LDDR - Niveau II : Fonctions

Exercice 1

a) Non car pour 1 x on a 2 images
b) Oui
c) Non car il y a x qui n'a pas d'image

Exercice 2

$f(x) = \dfrac{x+3}{x^2-2}$

a) oui avec $D_f = \mathbb{R} \setminus \{\pm\sqrt{2}\}$, oui,
b) $f(0) = -\dfrac{3}{2}$ $\quad f(-1) = \dfrac{2}{-1} = -2$
c) $f(x) = -4 \Leftrightarrow \dfrac{x+3}{x^2-1} = -4 \Rightarrow x+3 = -4x^2+4 \Leftrightarrow$

$\Leftrightarrow 4x^2+x-1=0 \quad \Delta = 1+8 = 9 \quad x_{1,2} = \dfrac{-1 \pm 3}{8} \begin{cases} -1/2 \\ 1/4 \end{cases}$

Exercice 3

a) $d(1)=1 \quad d(2)=2 \quad d(4)=3 \quad d(24)=8$
b) $d(n)=0$ impossible
$d(n)=1 \Rightarrow n=1$
$d(n)=2 \Rightarrow n=$ nombres premiers
c) $d(n)=$ impair $\Rightarrow n=$ impair, carré parfait
ex $1, 9, 25, 49, \ldots$

Exercice 4

a) $D_f = \mathbb{R}$, $D_g = \mathbb{R}$, $D_h =]1, +\infty[$
$D_i = \mathbb{R} \setminus \{-2\}$ $D_j = \mathbb{R}$ $D_k = \mathbb{R} \setminus \{\pm 3\}$
$D_l = \mathbb{R} \setminus \{1; -2\}$ $D_m = \mathbb{R} \setminus \{\pm 5/4\}$ $D_n =]-\infty; 1/2]$

b) $f(\frac{3}{2}) = -2 \quad f(\frac{3}{2}) = \dfrac{4 \cdot \frac{9}{4} - 9}{(\frac{3}{2}-1)(\frac{3}{2}+2)} = 0$

c) $g(x) = 5 \Rightarrow \frac{1}{2}x^2 + 4x + 8 = 8 \Rightarrow x(\frac{1}{2}x + 4) = 0 \Rightarrow$
$x = 0 \quad x = -8$
$h(x) = 5 \Rightarrow \sqrt{x-1} = 5 \Rightarrow x-1 = 25 \Rightarrow x = 26$
$m(x) = 0 \Leftrightarrow x - 2020 = 0 \Rightarrow x = 2020$

EXERCICE 5 $f(x)=\sin x \quad g(x)=\cos x \quad h(x)=\tan x$

a) $D_f=\mathbb{R} \quad Im f=[-1,1]$
$D_g=\mathbb{R} \quad Im g=[-1,1]$
$D_h=\mathbb{R}\setminus\left\{\frac{\pi}{2}+k\pi\right\} \quad Im h=\mathbb{R}$

b) $f(0)=\sin 0=0 \quad f\left(\frac{\pi}{6}\right)=\frac{1}{2} \quad f\left(-\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}$
$g(0)=1 \quad g\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2} \quad g\left(-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$
$h(0)=0 \quad h\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3} \quad h\left(-\frac{\pi}{4}\right)=-1$

c) $f(x)=1 \Rightarrow \sin x=1 \Rightarrow x=\frac{\pi}{2}+2k\pi$
$g(x)=1 \Rightarrow \cos x=1 \Rightarrow x=2k\pi$
$h(x)=1 \Rightarrow \tan x=1 \Rightarrow x=\pi/4+k\pi$

EXERCICE 6 car il y a de x pour les quels on a plusieurs images

EXERCICE 7 a) $f(-11)=1 \quad f(-7)=-3 \quad f(-4)=4, f(0)=4$
$f(1)=4,5 \quad f(9)=-1$

b) $Im f=[-3;5]$ Zéros: $x=-10,-8;-5.25,8,11.25$

c) $f(x)=5 \Rightarrow x=2$
$f(x)=2 \Rightarrow x=-12;[4;6]$
$f(x)=1 \Rightarrow x=-11;-9;-4,5,7,12$

d) $A=f(]5,7[)=[1,2]$
$B=\{x / f(x)>2\}=[-4;4[$

e)

x		-10		-8		-5.25		-4		8		$11,25$		12
$f(x)$	+	0	+	0	−	0	+	‖	+	0	−	0	+	

EXERCICE 8 a) $f_1(x)=3 \quad D_{f_1}=\mathbb{R}$ zéros $\emptyset$

x	$-\infty$		$+\infty$
f		+	

$f_2(x)=\frac{1}{2}x+1 \quad D_2=\mathbb{R} \quad \frac{1}{2}x+1=0 \Rightarrow x=-2 \quad (-2,0)$

x	$-\infty$		-2		$+\infty$
f		−	0	+	

$f_3(x) = x^2 + x - 2$ $D_f = \mathbb{R}$ $x^2 + x - 2 = 0 \Rightarrow x_1 = -2$, $x_2 = 1$

x	$-\infty$		-2		1		$+\infty$
f		$+$	0	$-$	0	$+$	

$f_4(x) = \frac{x+2}{x-1}$ $D_f = \mathbb{R}\setminus\{1\}$ $f(x) = 0 \Rightarrow x + 2 = 0 \Rightarrow x = -2$

x		-2		1		$+\infty$
f	$+$	0	$-$	$\nexists$	$+$	

b) f_1: $\mathrm{Im} f = \{3\}$ f_2: $\mathrm{Im} f = \mathbb{R}$ f_3: $\mathrm{Im} f = [-\frac{9}{4}, +\infty[$
f_4: $\mathrm{Im} f = \mathbb{R}\setminus\{1\}$

c) $f_1(x) = f_2(x) \Rightarrow \frac{1}{2}x + 1 = 3 \Rightarrow x + 2 = 6 \Rightarrow x = -4$ $I(-4; 3)$

$f_2(x) = f_3(x) \Rightarrow \frac{1}{2}x + 1 = x^2 + x - 2 \Rightarrow x + 2 = 2x^2 + 2x - 4 \Leftrightarrow$
$\Leftrightarrow 2x^2 + x - 6 = 0$ $\Delta = 1 + 48 = 49$ $x_{1,2} = \frac{-1 \pm 7}{4}$ $\begin{cases} x_1 = -2 \\ x_2 = \frac{3}{2} \end{cases}$

$I_1(-2; 0)$ $I_2(\frac{3}{2}; \frac{7}{4})$

$f_2(x) = f_4(x) \Rightarrow \frac{1}{2}x + 1 = \frac{x+2}{x-1} \Rightarrow x + 2 = \frac{2x+4}{x-1} \Leftrightarrow$
$\Leftrightarrow (x+2)(x-1) = 2x + 4 \Rightarrow$
$\Leftrightarrow x^2 + 2x - x - 2 = 2x + 4 \Leftrightarrow$
$\Leftrightarrow x^2 - x - 6 = 0$ $\Delta = 1 + 24 = 25$ $x_{1,2} = \frac{1 \pm 5}{2}$ $\begin{cases} x_1 = 3 \\ x_2 = -2 \end{cases}$

$I_1(3; \frac{5}{2})$ $I_2(-2; 0)$

EXERCIUL 9. $f_1(x) = \frac{x^4 - 11x^2 + 18}{x^7 - x + 2015} = 0 \Rightarrow x^4 - 11x^2 + 18 = 0$

$x^2 = t$ $t^2 - 11t + 18 = 0$ $\Delta = 121 - 72 = 49$

$t_{1,2} = \frac{11 \pm 7}{2}$ $\begin{cases} t_1 = 9 \\ t_2 = 2 \end{cases}$ $x^2 = 9 \Rightarrow x = \pm 3$ $x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

- $f_2(x) = \tan(x - \frac{\pi}{4})$
$\cos(x - \frac{\pi}{4}) \neq 0 \Rightarrow x - \frac{\pi}{4} \neq \frac{\pi}{2} + k\pi \Rightarrow x \neq \frac{3\pi}{4} + k\pi$
$f_2(x) = 0 \Rightarrow \sin(x - \frac{\pi}{4}) = 0 \Rightarrow x - \frac{\pi}{4} = 2k\pi \Rightarrow x = \frac{\pi}{4} + 2k\pi$
- $f_3(x) = \sqrt{4x+1} - x + 1$ $D_f = [-\frac{1}{4}, +\infty[$
$f_3(x) = 0 \Rightarrow \sqrt{4x+1} = x - 1 \Rightarrow 4x + 1 = (x-1)^2 \Rightarrow$
$4x + 1 = x^2 - 2x + 1 \Rightarrow x^2 - 6x = 0 \Rightarrow x(x-6) = 0$
$x = 0$ $x = 6$

$f_4(x)=0 \Rightarrow 5\cos x - 2\sin^2 x - 1 = 0 \Leftrightarrow$

$-2(1-\cos^2 x)+5\cos x - 1 = 0 \Leftrightarrow 2\cos^2 x + 5\cos x - 3 = 0$

$\Delta = 25+24 = 49 \quad \cos x = \dfrac{-5 \pm 7}{4} \begin{cases} -3 & \text{impossible} \\ \frac{1}{2} & \end{cases}$

