1.16 Exercises

1.1 : Find the domain and the range (except for the three last) of the following functions :

1) $y = x^2 + 1$
2) $y = -\dfrac{1}{x}$
3) $y = \dfrac{1}{(x-2)^2}$
4) $y = 1 + \sqrt{2x-5}$
5) $y = |\cos(x)|$
6) $y = \dfrac{1}{\sqrt{5-x}}$
7) $y = \dfrac{\tan(x)}{x}$
8) $y = \dfrac{1}{\tan(x)}$
9) $y = \dfrac{1}{\sqrt{x^3 - x^2 - 6x}}$

1. 2 : Determine the parity of the following functions :

1) $f(x) = |2x-5|$
2) $f(x) = x^3 \cdot \cos(x)$
3) $f(x) = 3x \cdot \sin(2x)$
4) $f(x) = \dfrac{3x^2+7}{x}$
5) $f(x) = \dfrac{\sqrt{x^2-1}}{x^4+5}$
6) $f(x) = \dfrac{\sin(5x)\cdot\cos(x)}{x^2}$

1. 3 : Prove the following results :

1) The product of two even functions is an even function.
2) The product of an even function and an odd function is an odd function.
3) The product of two odd functions is an even function.

Verify with the functions $f(x) = x^2$ and $g(x) = x^3$.

1. 4 : Establish the sign table of the following functions :

1) $y = 2x + 1$
2) $y = -x^2 + 7x - 6$
3) $y = x^2 - 3x + 4$
4) $y = \sqrt{x+3} - 2$
5) $y = x\cdot(x-3)\cdot(x+4)$
6) $y = \frac{-x+2}{x^2-9}$
7) $y = 2\cos(x) + 1$

1. 5 : Using a sign table, solve the inequation $\dfrac{x-3}{x^2-3x+2} > 0$.

1. 6 : Determine, if any, the period of the following functions :

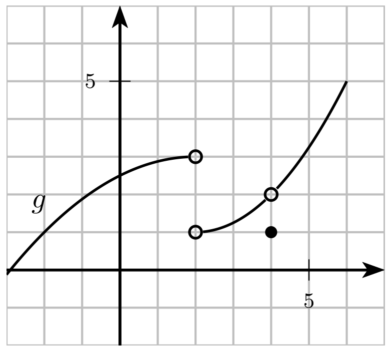
1) $y = 5\sin(4x)$
2) $y = -2\tan(3x)$
3) $y = \cos(5x)$
4) $y = x\cdot\sin(x)$
5) $y = \sin^2(x) + 2$
6) $y = \sin(4x) + \tan(3x)$

1. 7 : Study the function $f(x) = \dfrac{-2x+3}{x+1}$

1) Domain and range
2) Intersection with the axes
3) Table of signs
4) Behavior of the function when x tends to the excluded value
5) Behavior of the function when x tends to $+\infty$ and $-\infty$?
6) Graph
7) Coherence

1.8 : By using the graph of the function $g(x)$, determine, if they exist, the following limits :

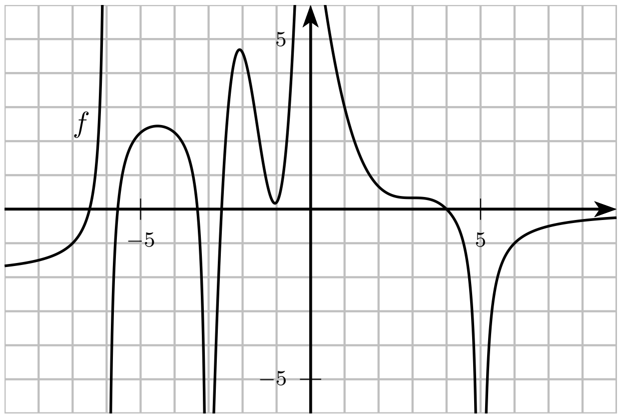
1) $\lim\limits_{x\to 2^-} g(x) =$
2) $\lim\limits_{x\to 2^+} g(x) =$
3) $\lim\limits_{x\to 2} g(x) =$
4) $\lim\limits_{x\to 4^-} g(x) =$
5) $\lim\limits_{x\to 4^+} g(x) =$
6) $\lim\limits_{x\to 4} g(x) =$
7) What is the value of $g(2)$ and $g(4)$?
8) Is the function g continuous ?



1.9 : By using the graph of $f(x)$, determine, if they exist, the following limits :

1) $\lim\limits_{x\to\infty} f(x) =$
2) $\lim\limits_{x\to 5} f(x) =$
3) $\lim\limits_{x\to -3} f(x) =$
4) $\lim\limits_{x\to -6} f(x) =$

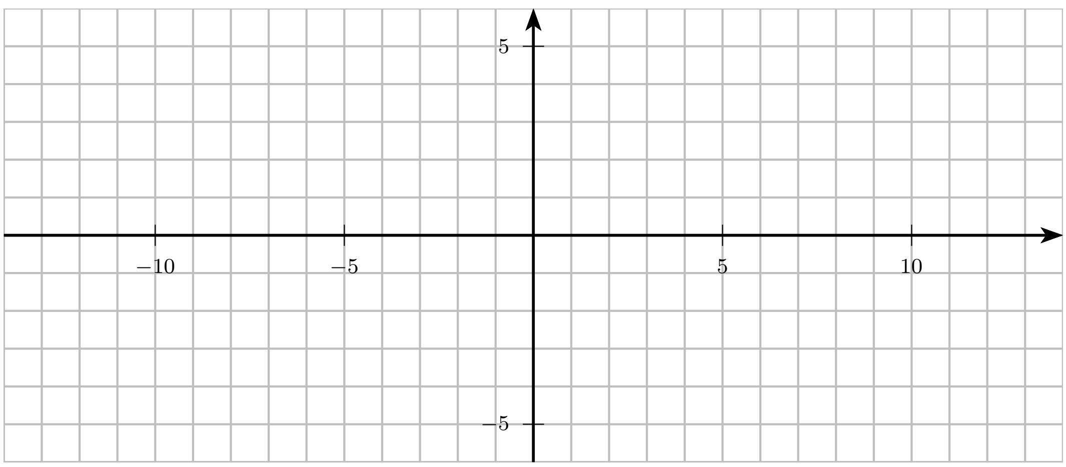
What is the domain of this function ? Determine the equation of the vertical and horizontal asymptotes.



1.10 : Sketch the graph of a function h such that :

1) $\lim\limits_{x\to+\infty} h(x) = \lim\limits_{x\to-\infty} h(x) = 3$
2) $\lim\limits_{x\to-7} h(x) = -\infty$
3) $\lim\limits_{x\to 4^-} h(x) = -\infty$
4) $\lim\limits_{x\to 4^+} h(x) = +\infty$
5) $h(-2) = 1$

What is the domain of **your** function h ?

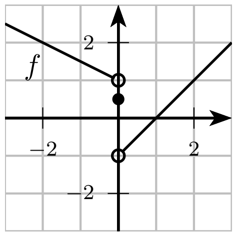


1.11 : Here opposite is the graph of the function $f(x)$. Whenever possible, calculate :

1) $\lim\limits_{x\to 2^-} f(x) =$ 4) $f(2) =$ 7) $\lim\limits_{x\to 0} f(x) =$

2) $\lim\limits_{x\to 2^+} f(x) =$ 5) $\lim\limits_{x\to 0^-} f(x) =$ 8) $f(0) =$

3) $\lim\limits_{x\to 2} f(x) =$ 6) $\lim\limits_{x\to 0^+} f(x) =$



Is the function f continuous ? Why ?

