LDDR Niveau 1 : TE 9 GEOMETRIE 3D

Exercice 1

$A(-3;0;4)\quad B(0;2;3)\quad C(6;2;-1)$

1) $\vec{AB}=\begin{pmatrix}3\\2\\-1\end{pmatrix}\quad \vec{AC}=\begin{pmatrix}9\\2\\-5\end{pmatrix}$

$\vec{n}_\Pi=\vec{AB}\wedge\vec{AC}=\begin{pmatrix}3\\2\\-1\end{pmatrix}\wedge\begin{pmatrix}9\\2\\-5\end{pmatrix}=\begin{pmatrix}-8\\6\\-12\end{pmatrix}\parallel\begin{pmatrix}4\\-3\\6\end{pmatrix}$

$\begin{matrix}3 & & 9\\2 & & 2\end{matrix}$

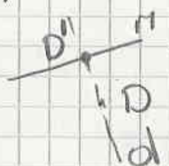
$\left.\begin{array}{l}\Pi:\ 4x-3y+6z+d=0\\ A:\ -12+24+d=0\Rightarrow d=-12\end{array}\right\}\ \Pi: 4x-3y+6z-12=0$

2) $\vec{n}_\Pi=\begin{pmatrix}4\\-3\\6\end{pmatrix}\quad \vec{n}_P=\begin{pmatrix}0\\1\\0\end{pmatrix}\quad \vec{n}_\Pi\cdot\vec{n}_P=-3$

$\|\vec{n}_\Pi\|=\sqrt{16+9+36}=\sqrt{61}\quad \|\vec{n}_P\|=1$

$\cos\varphi=\dfrac{\vec{n}_\Pi\cdot\vec{n}_P}{\|\vec{n}_\Pi\|\cdot\|\vec{n}_P\|}=\dfrac{-3}{\sqrt{61}\cdot1}\Rightarrow\varphi=112.6^\circ$

3) $D(-2;-1;13)\notin\Pi:\ -8+3+78-12=61\neq0$



On trouve la droite d; $\vec{d}=\vec{n}_\Pi=\begin{pmatrix}4\\-3\\6\end{pmatrix}$

$d:\begin{cases}x=-2+4\lambda\\y=-1-3\lambda\\z=13+6\lambda\end{cases}$

$D'=d\cap\Pi:\ 4(-2+4\lambda)-3(-1-3\lambda)+6(13+6\lambda)-12=0\Leftrightarrow$

$\Leftrightarrow -8+16\lambda+3+9\lambda+78+36\lambda-12=0\Leftrightarrow$

$\Leftrightarrow 61\lambda=-61\Rightarrow\lambda=-1\qquad D'(-6;2;7)$

Exercice 2

$d:\begin{cases}x=5+4\lambda\\y=1-3\lambda\\z=-1+\lambda\end{cases}\qquad S: x^2+y^2+z^2-4x+2y-10z+21=0\Rightarrow$

$\Rightarrow(x-2)^2-4+(y+1)^2-1+(z-5)^2-25+21=0$

$A(5;1;-1)\qquad (x-2)^2+(y+1)^2+(z-5)^2=9$

$C(2;-1;5)\quad r=3$

$\mathrm{dist}(C,d)=\dfrac{\|\vec{AC}\wedge\vec{d}\|}{\|\vec{d}\|}$

$\vec{AC}=\begin{pmatrix}-3\\-2\\6\end{pmatrix}\quad \vec{d}=\begin{pmatrix}4\\-3\\1\end{pmatrix}\qquad \begin{pmatrix}-3\\-2\\6\end{pmatrix}\wedge\begin{pmatrix}4\\-3\\1\end{pmatrix}=\begin{pmatrix}16\\27\\17\end{pmatrix}$

$\begin{matrix}-3 & & 4\\-2 & & -3\end{matrix}$

$\|\vec{AC}\wedge\vec{d}\|=\sqrt{16^2+27^2+17^2}=\sqrt{1274}=35.7\Rightarrow \mathrm{dist}=35.7-3=32.7$

EXERCICE 3

$S : (x-3)^2+y^2+(z+2)^2=81 \quad C(3;0;-2) \quad r=9$

1) $P(-1;7;2) : (-1-3)^2+7^2+(2+2)^2 = 16+49+16 = 81 \Rightarrow P \in S$

$\vec{n}_\alpha = \vec{CP} = \begin{pmatrix} -4 \\ 7 \\ 4 \end{pmatrix} \quad \alpha : -4x+7y+4z+d=0$