$\cos x = \dfrac{1}{2} \Rightarrow \begin{cases} x = \frac{\pi}{3} + 2k\pi \\ x = -\frac{\pi}{3} + 2k\pi \end{cases}$

EXERCICE 10

- $f(x) = \sqrt{1-x^2} \quad 1-x^2 \geq 0$

x	$-\infty$	-1		1	$+\infty$
$1-x^2$		$-$	0	$+$	0	$-$

$D_f = [-1, 1]$

- $g(x) = \sqrt{x^2-1}$

$D_g =]-\infty, -1] \cup [1, +\infty[$

EXERCICE 11

a) $\dfrac{-2x+3}{5} \leq 1 \Rightarrow -2x+3 \leq 5 \Rightarrow -2 \leq 2x \Rightarrow$

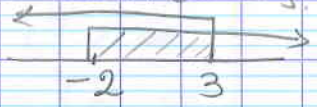
$x \geq -1 \quad x \in [-1, +\infty[$

b) $E = \{x \in \mathbb{R} : x+2 > 0 \text{ et } -2x+6 > 0\}$

$x+2 > 0 \Rightarrow x > -2$

$-2x+6 > 0 \Rightarrow 6 > 2x \Rightarrow x < 3$

$x \in]-2; 3[$



EXERCICE 12

a) $\dfrac{4x^3 - 2x^7}{3x+5} < 0 \Leftrightarrow \dfrac{2x^7(2x-1)}{3x+5} < 0$

x	$-\infty$		$-\frac{5}{3}$		0		$\frac{1}{2}$		$+\infty$
x^7		$-$		$-$	0	$+$		$+$	
$2x-1$		$-$		$-$		$-$	0	$+$	
$3x+5$		$-$	0	$+$		$+$		$+$	
Q		$-$	$\nexists$	$+$	0	$-$	0	$+$	

$x \in]-\infty; -\frac{5}{3}[\cup]0; \frac{1}{2}[$

b) $\frac{(3x+12)(x+2)}{x^2-6x+9} \leq 0 \Leftrightarrow \frac{3(x+4)(x+2)}{(x-3)^2} \leq 0$

	$-\infty$		-4		-2		3		$+\infty$
$x+4$		$-$	0	$+$		$+$		$+$	
$x+2$		$-$		$-$	0	$+$		$+$	
$x-3$		$-$		$-$		$-$	0	$+$	
Q		$-$	0	$+$	0	$-$	$\nexists$	$+$	

$x \in]-\infty, -4] \cup [-2; 3[$

c) $\frac{x+2}{x-2} > \frac{x-2}{x+2} \Leftrightarrow \frac{(x+2)^2-(x-2)^2}{(x+2)(x-2)} > 0 \Leftrightarrow$

$\frac{(x+2+x-2)(x+2-x+2)}{(x+2)(x-2)} > 0 \Leftrightarrow$

$\Leftrightarrow \frac{2x \cdot 4}{(x+2)(x-2)} > 0 \Leftrightarrow \frac{8x}{(x+2)(x-2)} > 0$

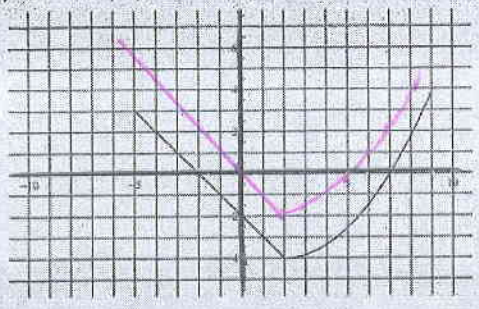
	$-\infty$		-2		0		$+2$		$+\infty$
x		$-$		$-$	0	$+$		$+$	
$x+2$		$-$	0	$+$		$+$		$+$	
$x-2$		$-$		$-$		$-$	0	$+$	
Q		$-$	$\nexists$	$+$	0	$-$	$\nexists$	$+$	

EXERCICE #4

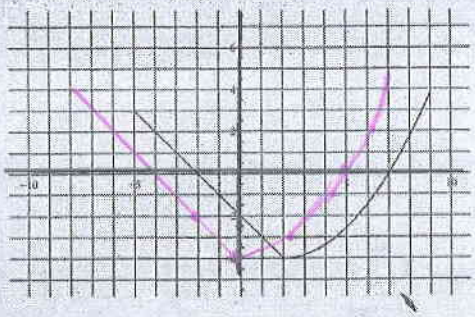
Exercice 14

Pour la fonction f dont le graphique est représenté ci-dessous, dessiner les fonctions suivantes. Remarques ?

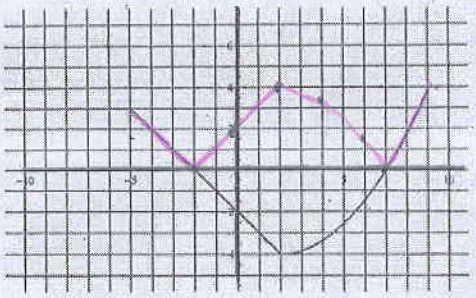
a) g : $g(x) = f(x) + 2$



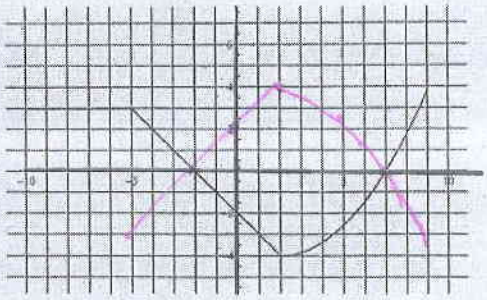
b) h : $h(x) = f(x + 2)$



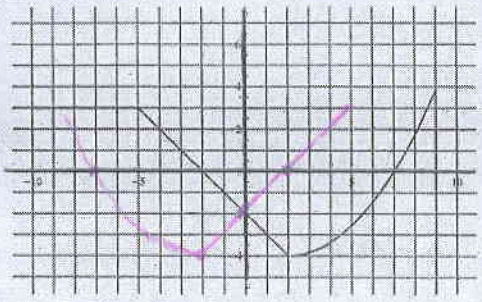
c) i : $i(x) = |f(x)|$



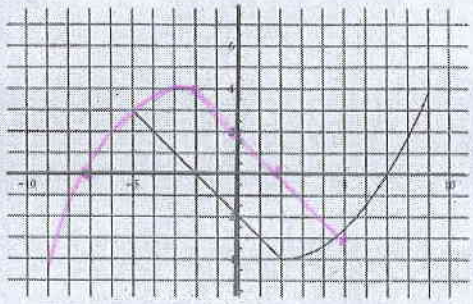
d) j : $j(x) = -f(x)$



e) k : $k(x) = f(-x)$

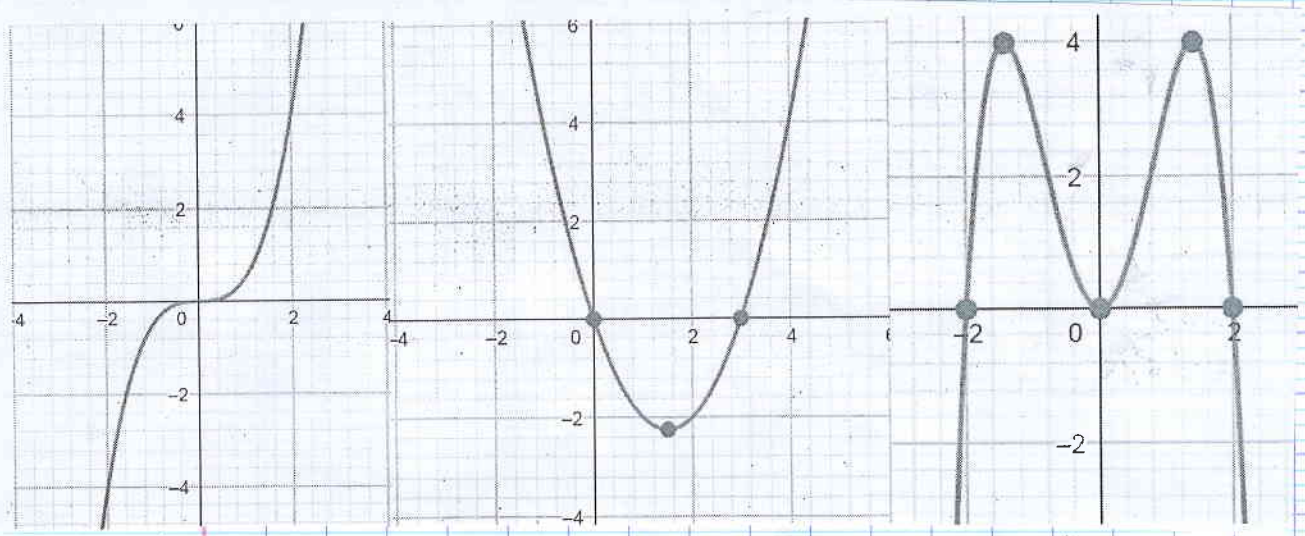


f) l : $l(x) = -f(-x)$



EXERCICE 15

- $f(x) = \frac{1}{2}x^3$ $D_f = \mathbb{R}$
  $f(-x) = \frac{1}{2}(-x)^3 = -\frac{1}{2}x^3 = -f(x)$ impaire
- $g(x) = x^2 - 3x$ $D_f = \mathbb{R}$
  $g(-x) = (-x)^2 - 3(-x) = x^2 + 3x$ ni-ni
- $h(x) = x^2(4 - x^2)$ $D_f = \mathbb{R}$
  $h(-x) = (-x)^2(4 - (-x)^2) = x^2(4 - x^2) = f(x)$ paire



