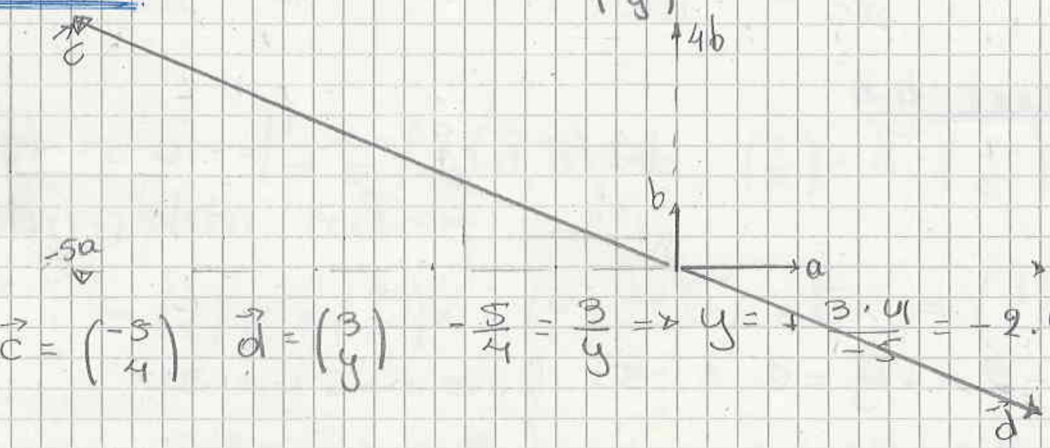
LDDR - Niveau 1 GEOMETRIE PLANE Classe E

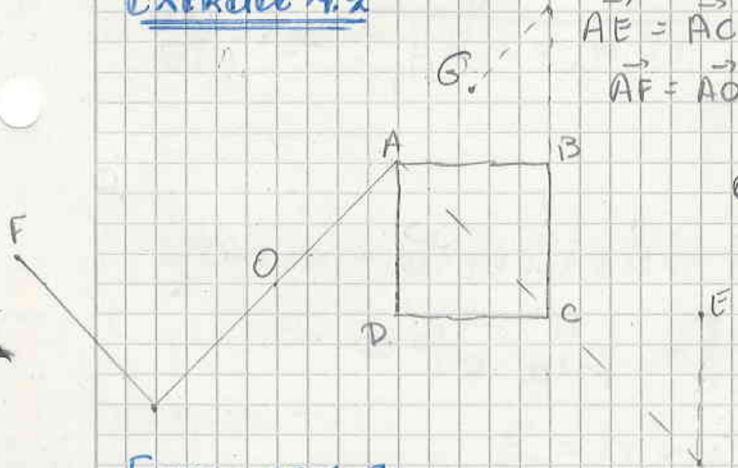
Exercice 4.1

$\vec{c} = -5\vec{a} + 4\vec{b}$ $\quad \vec{d} = \begin{pmatrix}3\\y\end{pmatrix}$



$\vec{c} = \begin{pmatrix}-5\\4\end{pmatrix}$ $\quad \vec{d} = \begin{pmatrix}3\\y\end{pmatrix}$ $\quad -\frac{5}{4} = \frac{3}{y} \Rightarrow y = +\frac{3\cdot 4}{-5} = -2.4$

Exercice 4.2



$\vec{AE} = \vec{AC} + \vec{BC} \Rightarrow AC + \vec{AC} - AB = 2\vec{AC} + \vec{BA}$

$\vec{AF} = \vec{AO} - \vec{OC} = \vec{AO} - \vec{AC} + \vec{AO} = 2\vec{AO} + \vec{CA}$

$\vec{CG} = 2\vec{CB} + \frac{1}{2}\vec{BD}$

Exercice 4.3

$\vec{a} = \begin{pmatrix}3\\4\end{pmatrix}$ $\quad \vec{b} = \begin{pmatrix}-2\\5\end{pmatrix}$ $\quad \vec{c} = \begin{pmatrix}-2\\6\end{pmatrix}$

a) $\vec{a} \parallel \begin{pmatrix}-6\\-8\end{pmatrix}$ $\quad \vec{b} \parallel \begin{pmatrix}7\\y\end{pmatrix}$ $\quad -\frac{2}{5} = \frac{7}{y} \Rightarrow -2y = 35 \Rightarrow y = -\frac{35}{2}$

