

1.6 Exercises

1.1 Calculate without calculator :

- | | | | |
|--------------------------------------|-----------------|--|-------------------------|
| 1) $\log_2(16)$ | 3) $\log_4(1)$ | 5) $\log_5(5)$ | 7) $\log_{16}(8)$ |
| 2) $\log_7\left(\frac{1}{49}\right)$ | 4) $\log_3(-9)$ | 6) $\log_{27}\left(\frac{1}{3}\right)$ | 8) $\log_{\sqrt{2}}(4)$ |

1.2 Rewrite the following by using $\log(p)$, $\log(q)$ and $\log(r)$.

- | | | |
|--------------------|---|--|
| 1) $\log(pqr)$ | 4) $\log\left(\sqrt{\frac{p}{q^2r}}\right)$ | 6) $\log\left(\sqrt{\frac{10p^{10}r}{q}}\right)$ |
| 2) $\log(pq^2r^3)$ | | |
| 3) $\log(100pr^5)$ | 5) $\log\left(\frac{qr^7p}{10}\right)$ | |

1.3 Transform the following by using the properties of logarithms, in one direction or the other.

- | | |
|--|---|
| 1) $\log\left((a^5\sqrt[3]{b})/c\right)$ | 3) $2 + 3\log(a)$ |
| 2) $\log\left(\sqrt{a \cdot \sqrt[3]{b}}\right)$ | 4) $2\log(a) - \frac{1}{2}\log(b) + 3\log(c)$ |

1.4 Solve the following equations :

- | | |
|---|---|
| 1) $10^x = 9.56$ | 12) $\log(0.5x - 3) = -1$ |
| 2) $10^{3x} = -5$ | 13) $2.8^x = 5$ |
| 3) $\log(x) = 2.5$ | 14) $3\log(2x) = 9$ |
| 4) $\log(4x - 1) = -2$ | 15) $1000^{x+2} = 10^{5x+8}$ |
| 5) $2\log(x) = -4$ | 16) $\log^2(x) - \log(x) - 2 = 0$ |
| 6) $\log(x^2 - 21) = 2$ | 17) $9 \cdot 3^{2x} - 82 \cdot 3^x + 9 = 0$ |
| 7) $10^{3x+2} = \sqrt{10}$ | 18) $2^{x^2-8} = \frac{1}{16}$ |
| 8) $\log(\log(x)) = 1$ | 19) $\log(99 + \log(8 + \log(x - 1))) = 2$ |
| 9) $2 \cdot 2^{2x} - 9 \cdot 2^x + 4 = 0$ | 20) $\log(x + 1) + \log(x + 5) = \log(96)$ |
| 10) $2^x = 10$ | 21) $\log x + 1 + \log x + 5 = \log(96)$ |
| 11) $1.03^x = 2$ | 22) $\log_x(0.0025) = 2$ |

1.5 Answer the following :

- 1) Determine the derivative of $f(x) = e^{3x}$, $g(x) = e^{-x}$, $h(x) = e^{3-2x}$ and $i(x) = e^{\sin(2x+3)}$
- 2) Determine the equation of the tangent to the given curves at the point whose abscissa is given.

a) $y = x - e^{2x}$ at $x = 0$ b) $y = e^{6-2x}$ at $x = 3$

- 3) Find the stationary point(s) of the function $y = 7x^2 - e^{x^2}$.

1.6 Determine the acute angle between the curves f and g at their point of intersection

- 1) $f(x) = e^{x+2}$ and $g(x) = e^{-x}$
- 2) $f(x) = e^{2x}$ and $g(x) = 2e^{3x}$

1.7 Differentiate the following :

- 1) $f(x) = x^2 e^{1-x}$
- 2) $f(x) = 5^{3x+1}$
- 3) $f(x) = \frac{e^x}{x+2}$
- 4) $f(x) = 3^x \cdot x^3$
- 5) $f(x) = \frac{e^x - 1}{e^x + 1}$
- 6) $f(x) = e^{\sqrt{x^2+1}}$
- 7) $f(x) = e^{x \cos(x)}$
- 8) $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