f g h

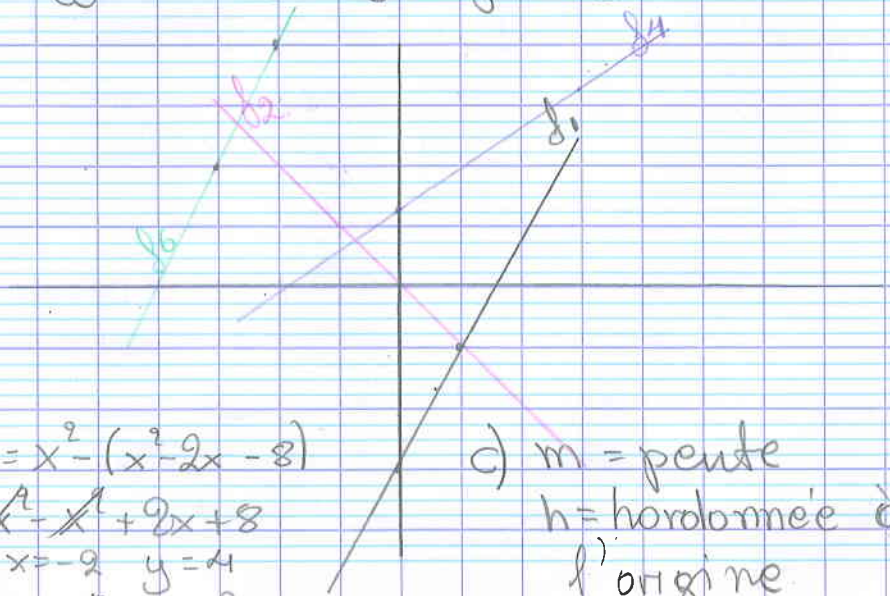
EXERCICE 16

$f(x) = \sin x$ $f(-x) = \sin(-x) = -\sin x = -f(x)$ impaire
$g(x) = \cos x$ $g(-x) = \cos(-x) = \cos x = g(x)$ paire
$h(x) = \tan x$ $h(-x) = \tan(-x) = -\tan x = -h(x)$ impaire

EXERCICE 17

a) Affines: f_1, f_4, f_6. f_2 (linéaire)

b)



$f_6(x) = x^2 - (x^2 - 2x - 8)$
$= \cancel{x^2} - \cancel{x^2} + 2x + 8$
$x = -2 \quad y = 4$
$x = -3 \quad y = 2$

c) m = pente
h = hordonnée à l'origine

EXERCICE 18 x km/h y m/s

$\frac{km}{h} = \frac{1000}{3600} = \frac{10}{36} = \frac{5}{18} \frac{m}{s} \Rightarrow y = \frac{5}{18}$ m/s.

EXERCICE 19 $f(x) = \frac{2x+2}{3}$ $g(x) = \frac{5}{4}x - \frac{5}{2}$

- $f(x) = g(x) \Rightarrow \frac{2x+2}{3} = \frac{5}{4}x - \frac{5}{2} \Leftrightarrow$

$\Leftrightarrow 4(2x+3) = 15x - 30 \Leftrightarrow 8x + 12 = 15x - 30 \Rightarrow$

$\Leftrightarrow 7x = 42 \Rightarrow x = 6 \quad y = \frac{14}{3} \quad I\left(6; \frac{14}{3}\right)$

- $\tan\varphi = \frac{2}{3} \Rightarrow \varphi = 33.7°$
- $\tan\alpha = \frac{5}{4} \Rightarrow \alpha = 51.34°$

$\left.\begin{matrix}\\\\\end{matrix}\right\} \alpha - \varphi = 17.64°$

EXERCICE 20 a) $y = 2x + h \quad A(2;3) \quad 3 = 4 + h \Rightarrow h = -1$

$y = 2x - 1$

b) $y = mx \quad P(-2;5) \quad 5 = -2m \Rightarrow m = -\frac{5}{2}$

$y = -\frac{5}{2}x$

c) $A(-4;1) \quad B(2;-2) \quad m = \frac{1+2}{-4-2} = \frac{3}{-6} = -\frac{1}{2}$

$y = -\frac{1}{2}x + h \quad A: 1 = 2 + h \Rightarrow h = -1$

$y = -\frac{1}{2}x - 1$

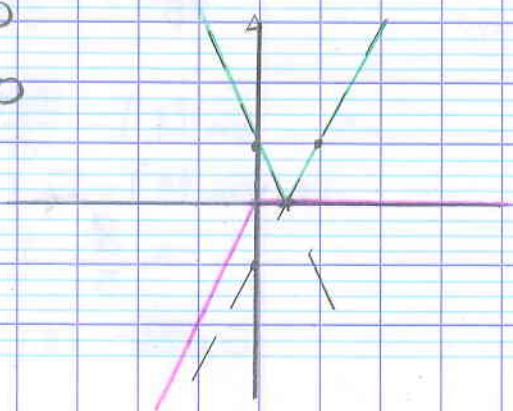
d) $f(3) = -2 \quad f(-5) = -2 \quad m = \frac{-2+2}{3+5} = 0$

$y = -2$

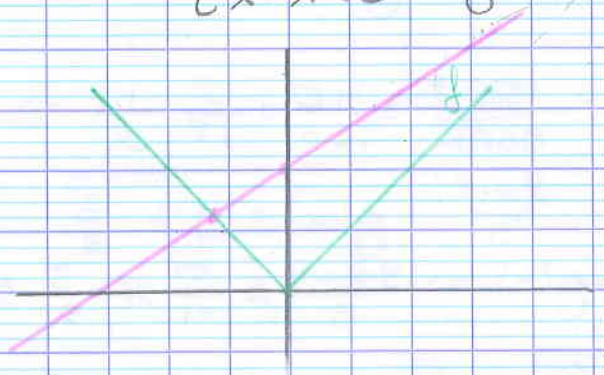
EXERCICE 21 a) $f(x) = |2x - 1| = \begin{cases} 2x - 1 & x \geq \frac{1}{2} \\ 1 - 2x & x < \frac{1}{2} \end{cases}$

$g(x) = x - |x| = \begin{cases} x - x = 0 & x \geq 0 \\ x + x = 2x & x < 0 \end{cases}$

$I\left(\frac{1}{2}; 0\right)$



b) $f(x)=|x|=\begin{cases} x & x\geq 0 \\ -x & x<0 \end{cases}$ $g(x)=\frac{2}{3}x+2$



$x\geq 0$: $x=\frac{2}{3}x+2 \Rightarrow 3x=2x+6 \Rightarrow x=6$ $I_1(6;6)$ $y=6$

$x<0$ $-x=\frac{2}{3}x+2 \Rightarrow -3x=2x+6 \Rightarrow 5x=-6 \Rightarrow$

$x=-\frac{6}{5}$ $y=\frac{6}{5}$ $I_2\left(-\frac{6}{5};\frac{6}{5}\right)$

EXERCICE 22 $f_1(x)=2x-3$ $f_2(x)=-x$ $f_4(x)=\frac{2}{3}x+\frac{4}{3}$ $f_6(x)=2x+8$

- $f_1 \cap f_2$ $2x-3=-x \Rightarrow 3x=3 \Leftrightarrow x=1$ $I(1;-1)$
- $f_1 \cap f_4$ $2x-3=\frac{2}{3}x+\frac{4}{3} \Rightarrow 6x-9=2x+4 \Rightarrow$

  $4x=13 \Rightarrow x=13/4$ $I\left(\frac{13}{4};\frac{7}{2}\right)$
- $f_1 \cap f_6$ $2x-3=2x+8 \Rightarrow -3=8$ imp $\Rightarrow f_1 \parallel f_6$
- $f_2 \cap f_4$ $-x=\frac{2}{3}x+\frac{4}{3} \Rightarrow -3x=2x+4 \Rightarrow 5x=-4 \Rightarrow$

  $x=-4/5$ $I\left(-\frac{4}{5};\frac{4}{5}\right)$
- $f_2 \cap f_6$ $-x=2x+8 \Rightarrow 3x=-8 \Rightarrow x=-8/3$ $I\left(-\frac{8}{3};\frac{8}{3}\right)$
- $f_4 \cap f_6$ $\frac{2}{3}x+\frac{4}{3}=2x+8 \Rightarrow 2x+4=6x+24 \Rightarrow$

  $\Leftrightarrow 4x=-20 \Rightarrow x=-5$ $I(-5;-2)$

EXERCICE 23 $v_A=\frac{500\,\text{m}}{60\,\text{min}}=\frac{250}{3}\,\text{m/min}$

$v_B=\frac{1500\,\text{m}}{60\,\text{min}}=750/3\,\text{m/min}$

$S_A(t)=\frac{250}{3}\cdot t$ m $S_B(t)=\frac{750}{3}(t-15)$ m

$S_A(t)=S_B(t) \Rightarrow \frac{250}{3}t=\frac{750}{3}(t-15) \Leftrightarrow$

$\Rightarrow \frac{1}{3}t=t-15 \Rightarrow \frac{2}{3}t=15 \Rightarrow t=22{,}5$.

