LDDR - Niveau 1 : TE 16 GEOMETRIE 3D

EXERCICE 1

$A(0;2;4)$ $B(1;-4;-3)$ $C(0;2;0)$ $D(1;1;-1)$

$d: \begin{cases} x = \lambda \\ y = 3-\lambda \\ z = -4+4\lambda \end{cases}$

a) $B_m(0;-4;-3)$

b) $P(4;-1;16) \in d$? $\quad 4 = \lambda$; $-1 = 3-\lambda \Rightarrow \lambda = 4$; $16 = -4+4\lambda \Rightarrow 4\lambda = 20 \Rightarrow \lambda = 5$ $\Rightarrow$ Non

c) $\vec{AB} = \begin{pmatrix} 1 \\ -6 \\ -7 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$ $\vec{AD} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$

linéairement indépendants $\Rightarrow$ ne sont pas sur le même plan

On cherche le plan de A, B, C

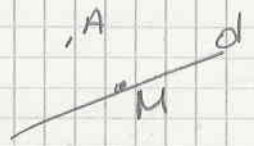
$\vec{AB} \wedge \vec{AC} = \begin{pmatrix} 1 \\ -6 \\ -7 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 24 \\ 4 \\ 0 \end{pmatrix} // \begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix} = \vec{n}_6$

$6x + y + d = 0$ $\quad A: 2 + d = 0 \Rightarrow d = -2$

$6: 6x + y - 2 = 0$

$D \in 6$? $\quad 6 + 1 - 2 = 5 \neq 0 \Rightarrow D \notin 6 \Rightarrow \vec{AB}, \vec{AC}, \vec{AC}$ lin ind

d) $M(1; -\frac{3}{2}; -2)$ $\quad d = \begin{cases} x = 1+\lambda \\ y = -\frac{3}{2} - 6\lambda \\ z = -2 - 7\lambda \end{cases}$

e) 

$\vec{d} = \begin{pmatrix} 1 \\ -6 \\ -7 \end{pmatrix} // M$

$\vec{AM} = \begin{pmatrix} 1 \\ -7/2 \\ -6 \end{pmatrix} // \begin{pmatrix} 2 \\ -7 \\ -12 \end{pmatrix}$ $\Big\} \vec{n}_M = \vec{d} \wedge \vec{AM}$

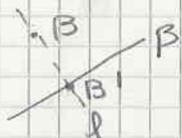
$\vec{n}_M = \begin{pmatrix} 1 \\ -6 \\ -7 \end{pmatrix} \wedge \begin{pmatrix} 2 \\ -7 \\ -12 \end{pmatrix} = \begin{pmatrix} 23 \\ -2 \\ 5 \end{pmatrix}$ $\quad 23x - 2y + 5z + d = 0$

$A: -4 + 20 + d = 0 \Rightarrow d = -16$

$M: 23x - 2y + 5z - 16 = 0$

f) $\alpha //$ paroi $\Rightarrow \vec{n}_\alpha = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\quad y + d = 0$; $B: -4 + d = 0 \Rightarrow d = 4$ $\Big\} \alpha: y + 4 = 0$

g) $\beta: 3x - y - 5 = 0$



On trouve la droite $l \perp \beta$

$\vec{l} = \vec{n}_\beta = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$

$l = \begin{cases} x = 1+3\lambda \\ y = -4-\lambda \\ z = -3 \end{cases}$ $\quad B' = l \cap \beta$

$3(1+3\lambda) - (-4-\lambda) - 5 = 0 \Rightarrow$ $\quad B'(\frac{2}{5}; -\frac{19}{5}; -3)$

$\Rightarrow 3 + 9\lambda + 4 + \lambda - 5 = 0 \Rightarrow 10\lambda = -2 \Rightarrow \lambda = -1/5$

EXERCICE 2

$$d: \begin{cases} x = 2 - \lambda \\ y = -2 - 3\lambda \\ z = -1 - 2\lambda \end{cases} \qquad e: \begin{cases} x = 2 + 4\mu \\ y = 3 + 2\mu \\ z = 1 + 4\mu \end{cases}$$