1.8 Differentiate the following :

- 1) $f(x) = \ln(2x - 1)$
- 2) $f(x) = \ln(ax + b)$
- 3) $f(x) = 3 \ln(x^{-2})$
- 4) $f(x) = \ln\left(\frac{1}{3x+1}\right)$
- 5) $f(x) = \ln(-x^2 + 6)$
- 6) $f(x) = \ln\left(\frac{1}{x}\right)$
- 7) $f(x) = \ln\left(\frac{2x+1}{3x-1}\right)$
- 8) $f(x) = \ln(\cos(x))$

1.9 Determine the value of $a > 0$ so that the equation $x^2 - 2x + 2 \log(a) = 0$ has two real solutions ?

1.10 Let's consider the function $f(x) = (x^2 + ax + b)e^x$. Find the value of a and b given that the graph of f has only one stationary point whose abscissa is 2.

1.11 Answer the following :

- 1) Determine the equation of the tangent to the curve $y = \ln(-x)$ at $x = -\frac{1}{3}$.
- 2) Determine the equation of the normal to the curve $y = \ln(2x - 3)$ at $x = 2$.

1.12 Find the stationary point(s) of the following functions.

- 1) $y = x^2 - \ln(x^2)$
- 2) $y = \frac{1}{2}x^2 - \ln(2x)$

1.13 Analyse the asymptotic behaviour (when $x \rightarrow \pm\infty$) of the following :

$$\begin{array}{lll} 1) \quad f(x) = \frac{\ln(x^2 + 1)}{x} & 3) \quad f(x) = \frac{e^x}{x^{1000} + 2} & 5) \quad f(x) = \frac{2e^x - 1}{e^x + 2} \\ 2) \quad f(x) = \frac{e^x}{x^2 - 3x + 2} & 4) \quad f(x) = \frac{\ln(x^2)}{\ln^2(x)} & 6) \quad f(x) = \frac{e^x - 2e^{-x}}{e^x + e^{-x}} \end{array}$$

1.14 Analyse the following :

$$\begin{array}{ll} 1) \quad f(x) = (x - 1)^2 \cdot e^x & 3) \quad f(x) = \frac{\ln^2(x)}{x} \\ 2) \quad f(x) = (2x^2 - 4) \cdot e^{-x} & 4) \quad f(x) = \frac{1}{x - 3} \cdot e^{-x} \end{array}$$

1.15 Determine the general expression for the function $f(x)$ given by its derivative :

$$\begin{array}{ll} 1) \quad f'(x) = 4x^3 & 4) \quad f'(x) = 9x^2 - 4x - 5 \\ 2) \quad f'(x) = 2x & 5) \quad f'(x) = 2x^2 - 3x - 4 \\ 3) \quad f'(x) = 10x^9 - 8x^7 - 1 & 6) \quad f'(x) = 1 - 5x - 7x^3 \end{array}$$

1.16 In each case, determine the general expression for $f(x)$:

$$\begin{array}{ll} 1) \quad f(x) \text{ passes through } (-4; 9) \text{ and is such that } f'(x) = \frac{1}{2}x^3 + \frac{1}{4}x + 1 & \\ 2) \quad f(x) \text{ is such that } f'(x) = 15x^2 - 6x + 4 \text{ and } f(1) = 0 & \end{array}$$

1.17 Calculate the indefinite integrals here below :

$$\begin{array}{lll} 1) \quad \int 4x \, dx & 7) \quad \int \frac{1}{2}x^8 \, dx & 12) \quad \int \frac{1}{\sqrt{x}} \, dx \\ 2) \quad \int 1 \, dx & 8) \quad \int \frac{1}{x^3} \, dx & 13) \quad \int e^{3x} \, dx \\ 3) \quad \int 9 \, dx & 9) \quad \int 6x^2 - 2x - 5 \, dx & 14) \quad \int e^{-x} \, dx \\ 4) \quad \int a \, dx & 10) \quad \int x(x + 2)(x - 2) \, dx & 15) \quad \int e^{3-2x} \, dx \\ 5) \quad \int x^2 \, dv & 11) \quad \int \sqrt{x} \, dx & 16) \quad \int \sin(5x) \, dx \\ 6) \quad \int t^5 \, dt & & \end{array}$$

