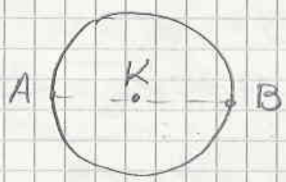
LDDR - Niveau 1 : TE 12 GEOMETRIE 3D

1) La plus petite sphère $\Rightarrow$ AB diamètre



$A(4;-18;5) \quad B(-8;10;7)$

$K\left(\frac{4-8}{2};\ \frac{-18+10}{2};\ \frac{5+7}{2}\right) \Rightarrow K(-2;-4;6)$

$r = \frac{\|\vec{AB}\|}{2} \qquad \vec{AB} = \begin{pmatrix}-12\\28\\2\end{pmatrix} \qquad \|\vec{AB}\| = \sqrt{12^2+28^2+2^2} = \sqrt{932}$

$r = \frac{\sqrt{932}}{2} \qquad (x+2)^2+(y+4)^2+(z-6)^2 = \frac{932}{4} = 233$

2) $x^2+y^2+z^2+4y-10z=7 \Rightarrow$

$\Rightarrow x^2+(y+2)^2-4+(z-5)^2-25=7 \Leftrightarrow$

$\Rightarrow x^2+(y+2)^2+(z-5)^2=36$ c'est une sphère

$K(0;-2;5) \quad r=6$

3) $K(7;-1;0) \qquad d: \begin{cases} x = 7-2\lambda \\ y = 3+\lambda \\ z = 5 \end{cases} \qquad \vec{d} = \begin{pmatrix}-2\\1\\5\end{pmatrix} \quad A(7;3;5)$

$r = \text{dist}(K,d) = \frac{\|\vec{d}\wedge\vec{AK}\|}{\|\vec{d}\|}$

$\vec{AK} = \begin{pmatrix}0\\-4\\-5\end{pmatrix} \qquad \vec{d}\wedge\vec{AK} = \begin{pmatrix}-2\\1\\5\end{pmatrix}\wedge\begin{pmatrix}0\\-5\\-5\end{pmatrix} = \begin{pmatrix}20\\-10\\10\end{pmatrix}$

$\begin{matrix}-2 & 0\\ 1 & -5\end{matrix}$

$\|\vec{d}\wedge\vec{AK}\| = \sqrt{20^2+10^2+10^2} = \sqrt{600}$

$\|\vec{d}\| = \sqrt{4+1+25} = \sqrt{30} \qquad r = \frac{\sqrt{600}}{\sqrt{30}} = \sqrt{20} = 2\sqrt{5}$

$S:\ (x-7)^2+(y+1)^2+z^2=20$

4) $S\ x^2+(y-3)^2+(z+1)^2=12 \qquad \ell: \begin{cases} x=-2 \\ y=3+\lambda \\ z=-1+\lambda \end{cases}$

$S\cap\ell\ \ (-2)^2+(3+\lambda-3)^2+(-1+\lambda+1)^2=12$

$\Rightarrow 4+\lambda^2+\lambda^2=12 \Rightarrow 2\lambda^2=8 \Rightarrow \lambda^2=4 \Rightarrow \lambda=\pm2$

$P_1(-2;5;1) \qquad P_2(-2;1;-3)$

$P_1:\ \vec{n}_1 = \vec{KP_1} = \begin{pmatrix}-2\\2\\2\end{pmatrix} \parallel \begin{pmatrix}1\\-1\\-1\end{pmatrix} \qquad K(0;3;-1)$

$\Pi_1:\ x-y-z+d=0 \qquad P_1:\ -2-5-1+d=0 \Rightarrow d=8$

$\Pi_1:\ x-y-z+8=0$

$P_2:\ \vec{n}_2 = \vec{KP_2} = \begin{pmatrix}-2\\-2\\-2\end{pmatrix} \parallel \begin{pmatrix}1\\1\\1\end{pmatrix}$

$\Pi_2:\ x+y+z+d=0 \qquad P_2:\ -2+1-3+d=0 \Rightarrow d=4$

$\Pi_2:\ x+y+z+4=0$

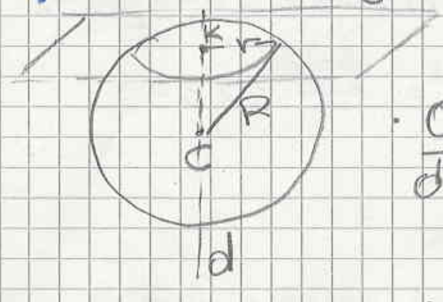
5) $C(k;k,k)$ $\quad r=4 \quad$ $\Pi: 2x+2y+z-8=0$

$\text{dist}(C,\Pi) = r \Rightarrow \dfrac{|2k+2k+k-8|}{\sqrt{2^2+2^2+1^2}} = 4 \Leftrightarrow$

$\Rightarrow |5k-8| = 12 \Rightarrow \begin{cases} 5k-8=12 \Rightarrow 5k=20 \Rightarrow \underline{k_1=4} \\ 5k-8=-12 \Rightarrow 5k=-4 \Rightarrow \underline{k_2=-\frac{4}{5}} \end{cases}$

6) $\Pi: 2x+2y+z-8=0 \quad S: (x+15)^2+y^2+(z-2)^2=169$

$C(-15;0;2) \quad R=13$



· On trouve la droite d

$\vec{d} = \vec{n}_\Pi = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad C \in d \quad d: \begin{cases} x = -15+2\lambda \\ y = 2\lambda \\ z = 2+\lambda \end{cases}$

· $K = d \cap \Pi$. $2(-15+2\lambda)+2\cdot 2\lambda + 2+\lambda-8=0 \Leftrightarrow$

$\Rightarrow -30+4\lambda+4\lambda+2+\lambda-8=0 \Leftrightarrow 9\lambda=36 \Rightarrow$

$\lambda = 4 \quad \underline{K(-7;8;6)} \quad \vec{KC} = \begin{pmatrix} -8 \\ -8 \\ -4 \end{pmatrix}$

$\|\vec{KC}\| = \sqrt{8^2+8^2+4^2} = \sqrt{144} = 12$

· $r^2 = R^2 - \|\vec{KC}\|^2 \Rightarrow r^2 = 13^2-12^2 = 25 \Rightarrow \underline{r=5}$

Bonus $\quad x^2+y^2+z^2=9 \quad K(0;1;0)$

$O(0,0,0) \quad R=3$

$\vec{OK} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \|\vec{OK}\| = 1$

$r = \sqrt{3^2-1^2} = \underline{\sqrt{8} = r}$

$\left.\begin{array}{l} \vec{n}_\alpha = \vec{OK} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \alpha: y+d=0 \\ K: 1+d=0 \Rightarrow d=-1 \end{array}\right\} \alpha: \underline{y-1=0}$