$S_A(22{,}5)=1875$ m.

EXERCICE 24 $f(x) = -x^2 - 1 \quad g(x) = \frac{1}{2}x^2 + 2x + 2 \quad h(x) = 2x^2 + 3x - 5$

$f(x)$: $S(0; -1)$. $f(x) = 0 \Rightarrow x^2 = -1$ imp

$g(x)$: $x_S = -\frac{2}{2 \cdot \frac{1}{2}} = -2 \quad y_S = \frac{1}{2} \cdot 4 - 4 + 2 = 0 \quad S(-2, 0)$

$g(x) = 0 \Rightarrow \frac{1}{2}x^2 + 2x + 2 = 0 \Leftrightarrow x^2 + 4x + 4 = 0 \Rightarrow$

$(x+2)^2 = 0 \Rightarrow x = -2 \quad (-2; 0)$

$h(x)$: $x_S = \frac{-3}{4} \quad y_S = 2 \cdot \frac{9}{16} - \frac{9}{4} - 5 = \frac{9}{8} - \frac{18}{8} - \frac{40}{8} =$

$= -\frac{49}{8} \quad S\left(-\frac{3}{4}; -\frac{49}{8}\right)$

$h(x) = 0 \Rightarrow 2x^2 + 3x - 5 = 0 \quad \Delta = 9 + 40 = 49$

$x_{1,2} = \frac{-3 \pm 7}{4} \quad \begin{cases} -10/4 = -5/2 \\ 4/4 = 1 \end{cases}$

EXERCICE 25 $y = ax^2 + bx + c$

$x_S = -\frac{b}{2a} \quad y_S = a\left(\frac{b^2}{4a^2}\right) + b \cdot \left(-\frac{b}{2a}\right) + c =$

$= \frac{b^2}{4a} - \frac{b^2}{2a} + c = \frac{b^2}{4a} - \frac{2b^2}{4a} + \frac{4ac}{4a} =$

$= -\frac{b^2}{4a} + \frac{4ac}{4a} = \frac{-b^2 + 4ac}{4a}$

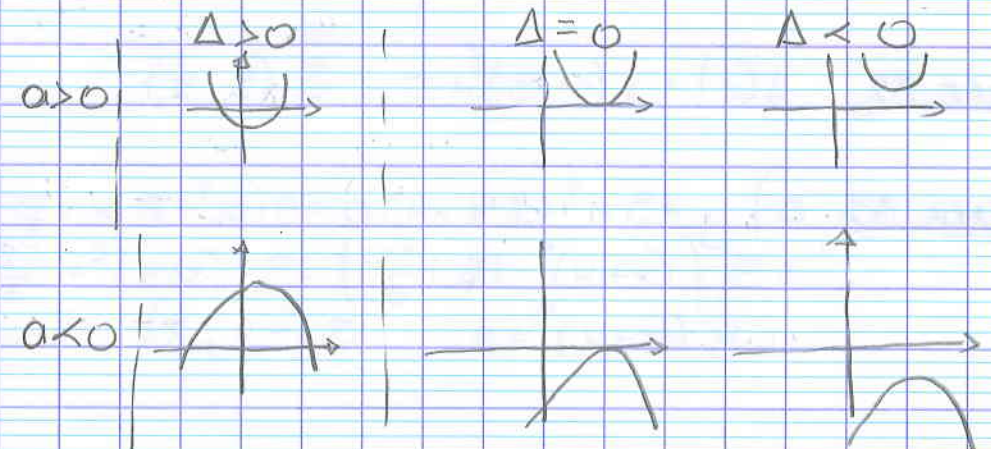
$S\left(-\frac{b}{2a}; \frac{-b^2 + 4ac}{4a}\right)$

EXERCICE 26 Si $a > 0$ convexe $a < 0$ concave

Si $b = 0$ sommet $\in Oy$

Si $c = 0$ $x = 0$

EXERCICE 27



EXERCICE 28 $f(x) = x^2 + x - 6 = x^2 - 3x + 2x - 6 = x(x-3) + 2(x-3)$

$= (x-3)(x+2) = 0 \Rightarrow x = 3 \quad x = -2$

$g(x) = \frac{1}{4}x^2 - 2x + 4 = \frac{1}{4}(x^2 - 8x + 16) = \frac{1}{4}(x-4)^2 = 0$

$\Rightarrow x = 4$

EXERCICE 29 $f(x) = x^2 - (m+1)x + m = 0$

$\Delta = (m+1)^2 - 4m = m^2 + 2m + 1 - 4m =$

$= (m^2 - 2m + 1 = (m-1)^2$

$x_{1;2} = \frac{m+1 \pm (m-1)}{2}$ $\begin{cases} \frac{m+1+m-1}{2} = m \\ \frac{m+1-m+1}{2} = 1 \end{cases}$

$x_1 = m \quad x_2 = 1$

EXERCICE 30 $f(x) = (x+2)(x-5) = x^2 - 3x - 10$.

EXERCICE 31 $f(x) = \frac{1}{\sqrt{4x - x^2}}$ $-x^2 + 4x > 0 \Rightarrow x(-x+4) > 0$

	0		4	
	− ∅	+	∅	−

$D_f =]0, 4[$

$g(x) = \sqrt{9x^2 - 6x + 1} = \sqrt{(3x-1)^2}$ $(3x-1)^2 \geq 0$

$D_g = \mathbb{R}$

$h(x) = \sqrt{x^2 - 4x + 3}$ $x^2 - 4x + 3 \geq 0$

	$-\infty$		1		3		$+\infty$
$x^2 - 4x + 3$		+	∅	−	∅	+	

$\Delta = 16 - 12 = 4 \quad x_{1;2} = \frac{4 \pm 2}{2} \begin{cases} 3 \\ 1 \end{cases}$

$D_h =]-\infty, 1] \cup [3, +\infty[$

EXERCICE 32 $f(x) = 2(x-3)^2 + 5 \quad S(3; 5)$

EXERCICE 33 a) $y = 3x^2 + 24x + 50 = 3\left(x^2 + 8x + \frac{50}{3}\right) =$

$= 3\left((x+4)^2 - 16 + \frac{50}{3}\right) = 3\left[(x+4)^2 + \frac{2}{3}\right] =$

$= 3(x+4)^2 + 2 \quad S(-4; 2)$

b) $y = -x^2 - 2x + 8 = -(x^2 + 2x - 4) =$
$= -((x+1)^2 - 1 - 4) = -(x+1)^2 + 3 \quad S(-1; 3)$

c) $y = ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) =$

$= a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right] = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{4ac}{4a^2}\right] =$

$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + \frac{4ac}{4a} = a\left(x + \frac{b}{2a}\right)^2 + \frac{-b^2+4ac}{4a}$

$S\left(-\frac{b}{2a}; \frac{-b^2+4ac}{4a}\right)$

EXERCICE 34 a) $f(x) = ax^2 + bx + c \quad A(1;9) \quad B(-2;-6) \quad C(-5;-3)$

$A: 9 = a + b + c$
$B: -6 = 4a - 2b + c$
$C: -3 = 25a - 5b + c$

$\left.\begin{array}{l} \\ \\ \end{array}\right\} \Leftrightarrow \left.\begin{array}{l} 15 = -3a + 3b \\ -3 = -21a + 3b \end{array}\right\} (-)$

$18 = 18a \Rightarrow a = 1$

$15 = -3 + 3b \Rightarrow 3b = 18 \Rightarrow b = 6$

$a = 1 \quad b = 6 \quad 9 = 1 + 6 + c \Rightarrow c = 2$

$f(x) = x^2 + 6x + 2$

b) $S(3; \frac{5}{2}) \quad A(2; -1)$

$y = a(x-3)^2 + \frac{5}{2} \quad A: -1 = a(2-3)^2 + \frac{5}{2} \Leftrightarrow$

$\Leftrightarrow -1 = a + \frac{5}{2} \Rightarrow a = -1 - \frac{5}{2} \Rightarrow a = -\frac{7}{2}$

$y = -\frac{7}{2}(x-3)^2 + \frac{5}{2}$

EXERCICE 35 a) $f(x) = -x^2 + 2x + 8 \quad g_1(x) = x + 6 \quad g_2(x) = x + 9$
$g_3(x) = -4x + 17$

· $f \cap g_1 \quad -x^2 + 2x + 8 = x + 6 \Rightarrow x^2 - x - 2 = 0$

$x_1 = 2 \quad y_1 = 8 \quad x_2 = -1 \quad y_2 = 5$

· $f \cap g_2 \quad -x^2 + 2x + 8 = x + 9 \Rightarrow x^2 - x + 1$

$\Delta = 1 - 4 = -3 < 0$ | impossible

· $f \cap g_3 : -x^2 + 2x + 8 = -4x + 17 \Rightarrow x^2 - 6x + 9 = 0 \Rightarrow$