1.12 : Compute, if they exist, the following limits :

1) $\lim\limits_{x\to 2} 5x =$ 4) $\lim\limits_{x\to 2} \dfrac{x^2-4}{x^2+4} =$ 7) $\lim\limits_{x\to 2} \dfrac{x^2-x-2}{x-2} =$

2) $\lim\limits_{x\to 2}(2x+3) =$ 5) $\lim\limits_{x\to 4} \sqrt{25-x^2} =$ 8) $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{(x-1)^2} =$

3) $\lim\limits_{x\to 3} \dfrac{x+2}{x-2} =$ 6) $\lim\limits_{x\to 4} \dfrac{x-4}{x^2-x-12} =$ 9) $\lim\limits_{x\to 2} \dfrac{x^2-2x}{x} =$

1.13 : Compute, if they exist, the following limits :

1) $\lim\limits_{x\to\infty} (-4x^2) =$ 4) $\lim\limits_{x\to\infty} \dfrac{x^2+x-2}{4x^3-1} =$ 7) $\lim\limits_{x\to\infty} \dfrac{1+x}{x} =$

2) $\lim\limits_{x\to-\infty} 5x^3+2 =$ 5) $\lim\limits_{x\to\infty} \dfrac{4x-2}{5x+7} =$ 8) $\lim\limits_{x\to\infty} \dfrac{(x-1)^2}{x^2+3x} =$

3) $\lim\limits_{x\to\infty} \dfrac{3}{x} =$ 6) $\lim\limits_{x\to\infty} \dfrac{2x^3+1}{x^2+1} =$ 9) $\lim\limits_{x\to-\infty} \dfrac{(x^2+6x)(5x-2)}{-10x^3+13x-7} =$

1.14 : Given the function $f(x) = \dfrac{2x-4}{x^2+x-6}$. Calculate, if they exist, the following limits :

1) $\lim\limits_{x\to 2^-} f(x) =$ 4) $\lim\limits_{x\to 1^-} f(x) =$ 7) $\lim\limits_{x\to -3^-} f(x) =$

2) $\lim\limits_{x\to 2^+} f(x) =$ 5) $\lim\limits_{x\to 1^+} f(x) =$ 8) $\lim\limits_{x\to -3^+} f(x) =$

3) $\lim\limits_{x\to 2} f(x) =$ 6) $\lim\limits_{x\to 1} f(x) =$ 9) $\lim\limits_{x\to -3} f(x) =$

Determine the value of $f(2)$, $f(1)$ and $f(-3)$?

1.15 : 1) Determine the equation of the vertical asymptotes of the functions :

a) $f(x) = \dfrac{x}{x^2-x-2}$ b) $g(x) = \dfrac{2-2x}{2x^2-5x+3}$

2) Determine all the asymptotes of the following functions :

a) $h(x) = \dfrac{2x-5}{x^2+1}$ c) $j(x) = \dfrac{x^2+1}{2x-5}$

b) $i(x) = \dfrac{2x^2+4x+2}{-x^2+4}$ d) $k(x) = \dfrac{-2x^3+9x^2-13x+7}{x^2-4x+4}$

1. 16 : Study the function $f(x) = \dfrac{-2x^3 - 6x^2 + 20x}{5x^2 - 15x + 10}$ and sketch it.

1. 17 : Determine, in the form of your choice, the expression of the following functions :

1) $f_1(x)$ has a vertical asymptote at $x = 5$, a slant asymptote with equation $y = -x + 5$. Moreover, the curve is over the asymptote when x tends to $+\infty$.
2) $f_2(x)$ has a vertical asymptote at $x = 2$ and a hole with coordinates $(3; 11)$.
3) $f_3(x)$ has the slant asymptote $y = x+1$ and the graph of f cuts the asymptote at $x = 2$. The domain of f is $\mathbb{R}$.

1. 18 : Calculate, if they exist, the following limits :

1) $\lim\limits_{x\to 0} \dfrac{\sin(2x)}{x} =$
2) $\lim\limits_{x\to 0} \dfrac{\sin(3x)}{2x} =$
3) $\lim\limits_{x\to 0} \dfrac{1-\cos^2(x)}{x} =$
4) $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x^2} =$
5) $\lim\limits_{x\to 0} \dfrac{\tan(5x)}{2x} =$
6) $\lim\limits_{x\to 0} \dfrac{1-\cos(x)}{x^2} =$
7) $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{1-\sin(x)}{\cos^2(x)} =$
8) $\lim\limits_{x\to 0} x \cdot \sin\left(\frac{1}{x}\right) =$
9) $\lim\limits_{x\to 0} x^2 \cdot \sin\left(\frac{1}{x}\right) =$

1. 19 : Calculate, if they exist, the following limits :

1) $\lim\limits_{x\to 0} \dfrac{|x|}{x} =$
2) $\lim\limits_{x\to 4} \dfrac{\sqrt{x}-2}{x-4} =$
3) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{x} =$
4) $\lim\limits_{x\to 2} \dfrac{x^2+1}{|x^2-4|} =$
5) $\lim\limits_{x\to 0} \dfrac{x^2}{\sqrt{x^2+1}-1} =$
6) $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x+1}-\sqrt{2x-1}} =$
7) $\lim\limits_{x\to 5} \dfrac{x-5}{\sqrt{2x-1}-3} =$
8) $\lim\limits_{x\to 1} \dfrac{1-\sqrt{2-\sqrt{x}}}{x-1} =$

1. 20 : Determine the domain and the asymptotes of the following :

1) $f_1(x) = \sqrt{4x^2-5}$
2) $f_2(x) = \sqrt{x^2+x+1}$
3) $f_3(x) = \dfrac{x-\sqrt{x^2+1}}{x+1}$
4) $f_4(x) = 5x - \sqrt{4x^2+2x}$

1. 21 : Calculate, if they exist, the following limits :

1) $\lim\limits_{x\to 0} \dfrac{1-\cos(x)}{x} =$
2) $\lim\limits_{x\to 0} \dfrac{\sin(\frac{x}{3})}{2x} =$
3) $\lim\limits_{x\to 0} \dfrac{1-\cos^2(x)}{\tan(x)} =$
4) $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{2x-2} =$
5) $\lim\limits_{x\to 0} \dfrac{1-\cos^2(x)}{\tan^2(x)} =$
6) $\lim\limits_{x\to 0} \dfrac{5x^2}{\sqrt{x^2+4}-2} =$
7) $\lim\limits_{x\to -\infty} \sqrt{x^2-2x+4}+2x =$
8) $\lim\limits_{x\to -\infty} \sqrt{x^2-2x+4}+x =$
9) $\lim\limits_{x\to \infty} \dfrac{\sqrt{x^2+2x+3}}{x} =$
10) $\lim\limits_{x\to \infty} \sqrt{x^2+2x}-\sqrt{x^2+4} =$
11) -
12) $\lim\limits_{x\to \infty} \sqrt{x+\sqrt{x}}-\sqrt{x} =$

1. 22 : What must the value of a and b be, so that the following functions are continuous ?

1) $f(x) = \begin{cases} 2x & x < 3 \\ ax + b & 3 \leq x \leq 5 \\ -x & x > 5 \end{cases}$

2) $f(x) = \begin{cases} \frac{x-|x|}{x} & x < 0 \\ a & x = 0 \\ \frac{2x-1}{b} & x > 0 \end{cases}$

3) $f(x) = \begin{cases} x^2 + ax + 6 & x < 1 \\ 5x + b & 1 \leq x \leq 2 \\ \frac{x^2+5x-14}{x-2} & 2 < x \end{cases}$

4) $f(x) = \begin{cases} 0.5^x - 6 & x < -3 \\ \sqrt{x - a} & -3 \leq x \leq 9 \\ \frac{-\sqrt{x^2+19}}{b} & 9 < x \end{cases}$

1. 23 : By using the definition of the **derivative**, determine the derivative of :

1) $f(x) = h$
2) $f(x) = mx$
3) $f(x) = mx + h$
4) $f(x) = x^2$
5) $f(x) = x^3$
6) $f(x) = \dfrac{1}{x}$
7) $f(x) = \sqrt{x}$
8) $f(x) = x^n$ $(n \in \mathbb{N}^*)$

1. 24 : We consider the function $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$

1) Give its domain, graph it and give its range.
2) Is the function continuous at $x = 0$? Why ?
3) What can you say about the derivative of the function at $x = 0$?

