LJP TE 22 ANALYSE DERIVEES

EXERCICE 1.

$f(x) = \dfrac{ax-2b}{8-3x} \quad a, b \in \mathbb{R}$

1. $P(1;\frac{1}{3}) \quad f(1) = \frac{1}{3} \Leftrightarrow \dfrac{a-2b}{5} = \dfrac{1}{3} \Leftrightarrow 3a-6b=5$

2. $f'(2) = 1$

$$f'(x) = \frac{a(8-3x)-(ax-2b)(-3)}{(8-3x)^2} = \frac{8a-\cancel{3ax}+\cancel{3ax}-6b}{(8-3x)^2}$$

$$= \frac{8a-6b}{(8-3x)^2}$$

$f'(2) = 1 \Leftrightarrow \dfrac{8a-6b}{4} = 1 \Rightarrow 8a-6b=4 \Leftrightarrow 4a-3b=2$

On a le système:

$$\begin{cases} 3a-6b=5 \ (1) \\ 4a-3b=2 \quad \times(-2) \end{cases} \quad + \begin{cases} 3a-6b=5 \\ -8a+6b=-4 \end{cases}$$

$$-5a = 1 \Rightarrow a = -\tfrac{1}{5}$$

$-\dfrac{3}{5} - 6b = 5 \Leftrightarrow -3-30b=25 \Leftrightarrow$

$\Leftrightarrow -28 = 30b \Rightarrow b = -\dfrac{28}{30} \Rightarrow b = -\dfrac{14}{15}$

EXERCICE 2

1. $D_f = \mathbb{R}\setminus\{2\}$

2. AV $x=2$ AO: $y = x+1$

3. PI $(1;0)$

4. a) $\lim\limits_{x\to 2} f(x) = +\infty$ c) $\lim\limits_{x\to +\infty} f(x) = +\infty$

b) $\lim\limits_{x\to 2^+} f'(x) = -\infty$ d) $\lim\limits_{x\to+\infty} f'(x) = 1$

5.

x	$-\infty$		1		2		$+\infty$
f''		$-$	0	$+$	‖	$+$	
f		$\cap$	PI	$\cup$		$\cup$	

Exercice 3

$$f(x) = \frac{x^2-6x+8}{2x+1}$$

1. $D_f = \mathbb{R} \setminus \{-\frac{1}{2}\}$

2. zéros (O_x) $f(x)=0 \Rightarrow x^2-6x+8=0 \Rightarrow \begin{matrix} x_1 = 2 & (2;0) \\ x_2 = 4 & (4;0) \end{matrix}$

$\cap Oy: x=0 \quad y = \frac{8}{1} \quad (0,8)$

3.

	$-\infty$	$-\frac{1}{2}$		2		4	$+\infty$
N(x)	+		+	0	−	0	+
D	−	0	+		+		+
f	−	//	+	0	−	0	+

4. AV $\lim\limits_{x \to -\frac{1}{2}} f(x) = \text{"} \frac{45/4}{0} \text{"} \begin{cases} x > -\frac{1}{2} & +\infty \\ x < -\frac{1}{2} & = -\infty \end{cases}$ AV $x = -\frac{1}{2}$

AO

$$\begin{array}{l|l} x^2-6x+8 & 2x+1 \\ \cline{2-2} -x^2-\frac{1}{2}x & \frac{1}{2}x - \frac{13}{4} \\ \hline -\frac{13}{2}x+8 & \\ +\frac{13}{2}x+\frac{13}{4} & \\ \hline \frac{45}{4} & \end{array}$$

AO : $y = \frac{1}{2}x - \frac{13}{4}$

5. $f'(x) = \frac{(2x-6)(2x+1)-(x^2-6x+8)\cdot 2}{(2x+1)^2} =$

$= \frac{4x^2-\cancel{12x}+2x-6-2x^2+\cancel{12x}-16}{(2x+1)^2} = \frac{2x^2+2x-32}{(2x+1)^2} = 0$

