LJP TE 18 ANALYSE

EXERCICE 1.

$f(x) = 2x^2 - 5x \qquad f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

1. $f(x+\Delta x) = 2(x+\Delta x)^2 - 5(x+\Delta x) =$

$= 2x^2 + 4x\Delta x + 2\Delta x^2 - 5x - 5\Delta x$

$f(x+\Delta x) - f(x) = 2x^2 + 4x\Delta x + 2\Delta x^2 - 5x - 5\Delta x - 2x^2 + 5x$

$= \Delta x (4x + 2\Delta x - 5)$

$\frac{f(x+\Delta x) - f(x)}{\Delta x} = 4x + 2\Delta x - 5$

$f'(x) = \lim_{\Delta x \to 0} (4x + 2\Delta x - 5) = 4x - 5$

pour $x = -3 \quad f'(-3) = -12 - 5 = -17$

2. $t: \; y = mx + h$

$f(-3) = 18 + 15 = 33$

$f'(-3) = -17$

$f(-3) = f'(-3) \cdot (-3) + h$

$33 = -17 \cdot (-3) + h$

$33 = 51 + h \Rightarrow h = -18.$

$t: \; y = -17x - 18$

EXERCICE 2.

1. $y = 4x^2 - 5x^9 - 30000 \Rightarrow y' = 8x - 45x^8$

2. $y = \frac{15}{x^3} - \frac{16}{x^4} + \frac{10}{10^{-8}} = 15x^{-3} - 16x^{-4} + 10^9$

$y' = -45x^{-4} + 64x^{-5} = -\frac{45}{x^4} + \frac{64}{x^5}$

3. $y = 3\sqrt[3]{x} + 2\sqrt{x} + \sqrt{10} = 3x^{1/3} + 2x^{1/2} + \sqrt{10}$

$y' = 3 \cdot \frac{1}{3} x^{-2/3} + 2 \cdot \frac{1}{2} x^{-1/2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{\sqrt{x}}$

4. $y = \dfrac{3x^2 - 2x^3}{2x^2 + 3x^3}$

$$y' = \frac{(6x - 6x^2)(2x^2 + 3x^3) - (3x^2 - 2x^3)(4x + 9x^2)}{(2x^2 + 3x^3)^2}$$

$$= \frac{12x^3 - 12x^4 + 18x^4 - 18x^5 - 12x^3 + 8x^4 - 27x^4 + 18x^5}{(x^2)^2(2 + 3x)^2}$$

$$= \frac{-13x^4}{x^4(2+3x)^2} = \frac{-13}{(2+3x)^2}$$

5. $y = \sin(2x - 4x^2 + \pi)$

$$y' = \cos(2x - 4x^2 + \pi) \cdot (-8x + 2) = 2(-4x + 1)\cos(2x - 4x^2 + \pi)$$

6. $y = 6x - \frac{1}{4}(x + \sqrt{3x})^4$

$$y' = 6 - \frac{1}{4} \cdot 4(x + \sqrt{3x})^3 \cdot \left(1 + \frac{3}{2\sqrt{3x}}\right) =$$

$$= 6 - (x + \sqrt{3x})^3 \left(1 + \frac{3}{2\sqrt{3x}}\right)$$

7. $y = \pi + \sqrt[5]{\dfrac{7-x}{\pi}} - 1 = \pi + \left(\dfrac{7-x}{\pi}\right)^{1/5} - 1$

$$y' = \frac{1}{5}\left(\frac{7-x}{\pi}\right)^{-4/5} \cdot \left(-\frac{1}{\pi}\right) = -\frac{1}{5\pi} \frac{1}{\sqrt[5]{\left(\frac{7-x}{\pi}\right)^4}}$$

EXERCICE 3

1. $f(x) = \dfrac{13x - 5x^3}{13x - 5}$ $\quad D_f = \mathbb{R} \setminus \{5/13\}$

AV $\quad \lim_{x \to 5/13} f(x) = \dfrac{nb}{0}$ $\quad x > 5/13 \Rightarrow \dfrac{nb^+}{0^+} = +\infty$ $\quad x < 5/13 \Rightarrow \dfrac{nb^+}{0^-} = -\infty$

AV $x = \frac{5}{13}$

AO/AH $\quad \emptyset$

2. $g(x) = \frac{1+x-5x^2}{10+x}$ $\quad D_g = \mathbb{R} \setminus \{-10\}$

AV $\lim_{x \to -10} g(x) = \frac{-509}{0}$ $\begin{cases} x > -10 & \frac{-509}{0^+} = -\infty \\ x < -10 & \frac{-509}{0^-} = +\infty \end{cases}$ $\quad AV\ x = -10$

AO

$$\begin{array}{r|l} -5x^2+x+1 & x+10 \\ +5x^2+50x & \hline & -5x+51 \\ \hline 51x+1 & \\ -51x-510 & \\ \hline -509 & \end{array}$$

AO: $y = -5x+51$

BONUS

$k(x) = \frac{6x-9x^2}{3x-2}$

$$\begin{array}{r|l} -9x^2+6x & 3x-2 \\ +9x^2-6x & \hline & -3x \\ \hline 0 & \end{array}$$

AO $y = -3x$

EXERCICE 4

$f(x) = \frac{x^2-x-2}{x-3}$ $\quad D_f = \mathbb{R} \setminus \{3\}$

$f'(x) = \frac{(2x-1)(x-3)-(x^2-x-2)}{(x-3)^2} =$

$= \frac{2x^2-x-6x+3-x^2+x+2}{(x-3)^2} = \frac{x^2-6x+5}{(x-3)^2}$

$f'(x) = 0 \Leftrightarrow x^2-6x+5=0 \Rightarrow x=3$ (impossible) $\quad x=2$

	$-\infty$		2		3		$+\infty$
f'		+	0	−	∄	+	
f		↗	Max	↘		↗	

Max $= f(2) = -6$.