$(x-3)^2 = 0 \Rightarrow x = 3 \quad y = 5$

b) $f(x) < g_1(x) \Rightarrow -x^2+2x+8 < x+6 \Rightarrow x^2 - x - 2 > 0$

	-1		2	
	$+$	0	$-$	0

$x \in]-\infty, -1[\cup]2, +\infty[$

EXERCICE 36

$y = 4x+h \quad f(x) = x^2-1$

$x^2-1 = 4x+h \Rightarrow x^2-4x-h-1=0$

$\Delta = 0 \Rightarrow 16 - 4(-h-1) = 0 \Rightarrow 16+4h+4=0$

$\Rightarrow 4h = -20 \Rightarrow h = -5$

$y = 4x-5$

EXERCICE 37

a) $S(2;0) \quad A(0;3)$

$y = a(x-2)^2 \quad A: 3 = a(-2)^2 \Rightarrow 3 = 4a \Rightarrow a = \frac{3}{4}$

$y = \frac{3}{4}(x-2)^2$

b) $S(-4;3) \quad A(-2;-1)$

$y = a(x+4)^2+3 \quad A: -1 = a(-2+4)^2+3 \Leftrightarrow$

$\Rightarrow -1 = 4a+3 \Rightarrow 4a = -4 \Rightarrow a = -1$

$y = -(x+4)^2+3$

EXERCICE 38

$f(x) = -2x^2+2x-1 \quad x_S = \frac{-2}{-4} = \frac{1}{2}$

$y_S = -2\frac{1}{4} + 2\frac{1}{2} - 1 = -\frac{1}{2} \quad S(\frac{1}{2}; -\frac{1}{2})$

$Im f =]-\infty; -\frac{1}{2}]$

EXERCICE 39

$d(u) = 0.006u^2 + 0.2u \leq 11.4 \Leftrightarrow$

$\Leftrightarrow 0.006u^2 + 0.2u - 11.4 \leq 0$

$\Delta = 0.3136 \quad u_{1,2} = \frac{-0.2 \pm 0.56}{0.012} \begin{cases} 30 \\ -633 \end{cases}$

	-633		30	
	$+$	0	$-$	0

$v \in [0, 30[$

EXERCICE 40

- $f(x) = x^2 + 3x - k$

$\Delta = 9 + 4k = 0 \Rightarrow 4k = -9 \Rightarrow k = -\frac{9}{4}$

- $f(x) = 2x^2 - kx + 2$

$\Delta = k^2 - 16 = 0 \Rightarrow k = \pm 4$

- $f(x) = x^2 - (k+6)x + k + 9$

$\Delta = (k+6)^2 - 4(k+9) = 0 \Rightarrow$

$\Rightarrow k^2 + 12k + 36 - 4k - 36 = 0 \Leftrightarrow$

$\Leftrightarrow k^2 + 8k = 0 \Leftrightarrow k(k+8) = 0 \Rightarrow \begin{matrix} k=0 \\ k=-8 \end{matrix}$

EXERCICE 41

$f(x) = 2x^2 - 4x + 3$
$g(x) = 3x^2 - 12x + 18$
$\Rightarrow 2x^2 - 4x + 3 = 3x^2 - 12x + 18$
$\Leftrightarrow x^2 - 8x + 15 = 0$

$\Delta = 64 - 60 = 4 \quad x_{1,2} = \frac{8 \pm 2}{2} \begin{matrix} x_1 = 5 \\ x_2 = 3 \end{matrix}$

$I_1(3;9) \quad I_2(5;33)$

EXERCICE 42

$f_1(x) = x^3 = 0 \Rightarrow x = 0$

$f_2(x) = -x^3 + 4x = 0 \Rightarrow x(-x^2 + 4) = 0 \Rightarrow \begin{matrix} x = 0 \\ x = \pm 2 \end{matrix}$

$f_3(x) = \frac{1}{5}x^4 - \frac{10}{3}x^2 + 3 = 0 \Rightarrow x^4 - 10x^2 + 9 = 0$

$x^2 = t \quad t^2 - 10t + 9 = 0 \quad \Delta = 100 - 36 = 64$

$t_{1,2} = \frac{10 \pm 8}{2} \begin{matrix} t_1 = 9 = x^2 \Rightarrow x = \pm 3 \\ t_2 = 1 = x^2 \Rightarrow x = \pm 1 \end{matrix}$

$f_4(x) = \frac{1}{4}x^3 - \frac{1}{2}x^2 - 2x = 0 \Rightarrow x^3 - 2x^2 - 8x = 0$

$\Rightarrow x(x^2 - 2x - 8) = 0 \quad \Delta = 4 + 32 = 36$

$x = 0 \quad x_1 = 4 ; x_2 = -2 \quad x_{1,2} = \frac{2 \pm 6}{2} \begin{matrix} x_1 = 4 \\ x_2 = -2 \end{matrix}$

EXERCICE 43 Vrai Théorème: Soit f fonction continue dans l'ensemble $[a,b]$ avec $f(a)f(b) < 0$

$\Rightarrow \exists\, x \in]a,b[: f(x) = 0$

Si f est de degré impair $\Rightarrow \exists\, a, b : f(a)f(b) <$

EXERCICE 44 a) $f(x) = (x-1)(x-2)(x-3)$

b) $f(x) = (x-1)^3(x-2)(x-3)$

EXERCICE 45 $x^3 - 7x - 6 \geq 0$

r.p. $\pm 1, \pm 2, \pm 3, \pm 6$

$x=1$ $1 - 7 - 6 \neq 0$ $x=-1$ $-1+7-6=0$

	1	0	-7	-6	-1
↓		-1	1	6	
	1	-1	-6	0	

$x^3 - 7x - 6 = (x+1)(x^2 - x - 6) \geq 0$

$(x+1)(x-3)(x+2) \geq 0$

	$-\infty$		-2		-1		3		$+\infty$
$x+1$		−		−	0	+		+	
$x-3$		−		−		−	0	+	
$x+2$		−	0	+		+		+	
f		−	0	+	0	−	0	+	

$x \in [-2; -1] \cup [3, +\infty[$

EXERCICE 46 $f(x) = \frac{1}{3}x^3$ $g(x) = \frac{13}{3}x - 4$

$f(x) = g(x) \Rightarrow \frac{1}{3}x^3 = \frac{13}{3}x - 4 \Leftrightarrow x^3 = 13x - 12$

$\Leftrightarrow x^3 - 13x + 12 = 0$ r.p. $\pm 1, \pm 2, \pm 3, \pm 4$

$x=-4$

	1	0	-13	12	-4
↓		-4	16	-12	
	1	-4	3	0	

$x^3 - 13x + 12 = 0 \Rightarrow (x+4)(x^2 - 4x + 3) = 0$

$(x+4)(x-3)(x-1) = 0$ $x = -4, x = 3, x = 1$

EXERCICE 47 a) Par Horner

b) $11^{10}-1 = (11-1)(11^9+11^8+\ \ +11+1) \Rightarrow 10 \mid 11^{10}-1$

c) $(1+2+2^2+\ \ +2^9)(2-1) = 2^{10}-1 \Rightarrow$

$\Rightarrow (1+2+\ \ +2^9) = 2^{10}-1$

EXERCICE 48 $f(x)=3x^2-7 \quad g(x)=2x-1 \quad h(x)=3x \quad i(x)=\frac{1}{x} \quad j(x)=\sqrt{x}$

$(f\circ g)(-2) = f(g(-2)) = f(-5) = 68$

$(g\circ f)(-2) = g(f(-2)) = g(5) = 9$

$(g\circ f)(x) = g(f(x)) = 2(3x^2-7)-1 = 6x^2-15$

$(f\circ g)(x) = f(g(x)) = 3(2x-1)^2-7$

$(g\circ h)(x) = 2(3x)-1 = 6x-1$

$(h\circ g)(x) = 3(2x-1) = 6x-3$

$(g\circ g)(x) = 2(2x-1) = 4x-2$

$(h\circ h)(x) = 3(3x) = 9x$

$(i\circ i)(x) = \frac{1}{\frac{1}{x}} = x \quad (h\circ i)(x) = 3\frac{1}{x}$

$(j\circ h)(x) = \frac{1}{3x} \quad (j\circ g\circ g)(x) = \frac{1}{\sqrt{2x-1}}$

EXERCICE 49 $(f\circ g\circ h)(x) = \frac{7}{x^3+1}$

$f(x)=7x \quad g(x)=\frac{1}{x} \quad h(x)=x^3+1$

EXERCICE 50

- $f(x) = -2x+3 \quad \mathbb{R}\to\mathbb{R} \quad y=-2x+3 \Rightarrow x=-\frac{y-3}{2}$

  $f^{-1}(x) = \frac{3-x}{2}$

- $h(x)=\frac{1}{x} \quad \mathbb{R}^*\to\mathbb{R}^* \quad h^{-1}(x)=\frac{1}{x}$

- $f(x)=x^2-4x+8 \quad x_S=\frac{4}{2}=2 \quad y_S=4$

  $[2,+\infty[\to [4,+\infty[$

  $y=x^2-4x+8 \Rightarrow x^2-4x+8-y=0 \quad \Delta = 16-4(8-y) =$

  $= -16+4y$

  $x_{1,2} = \frac{4\pm\sqrt{4y-16}}{2} \Rightarrow f^{-1}(x) = \frac{4+\sqrt{4x-16}}{2}$