1. 18 Calculate the definite integrals here below :

$$1) \int_2^5 x^2 dx$$

$$6) \int_1^4 \frac{1}{x^2} dx$$

$$2) \int_1^2 \frac{4}{3} x^3 dx$$

$$7) \int_0^\pi \sin(x) dx$$

$$3) \int_0^8 12\sqrt[3]{x} dx$$

$$8) \int_4^9 \frac{2\sqrt{x} + 3}{\sqrt{x}} dx$$

$$4) \int_1^8 \frac{1}{\sqrt[3]{x^2}} dx$$

$$9) \int_0^{\ln(2)} 2e^{1-2x} dx$$

$$5) \int_1^2 \frac{8}{x^3} + x^3 dx$$

$$10) \int_0^1 e^{\ln(2)x} dx$$

1. 19 Calculate the following :

$$1) \int_{-1}^1 e^{-x} dx$$

$$3) \int_{\ln(3)}^{\ln(9)} e^x dx$$

$$5) \int_{-3}^{-1} \frac{1}{x} dx$$

$$7) \int_0^8 \sqrt[3]{x} dx$$

$$2) \int_{-3}^9 e^{\ln(3)x} dx$$

$$4) \int_0^1 a^x dx$$

$$6) \int_1^3 \frac{1}{x-5} dx$$

1. 20 We consider the function $f(x) = e^{0.75x} - 3x + c$.

- 1) Determine the value of c so that f passes through the origin.
- 2) For $c = -1$ compute the coordinates of the stationary point.
- 3) For $c = -1$ determine the value of $\int_{-1}^0 f(x) dx$

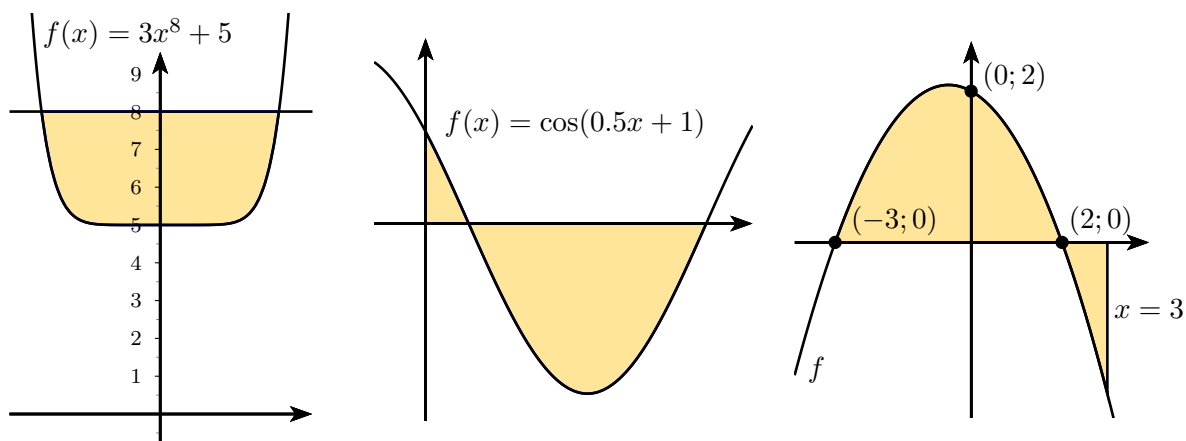
1. 21 Answer the following :

- 1) Calculate the area under the graph of $f(x) = x^4 + 5$ from $x = -1$ to $x = 1$.
- 2) Determine the area between the x -axis and the curve $f(x) = x^2 - 5$ from $x = -3$ to $x = 2$.
- 3) Calculate the area under the curve $y = \frac{6}{x^4}$ between $x = 1$ and $x = 2$.
- 4) Calculate the value of the area delimited by the curve $y = e^{2x}$, the axes and the line $x = 2$.