1. 25 : By using the linearity of the differentiation, find the derivative of :

1) $f(x) = 4x^3 - 7x^2 + 89$
2) $f(x) = (7 - 5x)^2$
3) $f(x) = 3\sqrt{x} + 8x^2 - 9x$
4) $f(x) = -5x^4 + \cos(60°)x^2 - \tan(45°)$
5) $f(x) = \dfrac{7}{2x} + 3\sin(x)$

1. 26 :

1) Determine the coordinates of the two points on the graph of the curve $y = f(x) = 2x^3 - 5x^2 + 9x - 1$ where the slope of the tangent is 13.
2) Determine the coordinates of the two points on the graph of the curve $y = f(x) = \frac{2}{x} - 3x$ where the slope of the tangent is -21.

1. 27 :

1) At the point P on the graph of the function $f(x) = x^2 + k$, the equation of the tangent is $y = 6x - 7$. Find the value of the constant k as well as the coordinates of the point P.
2) At the point P on the graph of the function $f(x) = 4\sqrt{x} + k$, the equation of the tangent is $y = 0.4x - 5$. Find the value of the constant k as well as the coordinates of the point P.

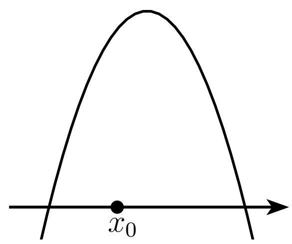
1. 28 : Determine the coefficients a, b and c so that the parabola with equation $y = ax^2 + bx + c$ passes through the point $(5; 8)$ and has at the point $(2; 2)$ a tangent with slope -4.

1. 29 : Determine the equation of the tangent to the following curves at the point whose abscissa is given.

1) $f(x) = -3x^2 + 5x - 2$ at $x = -2$
2) $g(x) = \dfrac{-3}{x}$ at $x = 5$
3) $h(x) = 5\sqrt{x} + 2$ at $x = 4$
4) $i(x) = \cos(x)$ at $x = \dfrac{3\pi}{4}$

1. 30 :

1) Precisely draw the tangent to the parabola here opposite at $x = x_0$ then, on the same graph, draw the secant line through the points with abscissa $x_0 - h$ and $x_0 + h$. Choose h randomly.



2) Show that for any value of h and x_0, the tangent to a parabola $f(x) = ax^2 + bx + c$ at a point x_0 has the same gradient as the secant line through the points with abscissa $x_0 + h$ and $x_0 - h$.

1. 31 :

1) We consider the function $f(x) = x^3 + ax^2 + bx$ where $a, b \in \mathbb{R}$. Find the value of a and b given that the point $T(1; 1)$ is a **stationary point** : a point at which the tangent to the graph is horizontal.
Determine the **type of the stationary point** among *maximum, minimum* and *level-point*.
2) Given that $f(x) = 2x^3 + ax^2 + bx - 5$ where $a, b \in \mathbb{R}$ is tangent to the line $t : y = -2.5x + 4$ at the point with abscissa $x = -1$, determine the value of a and b.

1. 32 : Differentiate the following functions using the product rule and then check your result by using the distributivity first :

1) $f(x) = (x + 1)(x - 1)$
2) $g(x) = (x^3 + 4)(x^2 + 3)$
3) $h(x) = x^m x^n$

1. 33 : Determine the equation of the tangent to $y = \dfrac{x^2 + 3}{x + 3}$ at $x = 1$.

1. 34 : Determine the slope of the tangent to $y = \dfrac{\sin(x)}{x^2}$ at $x = \pi$ (write your answer as a multiple of π).

1. 35 : For which value of a is the graph of the function $f(x) = \dfrac{2x + a}{x - 1}$ tangent to the line $y = -7x + 23$? Calculate the coordinates of the contact point.

1. 36 :

1) Determine the equation of the tangent to $y = \dfrac{3x+2}{(x+1)^2}$ at the point with abscissa 1.
2) Determine the equation of the tangent to the curve $y = \dfrac{1}{\sqrt{x}-1}$ at the point with ordinate 1.
3) Determine the equation of the tangent to the curve $y = (x^2-5)^3$ at the point with abscissa 2.

1. 37 : Establish the variation table of $f(x) = (x-6)(x+2)(x-1) = x^3 - 5x^2 - 8x + 12$. What are the extreme values of the slope of the tangent (maximum and minimum values = range of f'). Establish the table of convexity and find the inflection point's coordinates.

1. 38 : We consider the function $f(x) = -\frac{1}{2}x^3 - x^2 + 2x + 4$. Find the intervals on which : it is positive, it is negative, it increases, it decreases, it is convex, and it is concave.

1. 39 : Determine the domain of these functions and then determine if there graph has a vertical tangent, a cusp or a corner at $(0;0)$.

1) $f(x) = \sqrt[3]{x}$
2) $f(x) = \sqrt[5]{x^2}$
3) $f(x) = \sqrt[3]{x^2}$
4) $f(x) = \sqrt[4]{x^2}$

1. 40 : Determine the critical points (coordinates and type) of the following functions and establish their table of variation.

1) $y = x^3 - 3x^2 - 45x + 7$
2) $y = 3x^4 - 8x^3 + 6x^2$
3) $y = x^2 + \dfrac{54}{x}$
4) $y = 3x^5 - 20x^3 + 1$
5) $y = x - \sqrt{x}$
6) $y = x^{\frac{1}{3}}(4-x)$

1. 41 : Find the solutions of the following equations

1) $-2x^{\frac{3}{4}} + x^{\frac{7}{4}} + x^{\frac{11}{4}} = 0$
2) $-1.8\sqrt{x} + 3x^{\frac{3}{2}} = 0$
3) $-\dfrac{0.2}{x^{\frac{4}{3}}} - \dfrac{2}{x^{\frac{1}{3}}} - 5x^{\frac{2}{3}} = 0$
4) $\dfrac{8}{\sqrt[3]{x}} - x^{\frac{8}{3}} = 0$

1. 42 : Establish the table of variations of the following functions :

1) $y = x^3 - 12x$
2) $y = x^{\frac{3}{2}}(x-1)$
3) $y = x^{\frac{3}{4}} - 2x^{\frac{7}{4}}$
4) $y = x + \dfrac{3}{x}$
5) $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

1. 43 : Show that $f(x) = \dfrac{3x(x-a)}{x^2+9}$ (with $a \in \mathbb{R}^*$) always has two stationary points.

1. 44 :

1) Determine the equation of the tangents to $y = 3(x-1)^2 + 5$ that pass through the origin.
2) Determine the equation of the tangents to $f(x) = x^2 - 15x - 9$ that pass through $(-4;3)$.

1. 45 : Find a and b such that the following functions are smooth : continuous and differentiable everywhere

$$f(x)=\begin{cases} ax^2+bx & x\leq 2 \\ \dfrac{-x^2+3x-2}{2x^2-4x} & x>2 \end{cases} \qquad h(x)=\begin{cases} \dfrac{x^2-2x-3}{x^2+x} & x<-1 \\ \sqrt{ax+b} & x\geq -1 \end{cases}$$

$$g(x)=\begin{cases} \dfrac{-x^2+ax}{x} & x<0 \\ ax^3+bx+3 & x\geq 0 \end{cases}$$

1. 46 : Differentiate the following :

1) $f(x)=(3x+1)^3$
2) $f(x)=\cos(2x)$
3) $f(x)=\sqrt{8x^2-2x+3}$
4) $f(x)=-x^2+\dfrac{4}{2x+3}$
5) $f(x)=(7x^3-2x)^4$
6) $f(x)=\cos^3(x)$
7) $f(x)=\dfrac{x}{(2x+5)^2}$
8) $f(x)=\sin^4\left(\frac{x}{2}\right)$
9) $f(x)=\sqrt[5]{x^4}$
10) $f(x)=\dfrac{1}{\sqrt{3x}}$
11) $f(x)=\sqrt{\frac{1+x}{1-x}}$
12) $f(x)=\sqrt[3]{\tan(x)}$

1. 47 : Differentiate the following :

1) $f(x)=\arcsin(x)$
2) $f(x)=\arccos(x)$
3) $f(x)=\arctan(x)$

1. 48 : For each of the following, determine, at each point of intersection, the angle between the curve $y=f(x)$ and O_x.