- $g(x) = \sqrt{3x}$ $\quad [0, +\infty[\to [0, +\infty[$

$y^2 = 3x \Rightarrow x = \frac{y^2}{3}$ $\quad g^{-1}(x) = \frac{x^2}{3}$

- $k(x) = x^2 - 2x$ $\quad x_S = \frac{2}{2} = 1$ $\quad y_S = -1$

$[1, +\infty[\to [-1, +\infty[$

$y = x^2 - 2x \Rightarrow x^2 - 2x - y = 0$

$\Delta = 4 + 4y$ $\quad x = \frac{2 + \sqrt{4+4y}}{2} = \frac{2 + 2\sqrt{1+y}}{2} = 1 + \sqrt{1+y}$

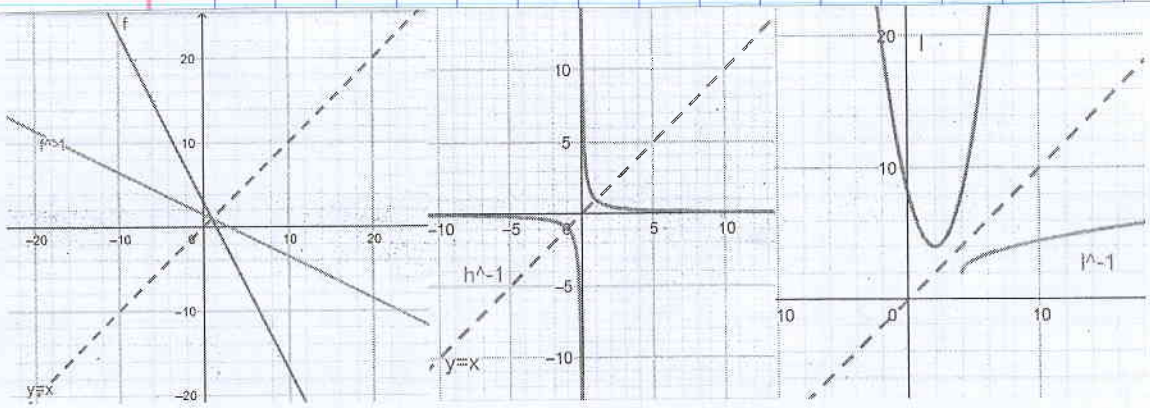
$k^{-1}(x) = 1 + \sqrt{1+x}$

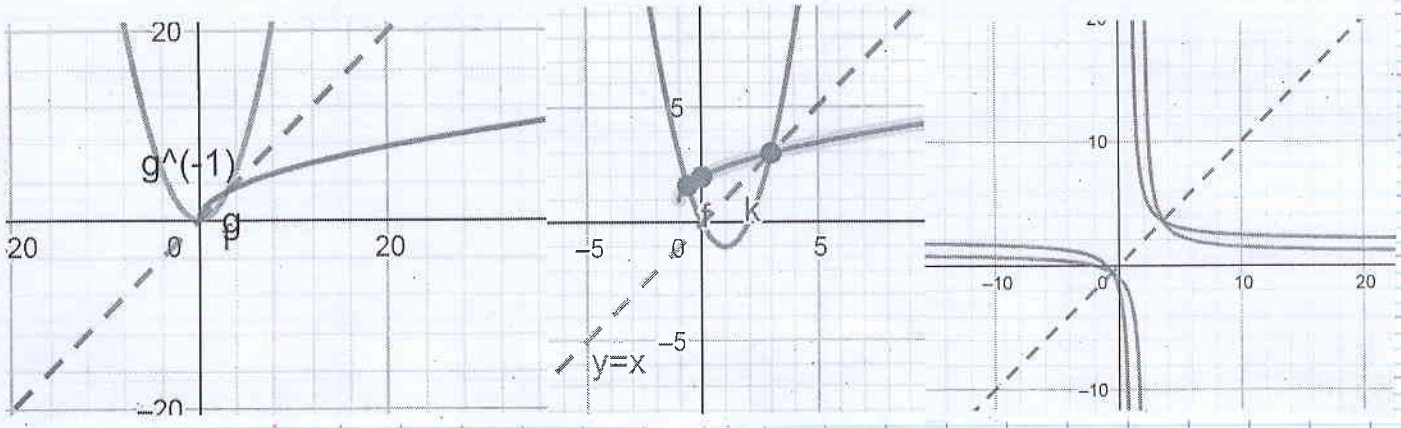
- $m(x) = \frac{x+2}{x-5}$ $\quad \mathbb{R} \setminus \{5\} \to \mathbb{R} \setminus \{1\}$

$y = \frac{x+2}{x-2} \Rightarrow yx - 2y = x + 2 \Rightarrow$

$\Rightarrow (y-1)x = 2y + 2 \Rightarrow x = \frac{2(y+1)}{y-1}$

$m^{-1}(x) = \frac{2(x+1)}{y-1}$





$(f \circ f^{-1})(x) = x = f^{-1} \circ f$ etc

EXERCICE 51 $f(x) = \frac{x+1}{2x+1}$ $D_f = \mathbb{R} \setminus \{-\frac{1}{2}\} \to \mathbb{R} \setminus \{\frac{1}{2}\}$

$y = \frac{x+1}{2x+1} \Rightarrow 2yx + y = x + 1 \Leftrightarrow (2y-1)x = 1 - y \Rightarrow$

$\Leftrightarrow x = \frac{1-y}{2y-1}$ $f^{-1}(x) = \frac{1-x}{2x-1}$

$g(x) = \frac{x+1}{2x-1}$ $D_g = \mathbb{R} \setminus \{\frac{1}{2}\} \to \mathbb{R} \setminus \{\frac{1}{2}\}$

$y = \frac{x+1}{2x-1} \Rightarrow 2yx - y = x + 1 \Rightarrow (2y-1)x = y + 1 \Rightarrow$

$x = \frac{y+1}{2y-1}$ $g^{-1}(x) = \frac{x+1}{2x-1}$

EXERCICE 52 $f(x) = \frac{x+1}{x-2}$

1) $D_f = \mathbb{R} \setminus \{2\}$ $\lim\limits_{x \to 2^+} f(x) = +\infty$ $\lim\limits_{x \to 2^-} f(x) = -\infty$

2) $\cap Oy$ $x = 0$ $f(0) = -\frac{1}{2}$ $(0, -\frac{1}{2})$

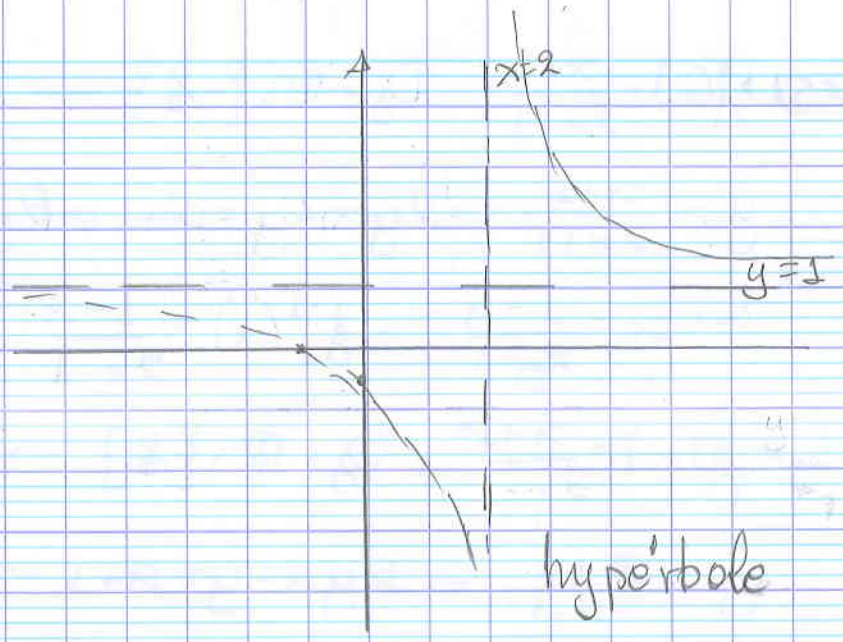
3) $f(x) = 0 \Rightarrow x = -1$ $(-1; 0)$

x	$-\infty$		-1		2	$+\infty$
$x+1$		$-$	0	$+$	$\|$	$+$
$x-2$		$-$	$\|$	$-$	0	$+$
f		$+$	0	$-$	$\|\|$	$+$

4) $y = \frac{x+1}{x-2} \Rightarrow yx - 2y = x + 1 \Rightarrow (y-1)x = 2y + 1 \Rightarrow$

$x = \frac{2y+1}{y-1}$ $f^{-1}(x) = \frac{2x+1}{x-1}$ $\text{Im} f = \mathbb{R} \setminus \{1\}$