$\vec{c} \parallel \begin{pmatrix}x\\-11\end{pmatrix}$ $\quad \frac{x}{-11} = -\frac{2}{6} \Rightarrow 6x = 22 \Rightarrow x = \frac{22}{6} = \frac{11}{3}$

b) $2\vec{a} - 3\vec{b} = \begin{pmatrix}6\\8\end{pmatrix} - \begin{pmatrix}-6\\15\end{pmatrix} = \begin{pmatrix}12\\23\end{pmatrix}$

$\frac{1}{3}\vec{a} + \frac{3}{2}\vec{c} = \begin{pmatrix}1\\4/3\end{pmatrix} + \begin{pmatrix}-3\\9\end{pmatrix} = \begin{pmatrix}-2\\\frac{31}{3}\end{pmatrix}$

$-4(\vec{a} - \vec{b}) + 3(-\vec{b} + \vec{c}) = -4\begin{pmatrix}5\\-1\end{pmatrix} + 3\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-20\\4\end{pmatrix} + \begin{pmatrix}0\\3\end{pmatrix} = \begin{pmatrix}-20\\7\end{pmatrix}$

c) $\det(\vec{a}, \vec{b}) = \begin{vmatrix}3 & -2\\4 & 5\end{vmatrix} = 15 + 8 = 22 \neq 0 \Rightarrow \vec{a} \nparallel b \Rightarrow$
ils forment une base

$\vec{c} = x\vec{a} + y\vec{b} \Rightarrow \begin{pmatrix}-2\\6\end{pmatrix} = x\begin{pmatrix}3\\4\end{pmatrix} + y\begin{pmatrix}-2\\5\end{pmatrix} \Rightarrow$

$\Rightarrow \begin{matrix} -2 = 3x - 2y & \cdot 5 \\ 6 = 4x + 5y & \cdot 2 \end{matrix} \Rightarrow \begin{cases} -10 = 15x - 10y \\ +\ 12 = 8x + 10y \end{cases}$

$2 = 23x \Rightarrow x = \frac{2}{23}$

$-2 = 3\cdot\frac{2}{23} + 2y \Rightarrow 2y = +\frac{52}{23}$

$\Rightarrow y = +\frac{26}{23}$ $\quad \vec{c} = \frac{2}{23}\vec{a} + \frac{26}{23}\vec{b}$

b) $\vec{c} = \frac{2}{23}\vec{a} + \frac{26}{23}\vec{b} \Rightarrow \frac{26}{23}\vec{b} = \vec{c} - \frac{2}{23}\vec{a} \Leftrightarrow$

$\Leftrightarrow \vec{b} = \frac{23}{26}\vec{c} - \frac{2}{26}\vec{a} \Rightarrow \vec{b} = -\frac{1}{13}\vec{a} + \frac{23}{26}\vec{c}$

EXERCICE 4.4

$\vec{a} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ $\det(\vec{a},\vec{b}) = \begin{vmatrix} -2 & 1 \\ 3 & 5 \end{vmatrix} = -10-3 = -13 \neq 0$

$\Rightarrow$ lin indépendants

$\vec{c} = \begin{pmatrix} 6 \\ 1 \end{pmatrix} = x\vec{a} + y\vec{b} \Rightarrow \begin{pmatrix} 6 \\ 1 \end{pmatrix} = x\begin{pmatrix} -2 \\ 3 \end{pmatrix} + y\begin{pmatrix} 1 \\ 5 \end{pmatrix} \Rightarrow$

$\Rightarrow \begin{cases} -2x + y = 6 & *(-5) \\ 3x + 5y = 1 \end{cases}$ $\begin{cases} 10x - 5y = -30 \\ 3x + 5y = 1 \end{cases}$