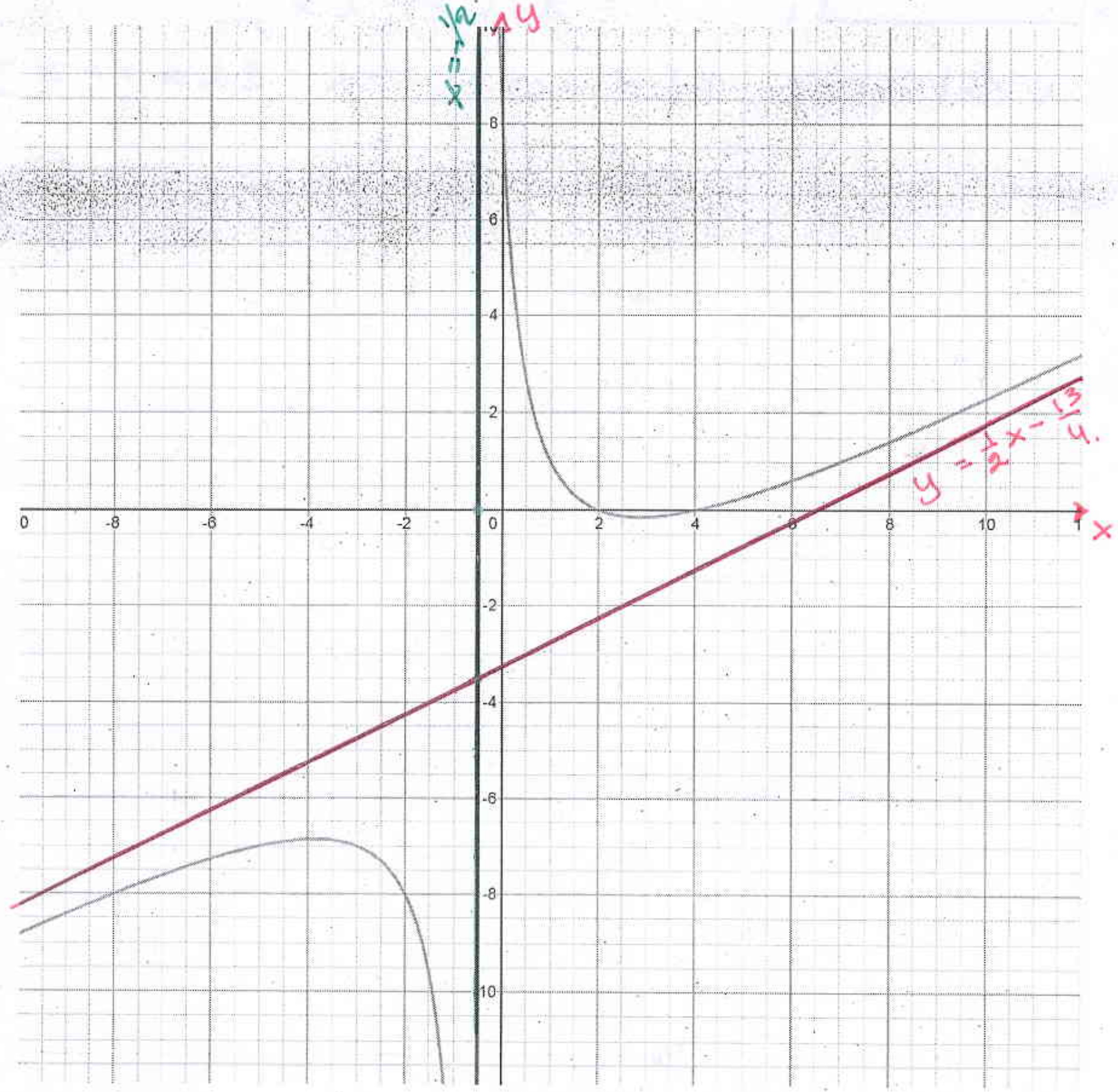
$\Rightarrow 2x^2+2x-32=0 \Leftrightarrow x^2+x-16=0$

$\Delta = 65 \quad x_{1,2} = \frac{-1 \pm \sqrt{65}}{2} \begin{cases} x_1 = 3.53 \\ x_2 = -4.53 \end{cases}$

	$-\infty$	-4.53		$-\frac{1}{2}$		3.53	$+\infty$
f'	+	0	−	//	−	0	+
f	↗	Max	↘		↘	Min	↗

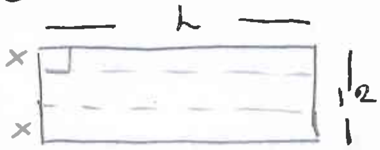
Max $= f(-4.53) = -6.92$

Min $= f(3.53) = -0.09$



$x = -1/2$
y
x
$y = \frac{1}{2}x - \frac{13}{4}$.

Exercice 4



$$V(x) = L\cdot(12-x)\cdot x =$$
$$= 12Lx - Lx^2$$

$$V'(x) = 12L - 2Lx = 0 \Rightarrow 12L = 2Lx \Rightarrow \underline{x = 6}$$