1) $y=x^2-1$
2) $y=x^4-5x^2+4$
3) $y=\dfrac{x}{\sqrt[3]{x^2-1}}$
4) $y=\dfrac{x^2-4}{x^2+1}$

1. 49 : Determine, at their point of intersection, the angle between the curves :

1) $y=x^2$ and $y=\dfrac{x^2}{4}+3$
2) $y=\sin(x)$ and $y=\cos(x)$
3) $y=x^3-4x$ and $y=x^3-2x^2$

1. 50 : Determine the value of the real number a so that the graphs of the functions f and g intersect themselves orthogonally.

1) $f(x)=0.25x$ and $g(x)=-x^2+a$
2) $f(x)=x^2$ and $g(x)=\dfrac{1}{2}-ax^2$
3) $f(x)=ax^2$ and $g(x)=\dfrac{1-x^2}{a}$
4) $f(x)=2x^2-a$ and $g(x)=\dfrac{x^2}{a}$

1. 51 : A rectangular enclosure is adjoining to a wall so that it is only enclosed along three of its sides.

1) What is the maximal area of this field, given that the enclosure's total length is 200m ?
2) What is the minimal length of its enclosure if its area is 300m^2 ?

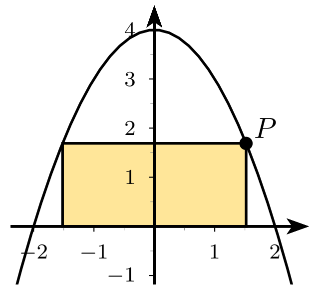
1. 52 : From a string with length 1m we build a rectangle. Determine largest area that can be obtained.

1. 53 : Find the two positive numbers x and y whose sum is 10 that are such that the number $z = x^3y^2$ is as large as possible.

1. 54 : A box (a parallelepiped without lid) is built from a 10cm by 8cm square cardboard. Four identical squares are cut up at each corner. Then the paper is folded and stuck together. Determine the measure of the sides of the squares so that the volume of the box is maximal. Determine the value of the maximal volume.

1. 55 : Let's consider the parabola with equation $y = -x^2 + 4$ and one of its point $P(x; y)$ located in the first quadrant. We then draw, under the parabola, a rectangle which has P as a vertex and which has two of its vertices on the x-axis.

1) Determine the value of x such that the area of the rectangle is maximal. What is the value of that area?
2) When such a rectangle is rotated around the y-axis we obtain a cylinder. Determine the dimensions of the cylinder whose volume is maximal.



3) When such a rectangle is rotated around the x-axis we also obtain a cylinder. Determine the dimensions of the cylinder whose volume is maximal and give that largest volume.
4) When such a rectangle is rotated around the line $x = 3$ we obtain a ring. Determine the dimensions of the ring whose volume is maximal and give that largest volume.

1. 56 : Find the points of the graph of $f(x) = \dfrac{4}{x}$ which are the closest to the origin.

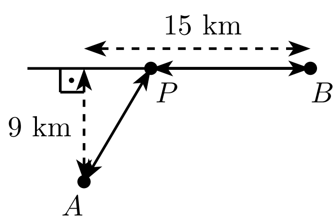
1. 57 : Find the coordinates of the point P on the curve $y = -x^2$ which is the closest to the line $x + y - 1 = 0$.

1. 58 : We consider the parabola $f(x) = 1 - x^2$ with M one of its point located in the first quadrant. The tangent to the parabola at M intersects O_x at A and O_y at B. Determine the coordinates of point M so that the area of the triangle OAB is minimal.

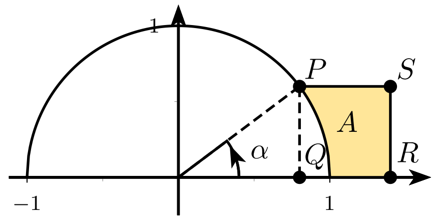
1. 59 : Given the function $f(x) = \dfrac{x^3}{x^2 + 1}$.

1) Represent the function f.
2) Determine the point of the graph of f whose abscissa is positive and whose distance to the asymptote is maximal. Calculate the value of this distance.

1. 60 : A lighthouse keeper (point A) must join its house on the coast (point B). First he rows at a speed of 4km/h and then he walks at a speed of 5km/h. Where must he come alongside (point P) so that the time of the route is as small as possible? The coast is supposed to be straight.



1.61 : The polygon $PQRS$ (in quadrant I) here opposite is a square. Determine the value of α for which the area A is the largest. Give the measure of that largest area.



1.62 : The color of a pixel is obtained with the RGB system : the color is obtained by choosing the intensity of red, green and blue. These intensities are natural numbers from 0 to 255. When the intensity of red, green and blue are identical the color obtained is a shade of gray. To each color we can associate the point $(r; g; b)$ of the space. Determine the shade of gray $(x; x; x)$ that is the closest to the given color $(r; g; b)$. That optimisation problem consists in determining the point $(x; x; x)$ whose distance to $(r; g; b)$ is minimal.

1.63 : Transform the following expressions using the "log rules" :

1) $\ln\left(\left(a^5\sqrt[3]{b}\right)/c\right)$
2) $2\ln(a) - \frac{1}{2}\ln(b) + 3\ln(c)$
3) $\ln\sqrt{a\cdot\sqrt[3]{b}}$
4) $\log\left(2.23\cdot 10^{23}\right)$
5) $\log_2(3)\cdot\log_3(4)\cdot\log_4(5)$

1.64 : Solve the following equations :

1) $\log(x) = \frac{1}{3}$
2) $7^x = 100$
3) $\ln(x+1) + \ln(x+5) = \ln(96)$
4) $\ln|x+1| + \ln|x+5| = \ln(96)$
5) $\log(12x+40) - \log(x-4) = 2$
6) $\log(99 + \log(8 + \log(x-1))) = 2$
7) $2^x = 3^{-x+1}$
8) $\log_x(5) = \log_4(7)$

1.65 : Find the domain and then differentiate the following functions :

1) $f(x) = \ln(x+3)$
2) $f(x) = \ln(2x)$
3) $f(x) = \ln\left(\frac{x}{2}\right)$
4) $f(x) = \ln(ax),\ a > 0$
5) $f(x) = \ln(-x)$
6) $f(x) = \ln(|x|)$
7) $f(x) = \ln(x^2)$
8) $f(x) = \ln\left(\frac{1}{x}\right)$
9) $f(x) = \ln\left(x^2-4\right)$
10) $f(x) = \ln\left(\sqrt[3]{x}\right)$
11) $f(x) = x\cdot(\ln(x)-1)$
12) $f(x) = \ln(ax+b)\,, a < 0$
13) $f(x) = \ln\left(\frac{x^2}{x-1}\right)$
14) $f(x) = \ln\left(\frac{x+1}{x-1}\right)$
15) $f(x) = \dfrac{x}{\ln(x)-1}$
16) $f(x) = \dfrac{\ln(2)}{\ln(x)}$
17) $f(x) = \ln(\ln(x))$
18) $f(x) = \ln(|\ln(x)|)$
19) $f(x) = x^2\left(2\ln(x)-1\right)$
20) $f(x) = \ln\left(x+\sqrt{1+x^2}\right)$
21) $f(x) = \ln(|\cos(x)|)$

1.66 : 1) Find the equation of the tangent to the curve :

- $y = \ln(-x)$ at $x = -\frac{1}{3}$
- $y = \ln(2x)$ at $x = \frac{1}{2}$

2) Find the stationary points of :

- $y = x - \ln(x)$
- $y = \frac{1}{2}x^2 - \ln(2x)$
- $y = x^2 - \ln(x^2)$

3) Find the equation of the normal to the curve : $y = \ln(2x - 3)$ at $x = 2$.