1.22 Answer the following :

- 1) Calculate $A = \int_1^b \frac{1}{x^2} dx$
- 2) Find the value of b so that $A = \frac{1}{2}$
- 3) Calculate the value of A when $b \rightarrow +\infty$.

1.23 In each case, determine the area of the shaded region :

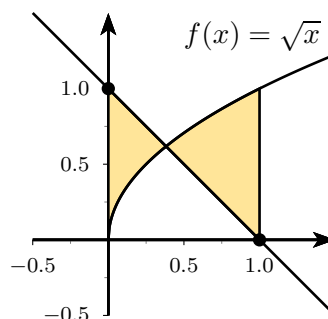


1.24 In each situation, plot the graphs of f and g on the same coordinate system, hatch the surface delimited by the two curves and determine its area.

- 1) $f(x) = \frac{1}{2}x^2$ and $g(x) = 4 - x$
- 2) $f(x) = 1 + \frac{1}{9}x^2$ and $g(x) = 3 - \frac{1}{9}x^2$

1.25 In the first quadrant we consider the area bounded by the axes, the line $y = 8$ and the graph of the function $f(x) = \frac{4 - x^2}{x^2}$. Sketch the situation and calculate the value of that area.

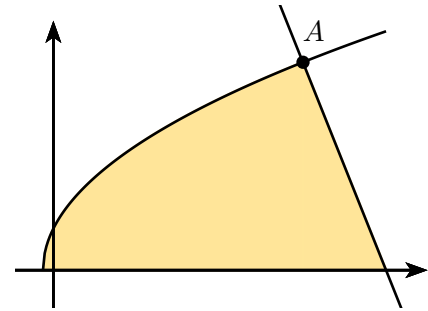
1.26 Calculate the value of the grey area :



1. 27

Here opposite is represented the curve $y = \sqrt{4x+1}$ as well as the normal line to this curve at the point $A(6; 5)$.

- 1) Determine the equation of that normal line.
- 2) Calculate the value of the grey area.

**1. 28**

Determine antiderivatives of the following functions, by using the integration by parts :

- | | | |
|-------------------------|-------------------------|-------------------------------|
| 1) $f(x) = x \cdot e^x$ | 4) $f(x) = x^2 \sin(x)$ | 7) $f(x) = x^2 \ln(x)$ |
| 2) $f(x) = \ln(x)$ | 5) $f(x) = x \ln(x)$ | 8) $f(x) = e^x \cdot \sin(x)$ |
| 3) $f(x) = x^2 e^{-2x}$ | 6) $f(x) = x(1-x)^4$ | 9) $f(x) = x\sqrt{x+1}$ |

1. 29

By substitution, find an antiderivative of the following :

- | | |
|------------------------------|---|
| 1) $f(x) = \cos(2x+1)$ | 9) $f(x) = \frac{e^x}{1+e^x}$ |
| 2) $f(x) = (2x+5)^4$ | 10) $f(x) = \frac{2x+1}{x^2+x+4}$ |
| 3) $f(x) = \sin(-5x+3)$ | 11) $f(x) = \frac{\sin(\sqrt{x})}{2\sqrt{x}}$ |
| 4) $f(x) = \sin(x) \cos(x)$ | 12) $f(x) = x(1-x^2)^4$ |
| 5) $f(x) = x e^{-x^2}$ | 13) $f(x) = x \cos(\pi x^2)$ |
| 6) $f(x) = x^2 e^{-x^3}$ | 14) $f(x) = x^3 \sqrt{1+x^4}$ |
| 7) $f(x) = \frac{3}{1-3x}$ | |
| 8) $f(x) = \frac{\ln(x)}{x}$ | |

1. 30

Find an antiderivative of the following :