$P : 4+49+8+d=0 \Rightarrow d=-61$

$\alpha : -4x+7y+4z-61=0$

2) $\pi : -4x+4y+7z=55$

$dist(C,\pi) = \dfrac{|-12-14-55|}{\sqrt{16+16+49}} = \dfrac{81}{\sqrt{81}} = \sqrt{81} = 9 = r \Rightarrow$

π est tangent à la Sphère

3) $\vec{L} = \vec{n}_\pi \wedge \vec{n}_\alpha = \begin{pmatrix} -4 \\ 4 \\ 7 \end{pmatrix} \wedge \begin{pmatrix} -4 \\ 7 \\ 4 \end{pmatrix} = \begin{pmatrix} -33 \\ -12 \\ -12 \end{pmatrix} // \begin{pmatrix} 11 \\ 4 \\ 4 \end{pmatrix} = \vec{L}$

$\begin{matrix} -4 & \quad & -4 \\ 4 & \quad & 7 \end{matrix}$

EXERCICE 4

$d : \begin{cases} x = 13+8\lambda \\ y = 2+\lambda \\ z = -4-3\lambda \end{cases} \qquad S : (x-3)^2+(y+2)^2+(z-5)^2=49$

1) $\vec{d} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} \quad \vec{n}_s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \vec{d}\cdot\vec{n}_s = -3$

$\|\vec{d}\| = \sqrt{8^2+1+9} = \sqrt{74} \quad \|\vec{n}_s\| = 1$

$\cos\varphi = \dfrac{|\vec{d}\cdot\vec{n}_s|}{\|\vec{d}\|\cdot\|\vec{n}_s\|} = \dfrac{|-3|}{\sqrt{74}} \Rightarrow \varphi = 69.6 \Rightarrow \alpha = 90-\varphi = 20.41°$

2) $d \cap S \quad (13+8\lambda-3)^2+(2+\lambda+2)^2+(-4-3\lambda-5)^2=49 \Leftrightarrow$

$\Leftrightarrow (10+8\lambda)^2+(4+\lambda)^2+(-9-3\lambda)^2=49 \Leftrightarrow$

$\Leftrightarrow 100+160\lambda+64\lambda^2+16+8\lambda+\lambda^2+81+54\lambda+9\lambda^2-49=0$

$\Leftrightarrow 74\lambda^2+222\lambda+148=0 \Leftrightarrow 37\lambda^2+111\lambda+74=0$

$\Delta = 1369 \quad \lambda_{1,2} = \dfrac{-111\pm37}{74}$ $\lambda_1=-2 \quad A(-3;0;2)$; $\lambda_2=-1 \quad B(5;1;-1)$

3) $\Omega(3;-2;5) \quad r=7$

$\vec{\Omega A} = \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} \quad \vec{\Omega B} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix} \quad \vec{\Omega A}\wedge\vec{\Omega B} = \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} \wedge \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix} = \begin{pmatrix} -3 \\ -42 \\ -22 \end{pmatrix} // \begin{pmatrix} 3 \\ 42 \\ 22 \end{pmatrix}$

$\begin{matrix} -6 & \quad & 2 \\ 2 & \quad & 3 \end{matrix}$

$\|\vec{\Omega A}\wedge\vec{\Omega B}\| = \sqrt{3^2+42^2+22^2} = \sqrt{2257}$

Aire $= \dfrac{1}{2}\sqrt{2257} \approx 23.75$

Exercice 5

Soit $P \in$ mur $P(0;y,z)$ $A(3;-4;2)$ $B(7,12;0)$

$\vec{PA} = \begin{pmatrix} 3 \\ -4-y \\ 2-z \end{pmatrix}$ $\vec{PB} = \begin{pmatrix} 7 \\ 12-y \\ -z \end{pmatrix}$

$\|\vec{PA}\| = \|\vec{PB}\| \Rightarrow \sqrt{9+(4+y)^2+(2-z)^2} = \sqrt{7^2+(12-y)^2+z^2} \Leftrightarrow$

$\Leftrightarrow 9+16+8y+\cancel{y^2}+4-4z+\cancel{z^2} = 49+144-24y+\cancel{y^2}+\cancel{z^2} \Rightarrow$

$\Rightarrow 32y-4z-164=0 \Rightarrow 8y-z-41=0$