1.67 : 1) Find the derivative of

- $f(x) = \mathrm{e}^{3x}$
- $h(x) = \mathrm{e}^{3-2x}$
- $g(x) = \mathrm{e}^{-x}$
- $i(x) = \mathrm{e}^{\sin(2x+3)}$

2) Find the equation of the tangent to the curves :

- $y = x - \mathrm{e}^{2x}$ at $x = 0$
- $y = \mathrm{e}^{6-2x}$ at $x = 3$

3) Calculate the coordinates of the stationary points of $y = 7x^2 - \mathrm{e}^{x^2}$.

1.68 : Calculate the acute angle between the curves f and g at their intersection point.

1) $f(x) = \mathrm{e}^{x+2}$ and $g(x) = \mathrm{e}^{-x}$
2) $f(x) = \mathrm{e}^{2x}$ and $g(x) = 2\mathrm{e}^{3x}$

1.69 : Differentiate the following :

1) $f(x) = 3^x \cdot x^3$
3) $f(x) = 2^{\sqrt{x^2+1}}$
2) $f(x) = \dfrac{\mathrm{e}^x - 1}{\mathrm{e}^x + 1}$
4) $f(x) = \dfrac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}}$

1.70 : Given the function $f(x) = (x^2 + ax + b)\,\mathrm{e}^x$. Find the value of a and b such that the graph of f has a unique stationary point with abscissa 2.

1.71 :

1) Determine the value of a so that the equation $x^2 - 2x + 2\log(a) = 0$ has exactly two real solutions.
2) Determine the value of a so that the equation $x^2 - 2\log(a)x + 4 = 0$ has exactly two real solutions.

1.72 : Solve :

1) $3 \cdot 2^x > 5^x$
2) $\dfrac{1}{2}(\mathrm{e}^x + \mathrm{e}^{-x}) = 2$
3) $\mathrm{e}^{2x} = 6 - \mathrm{e}^x$

1.73 : Determine the horizontal and slant asymptotes of the graphs of :

1) $f(x) = \dfrac{\ln(x^2)}{x}$
3) $f(x) = \dfrac{\mathrm{e}^x}{x^{1000} + 2}$
5) $f(x) = \dfrac{2\mathrm{e}^x - 1}{\mathrm{e}^x + 2}$
2) $f(x) = \dfrac{\mathrm{e}^x}{x^2 + ax + a}$
4) $f(x) = \dfrac{\ln(x^2)}{\ln^2(x)}$
6) $f(x) = \dfrac{\mathrm{e}^x - 2\mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}}$

1.74 : Study the following functions :

1) $f(x) = \ln(x^2 + 4)$
2) $f(x) = x\ln(x)$
3) $f(x) = \log_x(2)$
4) $f(x) = \ln(x) - \ln(\ln(x))$

1.75 : Study the following function :

1) $f(x) = \dfrac{\ln^2(x)}{x}$
2) $f(x) = (x-1)^2 \cdot e^x$
3) $f(x) = (2x^2 - 4) \cdot e^{-x}$
4) $f(x) = \dfrac{1}{x-3} \cdot e^{-x}$

1.76 : Study the function $f(x) = \dfrac{x}{\ln(x)} - 3$. Carefully determine the behavior of f and f' close to 0.

1.77 : Let's consider the function $f(x) = e^{0.75x} - 3x + c$.

1) Determine the value of c such that f passes through the origin.
2) For $c = -1$: determine the coordinates and type of the stationary point.
3) For $c = -1$: thanks to the asymptotic behavior, sketch the graph of f.

1.78 : 1) Use *Geogebra* to study the "hyperbolic functions" :

a) $f(x) = \sinh(x) = \dfrac{e^x - e^{-x}}{2}$

b) $g(x) = \cosh(x) = \dfrac{e^x + e^{-x}}{2}$

c) $h(x) = \tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$

d) $i(x) = \coth(x) = \dfrac{\cosh(x)}{\sinh(x)}$

2) Check that $\cosh^2(x) - \sinh^2(x) = 1 \quad \forall x \in \mathbb{R}$ (means : "for all $x \in \mathbb{R}$")

1.1 :

1.2 :

1) else
2) even
3) even
4) odd
5) odd
6) odd

1.3 : —

1.4 :

1)

x		$-\frac{1}{2}$	
$f(x)$	$-$	0	$+$

2)

x		1		6	
$f(x)$	$-$	0	$+$	0	$-$

3)

x	-3		1	
$f(x)$	$-$	$-$	0	$+$

4)

x	
$f(x)$	$+$

5)

x	0		$\frac{2}{3}\pi$		$\frac{4}{3}\pi$		2π
$f(x)$	$+$	$+$	0	$-$	0	$+$	$+$

6)

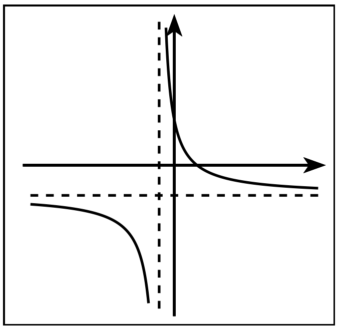
x		-4		0		3	
$f(x)$	$-$	0	$+$	0	$-$	0	$+$

1.5 : $x \in]1;2[\cup]3;+\infty[$

1.6 :

1) $\frac{\pi}{2}$
2) $\frac{\pi}{3}$
3) $\frac{2\pi}{5}$
4) No period
5) π
6) π

1.7 : $D = \mathbb{R} \setminus \{-1\}$ $\quad R = \mathbb{R} \setminus \{-2\}$ $\quad I_x(1.5;0)$ $\quad I_y(0;3)$ $\quad$ VA : $x = -1$ $\quad$ HA : $y = -2$



1.8 :

1) 3
2) 1
3) $\varnothing$
4) 2
5) 2
6) 2
7) $\varnothing$ 1
8) No

1.9 :

1) 0
2) $-\infty$
3) $-\infty$
4) $\varnothing$

$D = \mathbb{R} \setminus \{-6;-3;0;5\}$ $\quad$ VA : $x = -6$ $\quad x = -3$ $\quad x = 0$ $\quad x = 5$
HA : $y = -2$ $\quad y = 0$

1.10 : —

1.11 :

1) 2	3) 2	5) 1	7) $\varnothing$
2) 2	4) 2	6) -1	8) $\frac{1}{2}$

No

1.12 :

1) 10	3) 5	5) 3	7) 3	9) 0
2) 7	4) 0	6) $\frac{1}{7}$	8) $\varnothing$	

1.13 :

1) $-\infty$	3) 0	5) $\frac{4}{5}$	7) 1	9) $-\frac{1}{2}$
2) $-\infty$	4) 0	6) ∞	8) 1	

1.14 :

1) $\frac{2}{5}$	4) $\frac{1}{2}$	7) $-\infty$
2) $\frac{2}{5}$	5) $\frac{1}{2}$	8) ∞
3) $\frac{2}{5}$	6) $\frac{1}{2}$	9) $\varnothing$

$\varnothing$ $\frac{1}{2}$ $\varnothing$

1.15 :

1) a) VA : $x = 2$ $x = -1$
   b) VA : $x = 1.5$ Hole $(1; 2)$

2) a) HA : $y = 0$ SA : $y = 0.5x + 1.25$
   b) VA : $x = \pm 2$ HA : $y = -2$
   c) VA : $x = 2.5$
   d) VA : $x = 2$ SA : $y = -2x + 1$

1.16 : —

1.17 : For instance :