$+ \quad 13x = -29 \Rightarrow x = -\frac{29}{13}$

$+ 2 \cdot \frac{29}{13} + y = 6 \Rightarrow y = \frac{20}{13}$

$\vec{c} = -\frac{29}{13}\vec{a} + \frac{20}{13}\vec{b}$

$\begin{pmatrix} 7 \\ m \end{pmatrix} \parallel \vec{a} \Rightarrow \det\begin{vmatrix} 7 & -2 \\ m & 3 \end{vmatrix} = 0 \Rightarrow 21 + 2m = 0 \Rightarrow m = -\frac{21}{2}$

$\begin{pmatrix} n \\ -12 \end{pmatrix} \parallel (\vec{a}+\vec{b}) \Rightarrow \begin{pmatrix} n \\ -12 \end{pmatrix} \parallel \begin{pmatrix} -1 \\ 8 \end{pmatrix} \Rightarrow \begin{vmatrix} n & -1 \\ -12 & 8 \end{vmatrix} = 0 \Rightarrow$

$8n - 12 = 0 \Rightarrow n = \frac{12}{8} \Rightarrow n = \frac{3}{2}$

EXERCICE 4.5

$\vec{a} \parallel \vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\vec{b} \parallel (2\vec{e_1} + \vec{e_2}) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$3\vec{a} + \vec{b} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$

$\vec{a} \parallel \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{vmatrix} a_1 & 1 \\ a_2 & 0 \end{vmatrix} = 0 \Rightarrow -a_2 = 0 \Rightarrow a_2 = 0$

$\vec{b} \parallel \begin{pmatrix} 2 \\ 1 \end{pmatrix} \Rightarrow \begin{vmatrix} b_1 & 2 \\ b_2 & 1 \end{vmatrix} = 0 \Rightarrow b_1 - 2b_2 = 0$ (1)

$3\vec{a} + \vec{b} = \begin{pmatrix} 3a_1 \\ 0 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \end{pmatrix} \Rightarrow \begin{cases} 3a_1 + b_1 = 7 & (2) \\ b_2 = -1 \end{cases}$

(1) $\xrightarrow{b_2 = -1} b_1 = -2$ $\quad \vec{b} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

(2) $3a_1 - 2 = 7 \Rightarrow 3a_1 = 9 \Rightarrow a_1 = 3$ $\quad \vec{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

Exercices 4.6

$A(3;4) \quad B(-2;1)$

$\vec{OC} = \vec{AB} = \begin{pmatrix}-2-3\\1-4\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-5\\-3\end{pmatrix} \Rightarrow \begin{matrix}x=-5\\y=-3\end{matrix} \quad C(-5;-3)$

$\vec{OD} = -\vec{AB} = -\begin{pmatrix}-2-3\\1-4\end{pmatrix} \Rightarrow \vec{OD} = \begin{pmatrix}5\\3\end{pmatrix} \Rightarrow D(5;3)$

$\vec{BE} = \vec{OA} \Rightarrow \vec{OE} - \vec{OB} = \vec{OA} \Rightarrow \vec{OE} = \vec{OA} + \vec{OB} \Rightarrow$

$\vec{OE} = \begin{pmatrix}3-2\\4+1\end{pmatrix} \Rightarrow E(1;5)$

$\vec{BF} = -\vec{OA} \Rightarrow \begin{pmatrix}x+2\\y-1\end{pmatrix} = \begin{pmatrix}-3\\-4\end{pmatrix} \Rightarrow \begin{matrix}x=-5\\y=-3\end{matrix} \quad F(-5;-3)$

$\vec{AG} = \vec{OB} \Rightarrow \begin{pmatrix}x-3\\y-4\end{pmatrix} = \begin{pmatrix}-2\\1\end{pmatrix} \Rightarrow \begin{cases}x=1\\y=5\end{cases} \quad G(1;5)$

$\vec{AH} = -\vec{OB} \Rightarrow \begin{pmatrix}x-3\\y-4\end{pmatrix} = \begin{pmatrix}2\\-1\end{pmatrix} \Rightarrow \begin{matrix}x=5\\y=3\end{matrix} \quad H(5;3)$