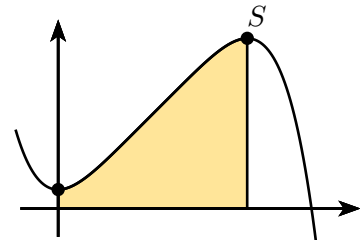
- | | |
|---------------------------------------|--|
| 1) $f(x) = \frac{2x^2 - 5x + 7}{x-3}$ | 2) $f(x) = \frac{5x^2 + 4x - 8}{3x-1}$ |
|---------------------------------------|--|

1. 31

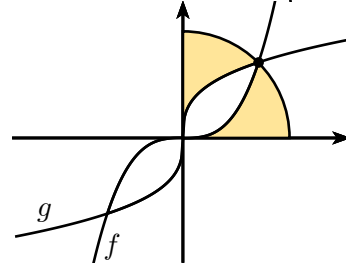
The antiderivative of

- 1) $f(x) = (2x^2 - 3x + 1)e^x$ is of the form $F(x) = (ax^2 + bx + c)e^x$. Find it !
- 2) $f(x) = (x^2 + 4x - 3)e^{-x}$ is of the form $F(x) = (ax^2 + bx + c)e^{-x}$. Find it !
- 3) $f(x) = (-5x^2 - 2x + 7)e^{2x}$ is of the form $F(x) = (ax^2 + bx + c)e^{2x}$. Find it !
- 4) $f(x) = 3 \cos(x) + 7 \sin(x)e^{-x}$ is of the form $F(x) = (a \cos(x) + b \sin(x))e^{-x}$. Find it !

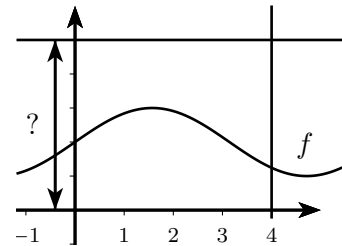
- 1.32** Here opposite is represented the curve $y = (1 - 4x)^5 + 20x$. Prove that this curve has a minimum located on the y -axis. Then calculate the value of the grey area.



- 1.33** Here opposite are partially represented the graphs of $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$. Calculate the grey area.



- 1.34** What must the height of that rectangular target be so that a dart thrown randomly has the same probability to hit the target above and below the curve $f(x) = \sin(x) + 2$?



1.8 1) $f'(x) = \frac{2}{2x-1}$

5) $f'(x) = \frac{-2x}{-x^2+6}$

2) $f'(x) = \frac{a}{ax+b}$

6) $f'(x) = -\frac{1}{x}$

3) $f'(x) = -\frac{6}{x}$

7) $f'(x) = \frac{-5}{6x^2+x-1}$

4) $f'(x) = \frac{-3}{3x+1}$

8) $f'(x) = -\tan(x)$

1.9 $a \in]0; \sqrt{10}[$

1.10 $a = -6 \quad b = 10$

1.11 1) $y = -3x + \ln\left(\frac{1}{3}\right) - 1 \cong -3x - 2.10$ 2) $y = -\frac{1}{2}x + 1$

1.12 1) $MIN(-1; 1) \quad MIN(1; 1)$ 2) $MIN(1; -0.19)$

1.13 1) $\lim_{x \rightarrow -\infty} f(x) = 0 \quad \lim_{x \rightarrow +\infty} f(x) = 0$

2) $\lim_{x \rightarrow -\infty} f(x) = 0 \quad \lim_{x \rightarrow +\infty} f(x) = +\infty$

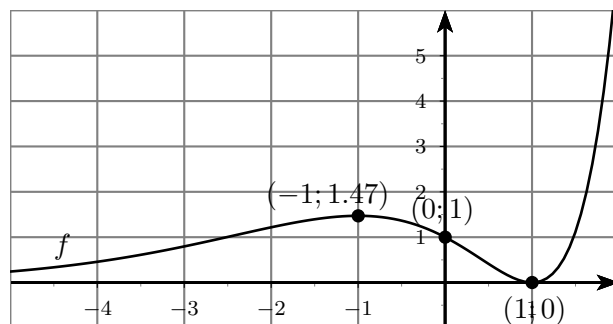
3) $\lim_{x \rightarrow -\infty} f(x) = 0 \quad \lim_{x \rightarrow +\infty} f(x) = +\infty$