a) $\underline{\underline{d}}$

T_s : $z = 0 \Rightarrow \lambda = -\frac{1}{2}$ $\quad T_s(2{,}5 ; -0{,}5 ; 0)$

T_m : $x = 0 \Rightarrow \lambda = 2$ $\quad T_m(0 ; -8 ; -5)$

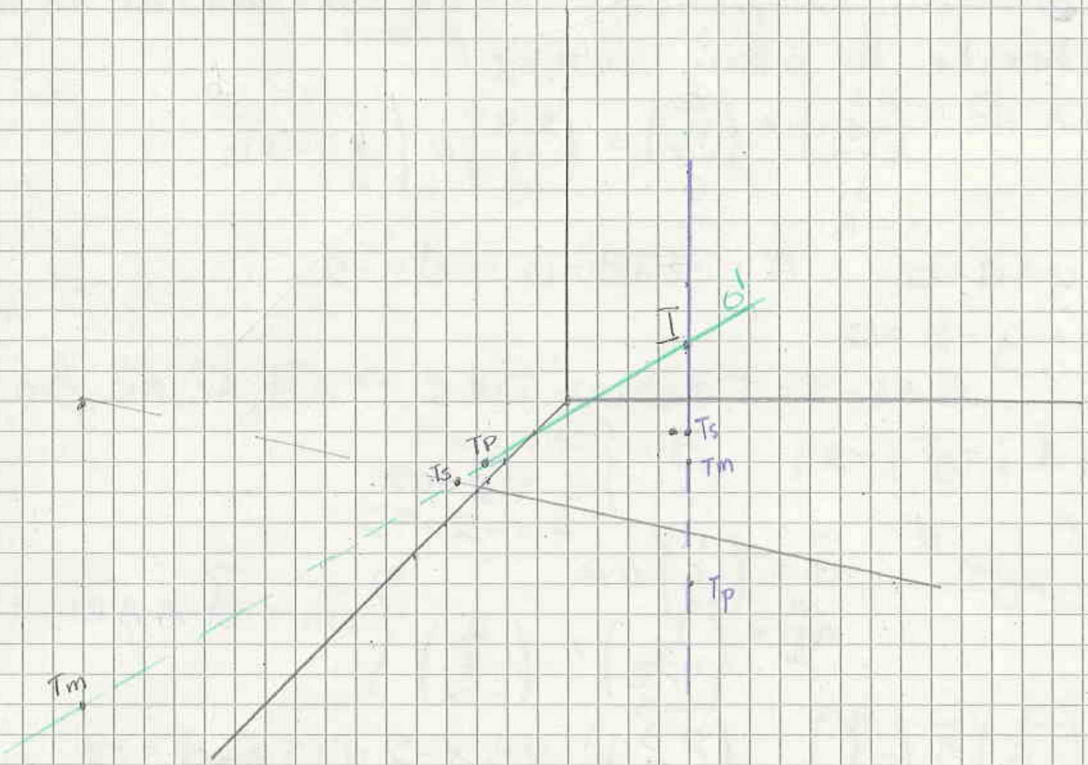
T_p : $y = 0 \Rightarrow \lambda = -\frac{2}{3}$ $\quad T_p\left(\frac{8}{3} ; 0, \frac{1}{3}\right)$

$\underline{\underline{e}}$:

T_s : $z = 0 \Rightarrow \mu = -\frac{1}{4}$ $\quad T_s(1 ; 2{,}5 ; 0)$

T_m : $x = 0 \Rightarrow \mu = -\frac{1}{2}$ $\quad T_m(0 ; 2 ; -1)$

T_p : $y = 0 \Rightarrow \mu = -\frac{3}{2}$ $\quad T_p(-4 ; 0 ; -5)$



b) $I = d \cap e$.

$2 - \lambda = 2 + 4\mu \Rightarrow \lambda + 4\mu = 0$ $\quad \cdot$

$-2 - 3\lambda = 3 + 2\mu \Rightarrow 3\lambda + 2\mu = -5$ $\quad \cdot(-2)$

$-1 - 2\lambda = 1 + 4\mu$ (3)

$$\begin{cases} \lambda + 4\mu = 0 \\ -6\lambda - 4\mu = 10 \end{cases}$$

$-5\lambda = 10 \Rightarrow \lambda = -2$

$-2 + 4\mu = 0 \Rightarrow \mu = \frac{1}{2}$

(3) $-1 + 4 \stackrel{?}{=} 1 + 2$ oui $\Rightarrow$ sécantes

$I(4 ; 4 ; 3)$

Exercice 3

$\Pi : 3x - 2y - 6 = 0$

$\Pi_x\ (2; 0; 0)$ $\Pi_y\ (0; -3; 0)$ Π_z n'existe pas

$\Rightarrow \Pi \parallel Oz$