LJP TE 16. FONCTIONS.

Exercice 1.

1. $f(x) = x^2+1$ $f(-x) = (-x)^2+1 = x^2+1 = f(x)$ paire.
$f(x) = 6x^3$ $f(-x) = 6(-x)^3 = -6x^3 = -f(x)$ impaire.
$f(x) = \sin x$ $f(-x) = \sin(-x) = -\sin x$ impaire.
$f(x) = \frac{x^2}{x^3-1}$ $f(-x) = \frac{x^2}{-x^3-1}$ ni-ni

2. $f(x) = \frac{2x+2}{4x-1}$ $D_f = \mathbb{R} \setminus \{1/4\}$

$f(x) = -\frac{2}{x^2-3x-4}$ $D_f = \mathbb{R} \setminus \{+4; -1\}$

$f(x) = \sqrt{4x-1}$ $D_f = [1/4, +\infty[$
$4x-1 \geq 0 \Rightarrow x \geq 1/4$

$f(x) = -\frac{2}{4x^2+3x-1}$ $D_f = \mathbb{R} \setminus \{-1; 1/4\}$
$4x^2+3x-1=0$ $\Delta = 9+16 = 25$ $x_{1,2} = \frac{-3 \pm 5}{8}$ $\begin{matrix} -1 \\ 1/4 \end{matrix}$

3. $f(x) = x^2-1$ $Im_f = [-1, +\infty[$
$f(x) = x^3$ $Im_f = \mathbb{R}$
$f(x) = \sqrt{x-3}$ $Im_f = [0, +\infty[$
$f(x) = x^2-3$ $Im_f = [-3, +\infty[$

Exercice 2

$f(x) = \frac{2x^3-2x^2}{2x^2+6x-8} = \frac{2x^2(x-1)}{2(x+4)(x-1)} = \frac{x^2}{x+4}$

1. $2x^2+6x-8=0 \Rightarrow x^2+3x-4=0$ $\begin{matrix} x_1 = -4 \\ x_2 = 1 \end{matrix}$
$D_f = \mathbb{R} \setminus \{-4; 1\}$

2. Ox zéros $f(x) = 0 \Rightarrow 2x^3-2x^2=0 \Rightarrow 2x^2(x-1)=0$
$x=0$ $x=1$ (imp) $(0,0)$
Oy $x=0$ $y=0$ $(0,0)$

3.

	$-\infty$		-4		0		1		$+\infty$
δ		$-$	‖	$+$	0	$+$	‖	$+$	

4. $2x^3 - 2x^2 \;\big|\; 2x^2+6x-8$

$-2x^3 - 6x^2 + 8x \;\big|\; x - 4$

$-8x^2 + 8x$

$+8x^2 + 24x - 32$

$-16x - 32$

$AO : y = x - 4$

EXERCICE 3

1. $\lim\limits_{\substack{x \to -2 \\ x < -2}} f(x) = -\infty$ $\qquad$ $\lim\limits_{x \to +\infty} f(x) = 1$

$\lim\limits_{x \to 0} f(x) = 1$ $\qquad$ $\lim\limits_{\substack{x \to 2 \\ x > 2}} f(x) = +\infty$

2.

	$-\infty$		-2		2		$+\infty$
δ'		$-$	‖	$-$	‖	$-$	
δ		↘		↘		↘	

EXERCICE 4

- $\lim\limits_{x \to 2} \dfrac{x^2-4}{x-2} = \text{"}\dfrac{0}{0}\text{"}$ $\lim\limits_{x \to 2} \dfrac{(x-2)(x+2)}{x-2} = 4$ Trou $(2,4)$

- $\lim\limits_{x \to 3} \dfrac{2x-3}{x^2+x-3} = \dfrac{3}{9} = \dfrac{1}{3}$

- $\lim\limits_{x \to -\infty} \dfrac{x^2}{x-1} = \lim\limits_{x \to -\infty} \dfrac{x^2}{x} = \lim\limits_{x \to -\infty} x = -\infty$