5) $\lim\limits_{x \to +\infty} f(x) = 1$



EXERCICE 53 $f(x) = \frac{x-5}{3x+4}$

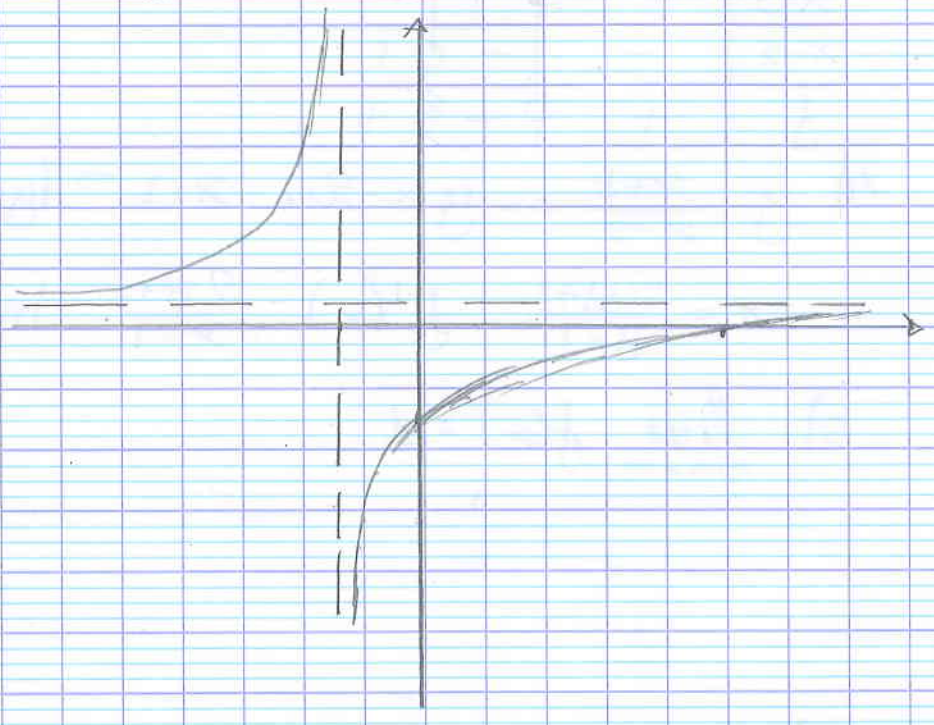
1) $D_f = \mathbb{R} \setminus \{-4/3\}$

2) $\cap Ox$ $y=0 \Rightarrow x=5$ $(5;0)$ $\cap Oy$ $x=0$ $(0;-5/4)$

3)

x	$-\infty$		$-4/3$		5		$+\infty$
$x-5$		$-$		$-$	0	$+$	
$3x+4$		$-$	0	$+$		$+$	
f		$+$	‖	$-$	0	$+$	

4) AO $y = 1/3$ $Im f = \mathbb{R} \setminus \{1/3\}$



EXERCICE 54 $f(x) = \frac{ax+b}{cx+d}$

a) AV $x = -\frac{b}{c}$ AH $y = \frac{a}{c}$

b) Si $c=0$ c'est une fonction affine
Si $ad=bc \Rightarrow d = \frac{bc}{a}$

$$f(x) = \frac{ax+b}{cx+\frac{bc}{a}} = \frac{ax+b}{\frac{acx+bc}{a}} = \frac{a(ax+b)}{c(ax+b)} = \frac{a}{c}$$

c) si $d=-a$ $f(x) = \frac{ax+b}{cx-a} = y \Rightarrow$

$$\Rightarrow ax+b = cyx - ay \Rightarrow$$
$$\Rightarrow (a-cy)x = -b-ay \Rightarrow$$
$$x = \frac{-b-ay}{a-cy} \Rightarrow f^{-1}(x) = \frac{-(ax+b)}{-(cx-a)} =$$
$$= \frac{ax+b}{cx+d} = f(x)$$

d) En général non
Le graphe passe par l'origine
si $f(0)=0 \Rightarrow b=0$

c) Si $a=0 \Rightarrow$ AH Ox

EXERCICE 55 $f(x) = \frac{ax+b}{x+d}$

AV $x=-1$ $-1+d=0 \Rightarrow d=1$ $f(x) = \frac{ax+b}{x+1}$

AH $y=3 = \frac{a}{1} \Rightarrow a=3$ $f(x) = \frac{3x+b}{x+1}$

Ox $f(x) = \frac{3x+4}{x+1}$

EXERCICE 56. $f(x)=\frac{x-2}{x+1}$ $g_1(x)=\frac{3x-5}{4}$ $g_2(x)=\frac{-3x+10}{4}$ $g_3(x)=\frac{x+4}{2}$

a) $f \cap g_1$ $\frac{x-2}{x+1}=\frac{3x-5}{4} \Rightarrow 4x-8=3x^2+3x-5x-5$

$\Leftrightarrow 3x^2-6x+13=0 \Rightarrow x^2-2x+1=0 \Rightarrow$

$(x-1)^2=0 \Rightarrow x=1 \quad (1;-1/2)$

$f \cap g_2$ $\frac{x-2}{x+1}=\frac{-3x+10}{4} \Rightarrow 4x-8=-3x^2-3x+10x+10$

$\Rightarrow 3x^2-3x-18=0 \Rightarrow x^2-x-6=0$

$\Delta=1+24=25 \quad x_{1,2}=\frac{1\pm5}{2}$ $\begin{cases} x_1=3 \\ x_2=-2. \end{cases}$

· $f \cap g_3$ $\frac{x-2}{x+1}=\frac{x+4}{2} \Leftrightarrow 2x-4=x^2+4x+x+4 \Leftrightarrow$

$\Leftrightarrow x^2+3x+8=0 \quad \Delta=9-32<0$ imp

b) $y=\frac{1}{3}x+h$

$\frac{1}{3}x+h=\frac{x-2}{x+1} \Leftrightarrow \left(\frac{1}{3}x+h\right)(x+1)=x-2$

$\Leftrightarrow \frac{1}{3}x^2+hx+\frac{1}{3}x+h=x-2 \Leftrightarrow$

$\Leftrightarrow x^2+3hx+x+3h=3x-6 \Leftrightarrow$

$\Leftrightarrow x^2+(3h-2)x+3h+6=0$

$\Delta=0 \Rightarrow (3h-2)^2-4(3h+6)=0 \Rightarrow$

$\Rightarrow 9h^2-12h+4-12h-24=0 \Leftrightarrow$

$\Leftrightarrow 9h^2-24h-20=0 \quad \Delta=1296$

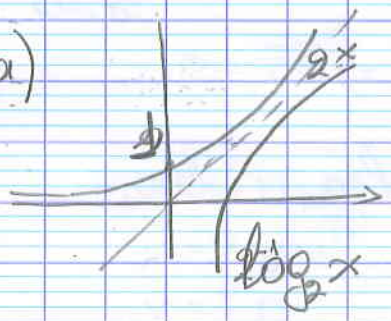
$h_{1,2}=\frac{24\pm36}{18}$ $\begin{cases} =\frac{60}{18}=\frac{20}{6}=\frac{10}{3}=h_1 \\ =\frac{-12}{18}=-\frac{2}{3}=h_2. \end{cases}$

$t_1: y=\frac{1}{3}x+\frac{10}{3} \qquad t_2: \frac{1}{3}x-\frac{2}{3}$

EXERCICE 57 $f(x) = \frac{3-x}{2x-4}$

1) $D_f = \mathbb{R} \setminus \{2\}$
2) $\cap Ox$ $y=0 \Rightarrow x=3$ $(3;0)$
   $\cap Oy$ $x=0$ $y=-\frac{3}{4}$ $(0;-3/4)$
3) AV: $y=2$ AH: $y=-1/2$

EXERCICE 58 a)



EXERCICE 59 a) $\log 100 = \log 10^2 = 2$
b) $\log_2 128 = \log_2 2^7 = 7$
c) $\log_7 1 = 0$
d) $\log_3(-2)$ impossible
e) $\log_2(\frac{1}{32}) = \log_2 2^{-5} = -5$
f) $\log \frac{1}{10} = \log 10^{-1} = -1$
g) $\log_9 3 = x \Rightarrow 9^x = 3 \Rightarrow 3^{2x} = 3 \Rightarrow 2x = 1 \Rightarrow x = 1/2$
h) $\log_{16}(4) = x \Rightarrow 16^x = 4 \Rightarrow 4^{2x} = 4 \Rightarrow 2x = 1 \Rightarrow x = 1/2$
i) $\log_3(0)$ impossible
j) $\log_{123}(123) = 1$
k) $\log \sqrt{10} = \log 10^{1/2} = 1/2$
l) $\log_2 \sqrt[3]{4} = \log_2 2^{2/3} = 2/3$
m) $\log_3 \frac{1}{\sqrt{27}} = \log_3 3^{-3/2} = -3/2$

EXERCICE 60

$x>0$

a) $\log_4(x)=3 \Rightarrow 4^3=x=64$

b) $10^x=5 \Rightarrow x=\log 5$

c) $\log x=\frac{1}{2} \Rightarrow x=10^{1/2}=\sqrt{10}$

$x>0$

d) $\log_x(125)=3$ $\quad x>0 \quad x\neq 1$

$\Rightarrow x^3=125 \Rightarrow x=\sqrt[3]{125}=5$

EXERCICE 61

1) $\log_a(u\cdot v)=\log_a u+\log_a v$

Soit $\log_a u=x \quad \log_a v=y$

$\Rightarrow a^x=u \quad a^y=v$

$u\cdot v=a^x\cdot a^y=a^{x+y} \Rightarrow x+y=\log_a(u\cdot v)$

$\Rightarrow \log_a u+\log_a v=\log_a(u\cdot v)$

2) $\log_a\left(\frac{u}{v}\right)=\log_a u-\log_a v$ (le même)