1) $f_1(x) = \frac{-x^2+26}{x-5}$ 2) $f_2(x) = \frac{11(x-3)}{(x-2)(x-3)}$

1.18 :

1) 2	4) $\varnothing$	7) 0.5
2) 1.5	5) 2.5	8) 0
3) 0	6) 0.5	9) 0

1.19 :

1) $\varnothing$	3) 0.5	5) 2	7) 3
2) 0.25	4) ∞	6) $-2\sqrt{3}$	8) 0.25

1. 20 :

1) $D = \left]-\infty; -\frac{\sqrt{5}}{2}\right] \cup \left[\frac{\sqrt{5}}{2}; \infty\right[$
SA : $y = \pm 2x$

2) $D = \mathbb{R}$
SA : $y = |x + 0.5|$

3) $D = \mathbb{R}\backslash\{-1\}$
left-HA : $y = 2$
right-HA : $y = 0$

4) $D = \mathbb{R}\backslash\left]-\frac{1}{2}; 0\right[$
left-SA : $y = 7x + \frac{1}{2}$
right-SA : $y = 3x - \frac{1}{2}$

1. 21 :

1) 0	4) 0.25	7) $-\infty$	10) 1
2) $\frac{1}{6}$	5) 1	8) 1	11) -
3) 0	6) 20	9) 1	12) 0.5

1. 22 :

1) $a = -\frac{11}{2}$ $b = \frac{45}{2}$
2) $a = 2$ $b = -0.5$
3) $a = -3$ $b = -1$
4) $a = -7$ $b = -2.5$

1. 23 :

1) $f'(x) = 0$
2) $f'(x) = m$
3) $f'(x) = m$
4) $f'(x) = 2x$
5) $f'(x) = 3x^2$
6) $f'(x) = \frac{-1}{x^2}$
7) $f'(x) = \frac{1}{2\sqrt{x}}$
8) $f'(x) = n \cdot x^{n-1}$

1. 24 : —

1. 25 :

1) $f'(x) = 12x^2 - 14x$
2) $f'(x) = 50x - 70$
3) $f'(x) = \frac{3}{2\sqrt{x}} + 16x - 9$
4) $f'(x) = -20x^3 + x$
5) $f'(x) = \frac{-7}{2x^2} + 3\cos(x)$

1. 26 : 1) $P_1\left(-\frac{1}{3}; -\frac{125}{27}\right)$ $P_2(2; 13)$ 2) $P_1\left(-\frac{1}{3}; -5\right)$ $P_2\left(\frac{1}{3}; 5\right)$

1. 27 : 1) $k = 2$ $P(3; 11)$ 2) $k = -15$ $P(25; 5)$.

1. 28 : $a = 2$ $b = -12$ $c = 18$

1. 29 :

1) $y = 17x + 10$
2) $y = 0.12x - 1.2$
3) $y = 1.25x + 7$
4) $y = -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}\left(\frac{3\pi}{4} - 1\right)$

1. 30 : -

1. 31 : 1) $a = -3$ $b = 3$ level-point 2) $a = -5$ $b = -18.5$

1. 32 :

1) $f'(x) = 2x$
2) $g'(x) = 5x^4 + 9x^2 + 8x$
3) $h'(x) = (m + n)x^{m+n-1}$

1. 33 : $y = 0.25x + 0.75$

1. 34 : $m = \frac{-1}{\pi^2}$

1. 35 : $a = 5 \quad P(2;9)$

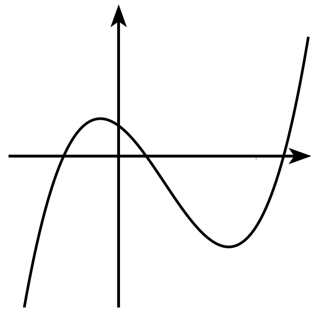
1. 36 : 1) $y = \frac{-1}{2}x + \frac{7}{4}$ 2) $y = -\frac{1}{4}x + 2$ 3) $y = 12x - 25$

1. 37 : Table of variation (TV)

x		$-\frac{2}{3}$		4	
y'	$+$	0	$-$	0	$+$
G_f	$\nearrow$	$\leftrightarrow$	$\searrow$	$\leftrightarrow$	$\nearrow$

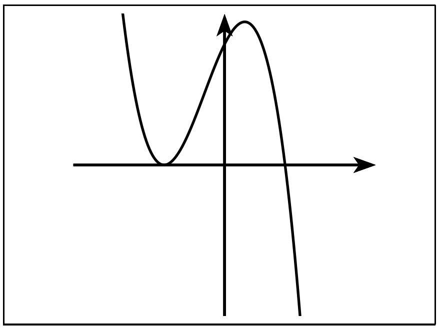
Table of convexity (TC)

x		$\frac{5}{3}$	
y''	$-$	0	$+$
G_f	$\frown$	IP	$\smile$



MAX($-\frac{2}{3}$;14.81) MIN(4;-36)
Maximal slope (at $x = \pm\infty$)= $+\infty$.
Minimal slope (at $x = \frac{5}{3}$)= $f'(\frac{5}{3}) \cong -16.3$.
Inflexion point at $(\frac{5}{3}; -10.59)$.

1. 38 : Positive on$]-\infty; 2[\setminus\{-2\}$
Negative on $]-2; 2[$
Increase on $\left]-2; \frac{2}{3}\right[$
Decrease on $]-\infty; -2[\cup\left]\frac{2}{3}; +\infty\right[$
Concave on $\left]-\frac{2}{3}; +\infty\right[$
Convex on $\left]-\infty; -\frac{2}{3}\right[$



1. 39 :

1) vertical tangency point
2) cusp
3) cusp
4) cusp

1. 40 :

1) $MAX(-3;88), MIN(5;-168)$

x		-3		5	
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$	$\leftrightarrow$	$\searrow$	$\leftrightarrow$	$\nearrow$

2) $MIN(0;0), LP(1;1)$

x		0		1	
$f'(x)$	$-$	0	$+$	0	$+$
$f(x)$	$\searrow$	$\leftrightarrow$	$\nearrow$	$\leftrightarrow$	$\nearrow$

3) $MIN(3;27)$

x		0		3	
$f'(x)$	$-$	✘	$-$	0	$+$
$f(x)$	$\searrow$	✘	$\searrow$	$\leftrightarrow$	$\nearrow$

4) $MAX(-2;65), LP(0;1), MIN(-2;63)$

x		-2		0		2	
$f'(x)$	$+$	0	$-$	0	$-$	0	$+$
$f(x)$	$\nearrow$	$\leftrightarrow$	$\searrow$	$\leftrightarrow$	$\searrow$	$\leftrightarrow$	$\nearrow$

5) $VTP(0;0), MIN(0.25;-0.25)$

x		0		$\frac{1}{4}$	
$f'(x)$	✘	✘	$-$	0	$+$
$f(x)$	✘	$\updownarrow$	$\searrow$	$\leftrightarrow$	$\nearrow$

6) $VTP(0;0,)MAX(1;3)$

x		0		1	
$f'(x)$	$+$	✘	$+$	0	$-$
$f(x)$	$\nearrow$	$\updownarrow$	$\nearrow$	$\leftrightarrow$	$\searrow$

1. 41 :

1) $x = 0$ $\quad x = 1$ $\quad x = 2$

2) $x = 0$ $\quad x = \frac{3}{5}$

3) $x = -0.2$

4) $x = 2$

1. 42 : 1) $MAX(-2;16), MIN(2;-16)$

x		-2		2	
$f'(x)$	$+$	0	$-$	0	$+$
G_f	$\nearrow$	$\leftrightarrow$	$\searrow$	$\leftrightarrow$	$\nearrow$

2) $MAX(0;0), MIN(0.6;-0.19)$

x		0		0.6	
$f'(x)$	✘	0	$-$	0	$+$
G_f	✘	$\leftrightarrow$	$\searrow$	$\leftrightarrow$	$\nearrow$