4) $\lim_{x \rightarrow -\infty} f(x)$ can't be considered $\lim_{x \rightarrow +\infty} f(x) = 0$

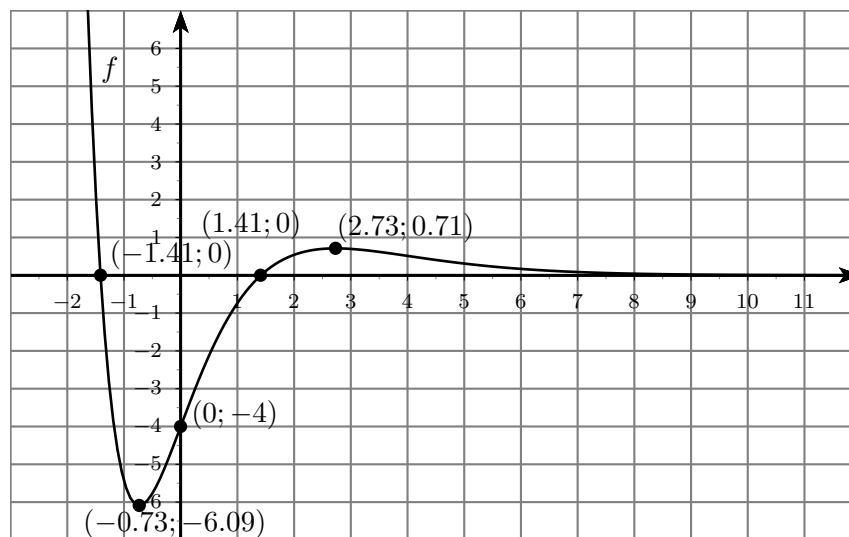
5) $\lim_{x \rightarrow -\infty} f(x) = -\frac{1}{2} \quad \lim_{x \rightarrow +\infty} f(x) = 2$

6) $\lim_{x \rightarrow -\infty} f(x) = -2 \quad \lim_{x \rightarrow +\infty} f(x) = 1$

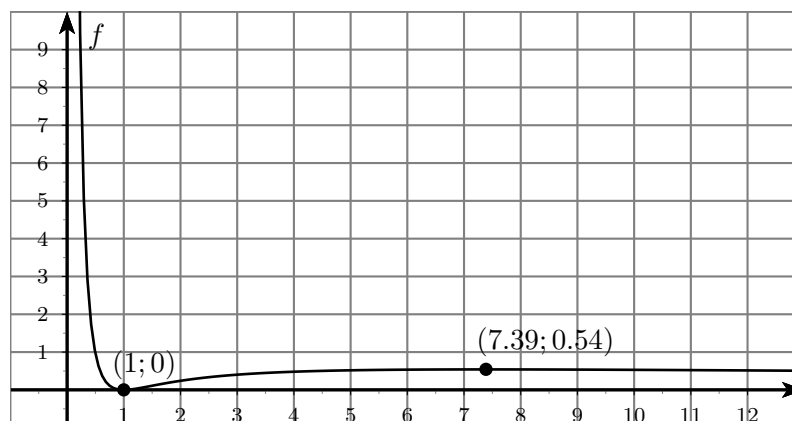
1.14 1)



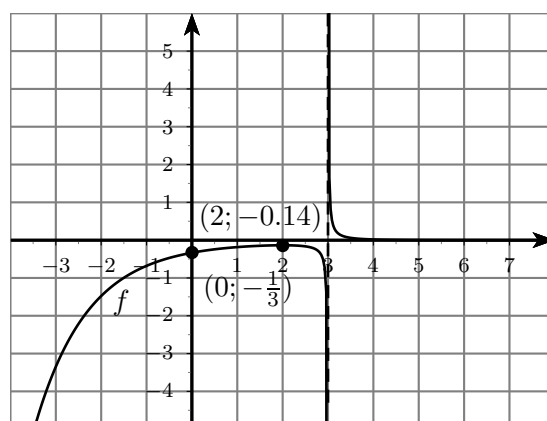
2)