3) $\log_a(u^p)=p\log_a u$

$\log_a u^p=\log_a(\underbrace{u\cdot u\ldots u}_{p})=\underbrace{\log_a u+\log_a u+\ldots+\log_a}_{p}$

$=p\log_a u$

4) $\log_b(u)=\frac{\log_a u}{\log_a b}$

$\log_b u=x \Rightarrow b^x=u \Rightarrow \log_a b^x=\log_a u \Rightarrow$

$\Rightarrow x\log_a b=\log_a u \Rightarrow x=\frac{\log_a u}{\log_a b}=\log_b u$

EXERCICE 62

a) $10^x = \pi \Rightarrow x = \log \pi$

b) $10,1562^x = 1 \Rightarrow x = 0$

c) $10^{2x-5} = \frac{3}{4} \Rightarrow 2x-5 = \log \frac{3}{4} \Rightarrow$

$x = \frac{\log \frac{3}{4} + 5}{2}$

g) $5^{3x} = 2 \cdot 5^{x+1} \Rightarrow \frac{5^{3x}}{5^{x+1}} = 2 \Rightarrow 5^{2x-1} = 2 \Rightarrow$

$\Rightarrow 2x-1 = \log_5 2 \Rightarrow x = \frac{\log_5 2 + 1}{2}$

d) $a^x a^3 = a \Rightarrow a^{x+3} = a \Rightarrow x+3 = 1 \Rightarrow x = -2$

e) $10 \cdot 2^x = 5 \Rightarrow 2^x = \frac{1}{2} = 2^{-1} \Rightarrow x = -1$

f) $4 \cdot 2^{x-3} = 5 \Rightarrow 2^2 \cdot 2^{x-3} = 5 \Rightarrow$

$\Rightarrow 2^{x-1} = 5 \Rightarrow x-1 = \log_2 5 \Rightarrow$

$x = \log_2 5 + 1$

h) $3^x = 4 \cdot 2^{x-1} \Rightarrow 3^x = 2^{x+1} \Rightarrow$

$\log 3^x = \log 2^{x+1} \Rightarrow x \log 3 = (x+1) \log 2 \Rightarrow$

$\Rightarrow x (\log 3 - \log 2) = \log 2 \Rightarrow$

$x = \frac{\log 2}{\log 3 - \log 2}$

i) $x = 2 \log_3 \frac{2}{3} - \log_3 12 = \log_3 \frac{4}{9} - \log_3 12$

$= \log_3 \left(\frac{\frac{4}{9}}{12}\right) = \log_3 \frac{1}{27} = \log_3 3^{-3} = -3$

j) $\log(x-1) = 2 \Rightarrow x-1 = 10^2 \Rightarrow x = 101 > 1$ $\frac{x>1}{}$ ✓

k) $\log x + \log(x+3) = 1 \Rightarrow \log(x(x+3)) = 1$

$\Rightarrow x^2 + 3x = 10 \Rightarrow x^2 + 3x - 10 = 0$

$x_1 = -5 < 0 \quad x_2 = 2 > 0 \Rightarrow \boxed{x = 2}$

$\left.\begin{matrix} x>0 \\ x>-3 \end{matrix}\right\} x>0$

l) $\log(x+1) - \log(x-1) = 1 \Rightarrow \log \frac{x+1}{x-1} = 1 \Rightarrow$

$\frac{x+1}{x-1} = 10 \Rightarrow x+1 = 10x-10 \Rightarrow 9x = 11 \Rightarrow x = \frac{11}{9} > 1$ ✓

$x>-1$
$x>1$

EXERCICE 64 a) $e^x = 4 \Rightarrow x = \ln 4$

b) $\ln x = \frac{3}{2} \Rightarrow x = e^{3/2}$

EXERCICE 65 a) $e^x + 3e^{-x} = 4 \Rightarrow e^x + \frac{3}{e^x} = 4 \Rightarrow$

$e^{2x} + 3 = 4e^x \Rightarrow e^{2x} - 4e^x + 3 = 0$

$e^x = t \quad t^2 - 4t + 3 = 0 \begin{cases} t_1 = 3 \\ t_2 = 1 \end{cases}$

- $e^x = 3 \Rightarrow x = \ln 3$
- $e^x = 1 \Rightarrow x = 0$

b) $e^{2x} - e^x - 2 = 0 \quad t^2 - t - 2 = 0 \begin{cases} t_1 = 2 \\ t_2 = -1 \end{cases}$

- $e^x = 2 \Rightarrow x = \ln 2$
- $e^x = -1$ impossible

$x > 0$, $x > -1$

c) $\ln x + \ln(x+1) = \ln 6 \Rightarrow$

$\Rightarrow \ln[x(x+1)] = \ln 6 \Rightarrow$

$\Rightarrow x^2 + x - 6 = 0 \begin{cases} x_1 = -3 \times \\ \boxed{x_2 = 2} > 0 \end{cases}$

EXERCICE 66 $f(x) = \ln x \quad g(x) = x^2 + x - 6$

$h = g \circ f = \ln^2 x + \ln x - 6 = 0$

$\ln x = t \quad t^2 + t - 6 = 0 \begin{cases} t_1 = -3 \\ t_2 = 2 \end{cases}$

$\ln x = -3 \Rightarrow x = e^{-3} \quad \ln x = 2 \Rightarrow x = e^2$

EXERCICE 67 $f(x) = \log(1 - x^2) \quad 1 - x^2 > 0$

	-1		1	
	$-$	0	$+$	0	$-$

$D_f =]-1, 1[$

$g(x) = \ln(x^2 - 2x - 8) \quad x^2 - 2x - 8 > 0$

	-2		4	
	$+$	0	$-$	0	$+$

$D_g =]-\infty; -2[\cup]4, +\infty[$

EXERCICE 69 $f(x) = \cos(2x)$ cou $T = \frac{2\pi}{2} = \pi$

$g(x) = \cos x \quad T = 2\pi$

$h(x) = \cos\frac{x}{2} \quad T = \frac{2\pi}{\frac{1}{2}} = 4\pi$

EXERCICE 70

- $\cos x \quad T = 2\pi$
- $\cos(2x) \quad T = \frac{2\pi}{2} = \pi$
- $\cos\left(\frac{x}{2}\right) \quad T = \frac{2\pi}{\frac{1}{2}} = 4\pi$
- $\sin 3x \quad T = \frac{2\pi}{3}$
- $\sin\frac{x}{4} \quad T = \frac{2\pi}{\frac{1}{4}} = 8\pi$
- $\sin(2\pi x) \quad T = \frac{2\pi}{x} = 1 \Rightarrow x = 2\pi$
- $\cos(\pi x) \quad T = \frac{2\pi}{x} = 2 \Rightarrow x = \pi$

EXERCICE 71

- $f(x) = \frac{1}{\cos x} \quad \cos x = 0 \Rightarrow x = \frac{\pi}{2} + k\pi$

$D_f = \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$

- $g(x) = \sqrt{\sin x} \quad \sin x \geq 0$

$D_g = \bigcup [0 + 2k\pi ; \pi + 2k\pi]$

- $h(x) = \frac{1}{2 - \sin x} \quad 2 - \sin x = 0 \Rightarrow \sin x = 2$ impossible

$D_h = \mathbb{R}$

EXERCICE 72

- $f(x) = \sin^2 x \quad \text{Im } f = [0, 1]$
- $g(x) = \frac{1}{2}\cos(2x) \quad \text{Im } g = \left[-\frac{1}{2}; \frac{1}{2}\right]$
- $h(x) = 1 + \cos x \quad \text{Im } h = [0, 2]$
- $k(x) = \frac{1}{\tan^2 x} \quad \text{Im } k =]0, +\infty[$

EXERCICE 73

a) $\text{Arctan } 1 = x \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$

b) $\text{Arcsin}(-1) = x \Rightarrow \sin x = -1 \Rightarrow x = -\frac{\pi}{2} + 2k\pi$

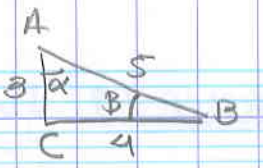
c) $\text{Arccos}\left(\frac{1}{2}\right) = x \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3} + 2k\pi$

EXERCICE 74

a) $\sin\left(\text{arc}\sin\frac{1}{2} - \text{arc}\cos 0\right) = \sin\left(\frac{\pi}{6} - \frac{\pi}{2}\right) = \sin\left(-\frac{2\pi}{6}\right) =$

$= \sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

b) $\cos\left(\arctan\frac{3}{4}+\arcsin\left(\frac{4}{5}\right)\right)$



$= \cos(\beta+\alpha) = \cos\frac{\pi}{2} = 0$

EXERCICE 15

a) $D_f = \mathbb{R}$ $\quad Im\,f = [-3;3]$

b) $T_f = \pi$

c) $\left\{\frac{\pi}{4}+k\frac{\pi}{2}\right\}$

d) $f(x) = 3\cos(2x)$

$g(x) = 3\cos\left(2x+\frac{\pi}{4}\right)$