3) $VTP(0;0), MAX(\frac{3}{14};0.18)$

x		0		$\frac{3}{14}$	
$f'(x)$	✘	✘	$+$	0	$-$
G_f	✘	$\updownarrow$	$\nearrow$	$\leftrightarrow$	$\searrow$

4) $MAX(-\sqrt{3};-3.46), MIN(\sqrt{3};3.46)$

x		$-\sqrt{3}$		0		$\sqrt{3}$	
$f'(x)$	$+$	0	$-$	✘	$-$	0	$+$
G_f	$\nearrow$	$\leftrightarrow$	$\searrow$	✘	$\searrow$	$\leftrightarrow$	$\nearrow$

5) $MIN(1;2)$

x		0		1	
$f'(x)$	✘	✘	$-$	0	$+$
G_f	✘	✘	$\searrow$	$\leftrightarrow$	$\nearrow$

1. 43 : —

1. 44 : 1) $t_{1,2} : y = (\pm 4\sqrt{6} - 6)x \quad (t_1 : y = 3.80x$ and $t_2 : y = -15.80x)$

2) $T_1(4;-53) \quad t_1 : y = -7x - 25 \qquad T_2(-12;315) \quad t_2 : y = -39x - 153$

1. 45 : $f : a = 0 \quad b = -\frac{1}{8} \qquad g : a = 3 \quad b = -1 \qquad h : a = 24 \quad b = 40$

1. 46 :

1) $f'(x) = 81x^2 + 54x + 9$
2) $f'(x) = -2\sin(2x)$
3) $f'(x) = \frac{8x-1}{\sqrt{8x^2-2x+3}}$
4) $f'(x) = -2x - \frac{8}{(2x+3)^2}$
5) $f'(x) = (84x^2 - 8)(7x^3 - 2x)^3$
6) $f'(x) = -3\sin(x)\cos^2(x)$
7) $f'(x) = \frac{-2x+5}{(2x+5)^3}$
8) $f'(x) = 2\sin^3\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$
9) $f'(x) = \frac{4}{5\sqrt[5]{x}}$
10) $f'(x) = \frac{-1}{2x\sqrt{3x}}$
11) $f'(x) = \frac{1}{\sqrt{1+x}(1-x)^{\frac{3}{2}}}$
12) $f'(x) = \frac{1}{3\cos^2(x)\sqrt[3]{\tan^2(x)}}$

1. 47 : 1) $f'(x) = \frac{1}{\sqrt{1-x^2}}$ 3) $f'(x) = \frac{1}{1+x^2}$
2) $f'(x) = \frac{-1}{\sqrt{1-x^2}}$

1. 48 : 1) $\alpha \simeq 63.43°$ 3) $\alpha \simeq 45°$
2) $\alpha_1 \simeq 80.54°$ $\alpha_2 \simeq 85.23°$ 4) $\alpha \simeq 38.66°$

1. 49 : 1) $\alpha \simeq 30.96°$ 3) $\alpha_1 \simeq 75.96°$ $\alpha_2 \simeq 6.91°$
2) $\alpha \simeq 70.53°$

1. 50 : 1) $a = 4.5$ 3) $a = \pm\sqrt{3}$
2) $a = 1$ 4) impossible

1. 51 : $A = 5000$ m (dim : 50x100) $L = 4\sqrt{150} \simeq 48.99m$

1. 52 : The rectangle is a square with side 0.25. The area is equal to 0.0625

1. 53 : $x = 6$ $y = 4$ $p = x^3y^2 = 3456$

1. 54 : $c = 1.47$ cm $V = 52.5$ cm^2

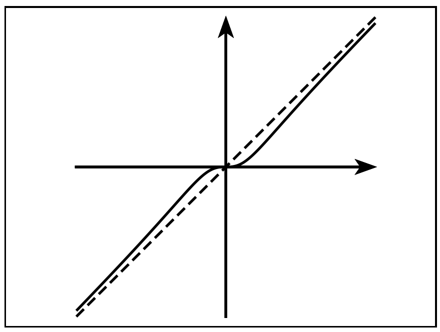
1. 55 : 1) $A(x) = -2x^3 + 8x$ $x = \frac{2\sqrt{3}}{3} \simeq 1.15$, $y \simeq 2.67$ $A \simeq 6.15$
2) $V(x) = -\pi x^4 + 4\pi x^2$ $x = \sqrt{2} \simeq 1.41$, $y = 2$ $V = 4\pi \simeq 12.57$
3) $V(x) = 2\pi x(-x^2+4)^2$ $x = \frac{2\sqrt{5}}{5} \simeq 0.89$, $y = 3.2$ $V = \frac{1024\sqrt{5}\pi}{125} \simeq 57.55$
4) $V(x) = 12\pi x(-x^2+4)$ $x = \frac{2\sqrt{3}}{3} \simeq 1.15$, $y \simeq 2.67$ $V = \frac{64\pi\sqrt{3}}{3} \simeq 116.08$

1. 56 : $P_{1,2}(\pm 2; \pm 2)$

1. 57 : $P(0.5; -0.25)$

1. 58 : $M(\frac{1}{\sqrt{3}}; \frac{2}{3})$

1. 59 : $P_1(1; \frac{1}{2})$ $P_2(-1; \frac{-1}{2})$
$distance = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \simeq 0.35$



1. 60 : He must come alongside at 12 km of the point of the cost which is the closest to the lighthouse.

1. 61 : $\alpha \simeq 63.44°$ and the length of the sides of the square is $\sin(\alpha) \simeq 0.89$

1. 62 :
$x = \frac{r+g+b}{3}$

1.63 :
1) $5\ln(a)+\frac{1}{3}\ln(b)-\ln(c)$
2) $\ln\left(\frac{a^2c^3}{\sqrt{b}}\right)$
3) $0.5\ln(a)+\frac{1}{6}\ln(b)$
4) $23\log(2.23)$
5) $\log_2(5)$

1.64 :
1) $x\simeq 2.15$
2) 2.37
3) $x=7$
4) $x_1=7$ et $x_2=-13$
5) $x=5$
6) $x=101$
7) $x\simeq 0.613$
8) $x\simeq 3.147$

1.65 :
1) $D=]-3;+\infty[\quad f'(x)=\frac{1}{x+3}$
2) $D=\mathbb{R}_+^*\quad f'(x)=\frac{1}{x}$
3) $D=\mathbb{R}_+^*\quad f'(x)=\frac{1}{x}$
4) $D=\mathbb{R}_+^*\quad f'(x)=\frac{1}{x}$
5) $D=\mathbb{R}_-^*\quad f'(x)=\frac{1}{x}$
6) $D=\mathbb{R}^*\quad f'(x)=\frac{1}{x}$
7) $D=\mathbb{R}^*\quad f'(x)=\frac{2}{x}$
8) $D=\mathbb{R}_+^*\quad f'(x)=-\frac{1}{x}$
9) $D=\mathbb{R}\setminus[-2;2]\quad f'(x)=\frac{2x}{x^2-4}$
10) $D=\mathbb{R}_+^*\quad f'(x)=\frac{1}{3x}$
11) $D=\mathbb{R}_+^*\quad f'(x)=\ln(x)$
12) $D=]-\infty;-\frac{b}{a}[\quad f'(x)=\frac{a}{ax+b}$
13) $D=]1;+\infty[\quad f'(x)=\frac{x-2}{x(x-1)}$
14) $D=\mathbb{R}\backslash]-1;1[\quad f'(x)=-\frac{2}{x^2-1)}$
15) $D=\mathbb{R}_+^*\setminus\{\mathrm{e}\}\quad f'(x)=\frac{\ln(x)-2}{(\ln(x)-1)^2}$
16) $D=\mathbb{R}_+^*\setminus\{1\}\quad f'(x)=-\frac{\ln(2)}{(x\ln^2(x))}$
17) $D=]1;+\infty[\quad f'(x)=\frac{1}{x\ln(x)}$
18) $D=\mathbb{R}_+^*\setminus\{1\}\quad f'(x)=\frac{1}{x\ln(x)}$
19) $D=\mathbb{R}_+^*\quad f'(x)=2x\ln(x)$
20) $D=\mathbb{R}\quad f'(x)=\frac{1}{\sqrt{1+x^2}}$
21) $D=\mathbb{R}\setminus\left\{\frac{\pi}{2}+k\pi,k\in\mathbb{Z}\right\}$
$f'(x)=-\tan(x)$

