20	21	0	

$P(x) = (x+2)\underbrace{(10x^3 - 39x^2 + 20x + 21)}_{Q(x)}.$

$x = 3 \quad Q(3) = 0$

				3
10	-39	20	21	
↓	30	-27	-21	
10	-9	-7	0	

$P(x) = (x+2)(x-3)(10x^2 - 9x - 7).$

$\Delta = 81 + 4 \cdot 10 \cdot 7 = 361 \qquad x_{1,2} = \frac{9 \pm 19}{20}$

$x_1 = \frac{28}{20} = \frac{7}{5} \qquad x_2 = \frac{-10}{20} = -\frac{1}{2}.$

Alors. $x = -2 \quad x = 3 \quad x = \frac{7}{5} \quad x = -\frac{1}{2}.$

$P(x) = 10(x+2)(x-3)\cdot(x - \frac{7}{5})(x + \frac{1}{2}).$

Bonus

1. Le quotient $Q(x)$ de la division $P(x)$ et $A(x)$ est un polynôme de degré 4 car degre $P(x)$ = degre $Q(x)$ + degre $A(x)$.
2. Non car $\deg R(x) < \text{degre } A(x) \Rightarrow \deg R(x) < 3$.