3)



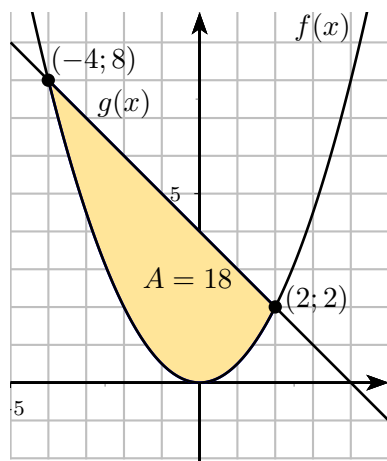
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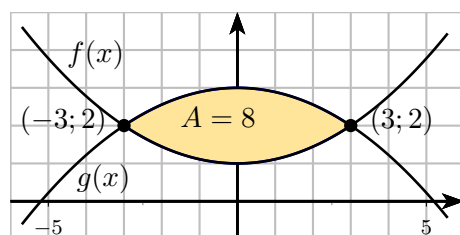
- 1.15** 1) $f(x) = x^4 + c$ 4) $f(x) = 3x^3 - 2x^2 - 5x + c$
 2) $f(x) = x^2 + c$ 5) $f(x) = \frac{2}{3}x^3 - \frac{3}{2}x^2 - 4x + c$
 3) $f(x) = x^{10} - x^8 - x + c$ 6) $f(x) = c + x - \frac{5}{2}x^2 - \frac{7}{4}x^4$
- 1.16** 1) $f(x) = \frac{1}{8}x^4 + \frac{1}{8}x^2 + x - 21$ 2) $f(x) = 5x^3 - 3x^2 + 4x - 6$
- 1.17** 1) $2x^2 + c$ 7) $\frac{1}{18}x^9 + c$ 12) $2x^{\frac{1}{2}} + c$
 2) $x + c$ 8) $-\frac{1}{2}x^{-2} + c$ 13) $\frac{1}{3}e^{3x} + c$
 3) $9x + c$ 9) $2x^3 - x^2 - 5x + c$ 14) $-e^{-x} + c$
 4) $ax + c$ 10) $\frac{1}{4}x^4 - 2x^2 + c$ 15) $-\frac{1}{2}e^{3-2x} + c$
 5) $x^2v + c$ 11) $\frac{2}{3}x^{\frac{3}{2}} + c$ 16) $-\frac{1}{5}\cos(5x) + c$
 6) $\frac{1}{6}t^6 + c$
- 1.18** 1) 39 4) 3 7) 2 10) $\frac{1}{\ln(2)} \cong 1.44$
 2) 5 5) $\frac{27}{4} = 6.75$ 8) 16
 3) 144 6) $\frac{3}{4} = 0.75$ 9) $\frac{3}{4}e \cong 2.04$
- 1.19** 1) $\frac{e^2-1}{e} \cong 2.35$ 4) $\frac{a-1}{\ln(a)}$ 6) $\ln\left(\frac{1}{2}\right) \cong -0.70$
 2) $\frac{531'440}{27\ln(3)} \cong 17'916.2$
 3) 6 5) $\ln\left(\frac{1}{3}\right) \cong -1.10$ 7) 12
- 1.20** 1) $c = -1$
 2) $S(1.85; -2.55)$
 3) $I \cong 1.2$
- 1.21** 1) $A = 10.4$ 3) $A = \frac{7}{4}$
 2) $A \cong 16.24$ 4) $A \cong 26.8$
- 1.22** 1) $A = -\frac{1}{b} + 1$ 2) $b = 2$ 3) $A = 1$
- 1.23** 1) $A = \frac{16}{3}$ 2) $A \cong 4.32$ 3) $A = \frac{71}{9}$