1.66 :
1) 1) $y=-3x+\ln\left(\frac{1}{3}\right)-1=-3x-2.1$ 2) $y=2x-1$
2) $S(1;1)$ min $\quad S(1;-0.19)$ min $\quad S_1(-1;1)\quad S_2(1;1)$ min
3) $y=-\frac{1}{2}x+1$

1.67 :
1) $f'(x)=3\mathrm{e}^{3x}\quad g'(x)=-\mathrm{e}^{-x}\quad h'(x)=-2\mathrm{e}^{3-2x}\quad i'(x)=2\cos(2x+3)\mathrm{e}^{\sin(2x+3)}$
2) $y=-x-1\qquad y=-2x+7$
3) $S_1(-1.39;6.62)$ max $\quad S_2(0;-1)$ min $\quad S_3(1.39;6.62)$ max

1.68 :
1) $\alpha\cong 40.4°$
2) $\alpha\cong 10.3°$

1.69 :
1) $f'(x)=3^x\cdot x^2(x\ln(3)+3)$
2) $f'(x)=\frac{2\mathrm{e}^x}{(\mathrm{e}^x+1)^2}$
3) $f'(x)=\frac{x\ln(2)2^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}$
4) $f(x)=\frac{4}{(\mathrm{e}^x+\mathrm{e}^{-x})^2}$

1.70 : $a=-6\quad b=10$

1.71 : $a\in]0;\sqrt{10}[\qquad a\in]0;0.01[\cup]100;+\infty[$

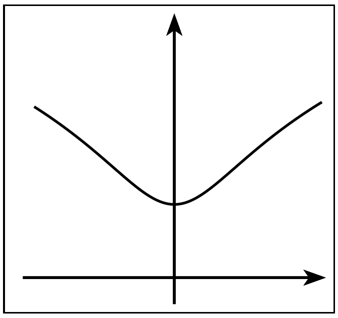
1.72 :
1) $x<-\frac{\ln(3)}{\ln(2/5)}$
2) $x_1=\ln(2+\sqrt{3})\quad x_2=\ln(2-\sqrt{3})$
3) $x=\ln(2)$

1.73 :

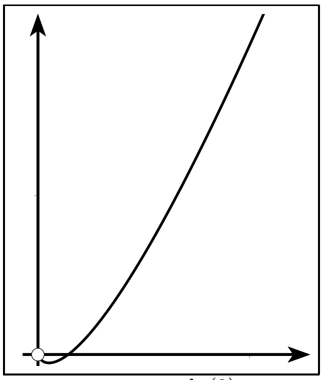
1) HA : $y = 0$ and VA : $x = 0$
2) HA left : $y = 0$
3) HA left : $y = 0$
4) HA left : $y = 0$
5) HA left : $y = -\frac{1}{2}$
   HA right : $y = 2$
6) HA left : $y = -2$
   HA right : $y = 1$

1.74 :

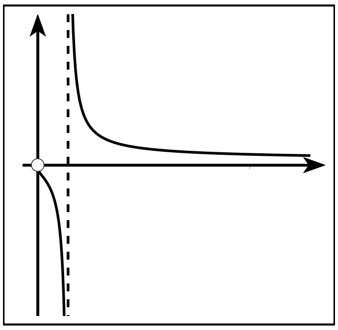
1) $P(0; 1.39)$ $f'(x) = \frac{2x}{x^2+4}$ $S(0; 1.39)$



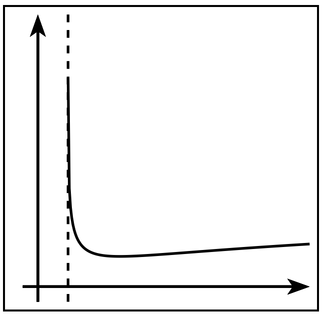
2) $P(1; 0)$ Hole $(0; 0)$ $f'(x) = \ln(x) + 1$ $S(0.37; -0.37)$



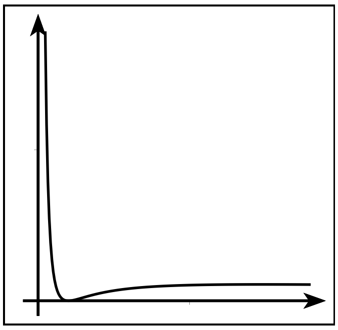
3) Hole $(0; 0)$ VA : $x = 1$ $f'(x) = -\frac{\ln(2)}{x \ln^2(x)}$



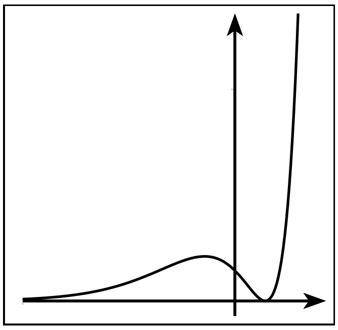
4) VA : $x = 1$ $f'(x) = \frac{\ln(x)-1}{x\ln(x)}$ $S(2.71; 1)$



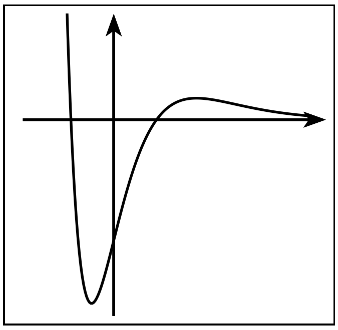
1.75 : 1) $P(1;0)$ VA : $x = 0$ HA : $y = 0$ $f'(x) = \frac{\ln(x)(2-\ln(x))}{x^2}$ $S_1(1;0)$ $S_2(7.39;0.54)$



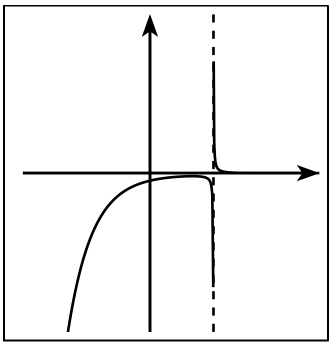
2) $P_1(1;0)$ $P_2(0;1)$ HA : $y = 0$ $f'(x) = (x^2-1)\mathrm{e}^x$ $S_1(-1;1.47)$ $S_2(1;0)$



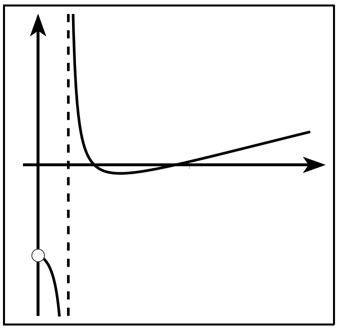
3) $P_{1,2}(\pm\sqrt{2};0)$ $P_3(0;-4)$ HA : $y = 0$ $f'(x) = (-2x^2+4x+4)\mathrm{e}^{-x}$ $S_1(-0.73;6.09)$ $S_2(2.73;0.71)$



4) VA : $x = 3$ HA : $y = 0$ $f'(x) = \frac{-x+2}{(x-3)^2} \cdot \mathrm{e}^{-x}$ $S(2;-0.14)$



1. 76 : $P_1(1.86;0)$ $P_2(4.54;0)$ Hole $(0;-3)$ VA : $x = 1$ $f'(x) = \frac{\ln(x)-1}{\ln(x)^2}$ $S(2.72;-0.28)$



1. 77 : 1) $c = -1$
2) $x = \frac{4}{3}\ln(4)$
3) —

1. 78 : -