1. 24

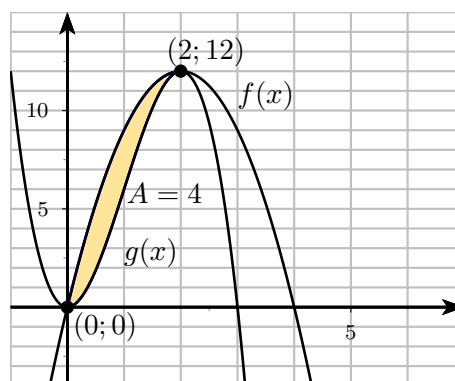
1)



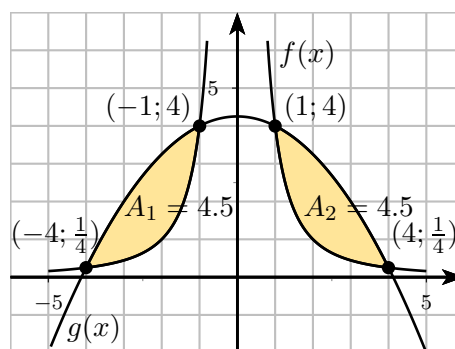
2)



3)



4)

**1. 25**

$$A = 8$$

1. 26

$$A \cong 0.47$$

1. 27

$$1) \quad y = -\frac{5}{2}x + 20$$

$$2) \quad A \cong 25.83$$

1. 28

$$1) \quad F(x) = (x - 1)e^x + c$$

$$2) \quad F(x) = x(\ln(x) - 1) + c$$

$$3) \quad F(x) = (x^2 - 3x + 4)e^x + c$$

$$4) \quad F(x) = 2x \sin(x) - (x^2 - 2) \cos(x) + c$$

$$5) \quad F(x) = \frac{x^2}{4}(2 \ln(x) - 1) + c$$

$$6) \quad F(x) = -\frac{1}{5}(1 - x)^5(x + \frac{1}{6}(1 - x)) + c$$

$$7) \quad F(x) = \frac{1}{9}x^3(3 \ln(x) - 1) + c$$

$$8) \quad F(x) = \frac{1}{2}e^x(\sin(x) - \cos(x)) + c$$

$$9) \quad f(x) = \frac{2}{3}x(x + 1)^{\frac{3}{2}} - \frac{4}{15}(x + 1)^{\frac{5}{2}} + c$$

1. 29

1) $F(x) = \frac{1}{2} \sin(2x + 1) + c$

8) $F(x) = \frac{1}{2} \ln^2(x) + c$

2) $F(x) = \frac{1}{10}(2x + 5)^5 + c$

9) $F(x) = \ln |1 + e^x| + c$

3) $F(x) = \frac{1}{5} \cos(-5x + 3) + c$

10) $F(x) = \ln |x^2 + x + 4| + c$

4) $F(x) = \frac{1}{2} \sin^2(x) + c$

11) $F(x) = -\cos(\sqrt{x}) + c$

5) $F(x) = -0.5e^{-x^2} + c$

12) $F(x) = -\frac{1}{10}(1 - x^2)^5 + c$

6) $F(x) = -\frac{1}{3}e^{-x^3} + c$

13) $F(x) = \frac{1}{2\pi} \sin(\pi x^2) + c$

7) $F(x) = -\ln |1 - 3x| + c$

14) $F(x) = \frac{1}{6}(1 + x^4)^{\frac{3}{2}} + c$

1. 30

1) $F(x) = x^2 + x + 10 \ln |x - 3| + c$

2) $F(x) = \frac{5}{6}x^2 + \frac{17}{9}x - \frac{55}{27} \ln |3x - 1| + c$

1. 31

1) $F(x) = (2x^2 - 7x + 8)e^x + c$

3) $F(x) = (-\frac{5}{2}x^2 + \frac{3}{2}x + \frac{11}{4})e^{2x} + c$

2) $F(x) = (-x^2 - 6x - 3)e^{-x} + c$

4) $F(x) = (-5 \cos(x) - 2 \sin(x))e^{-x}$

1. 32

$A = 2.5$

1. 33

$A = \frac{\pi-1}{2}$

1. 34

$h \cong 4.83$