Exercice 4.7

a) $A(7;5) \quad B(-4;1) \longrightarrow \vec{AB} = \begin{pmatrix}-11\\-4\end{pmatrix}$

b) $A(x,y) \quad B(2;\frac{1}{3}) \longrightarrow \vec{AB} = \begin{pmatrix}-3\\2\end{pmatrix} = \begin{pmatrix}2-x\\\frac{1}{3}-y\end{pmatrix} \Rightarrow \begin{matrix}x=5\\y=-\frac{5}{3}\end{matrix}$

c) $A(x;-2) \quad B(0;y) \longrightarrow \vec{AB} = \begin{pmatrix}v\\5\end{pmatrix} \parallel \begin{pmatrix}7\\-4\end{pmatrix} \Rightarrow \begin{vmatrix}v & 7\\5 & -4\end{vmatrix} = 0 \Rightarrow$

$\Rightarrow -4v - 35 = 0 \Rightarrow v = -\frac{35}{4}$

$\vec{AB} = \begin{pmatrix}-\frac{35}{4}\\5\end{pmatrix} = \begin{pmatrix}0-x\\y+2\end{pmatrix} \Rightarrow \begin{matrix}x = -\frac{35}{4}\\y=3\end{matrix}$

d) $A(\frac{1}{2};3) \quad B(x;-1) \longrightarrow \vec{AB} = \begin{pmatrix}v\\u\end{pmatrix} \parallel \begin{pmatrix}0\\1\end{pmatrix} = \vec{e_2} \Rightarrow$

$\begin{vmatrix}v & 0\\u & 1\end{vmatrix} = 0 \Rightarrow v = 0$

$\vec{AB} = \begin{pmatrix}0\\u\end{pmatrix} = \begin{pmatrix}x-\frac{1}{2}\\-1-3\end{pmatrix} \Rightarrow \begin{matrix}x=\frac{1}{2}\\u=-4\end{matrix}$

$B(\frac{1}{2};-1) \quad \vec{AB} = \begin{pmatrix}0\\-4\end{pmatrix}$

b. $A(2;-3) \quad B(-1,-2) \quad M\left(\frac{2-1}{2};\frac{-3-2}{2}\right) = \left(\frac{1}{2};-\frac{5}{2}\right)$

c. 

$\vec{BA} = \vec{AC} \Rightarrow \begin{pmatrix}3\\-1\end{pmatrix} = \begin{pmatrix}x-2\\y+3\end{pmatrix} \Rightarrow \begin{matrix}x=5\\y=-4\end{matrix}$

$C(5,-4)$

d. $P'(2a-x, 2b-y)$

EXERCICE 4.8

$A(4;-6)$ $\quad \vec{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\quad \vec{BC} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$ $\quad \vec{OD} = -\vec{CB}$

$\vec{OE} = \vec{OD} - \vec{BC}$

$\vec{AB} = \begin{pmatrix} x_B - 4 \\ y_B + 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \Rightarrow \begin{matrix} x_B = 7 \\ y_B = -4 \end{matrix} \quad B(7;-4)$

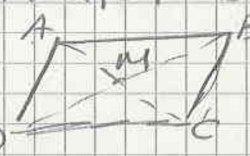
$\vec{BC} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} \Rightarrow \begin{pmatrix} x_C - 7 \\ y_C + 4 \end{pmatrix} \Rightarrow \begin{matrix} x_C = 2 \\ y_C = 0 \end{matrix} \quad C(2;0)$

$\vec{OD} = -\vec{CB} = \vec{BC} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} \Rightarrow D(-5;4)$

$\vec{OE} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} - \begin{pmatrix} -5 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow E(0;0)$

EXERCICE 4.9

$A(-2;5)$ $\quad B(1;-3)$ $\quad C\left(\frac{3}{2};\frac{3}{2}\right)$

a. 

$\vec{AB} = \vec{DC} \Rightarrow \begin{pmatrix} 3 \\ -8 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} - x \\ \frac{3}{2} - y \end{pmatrix} \Rightarrow$

$\Rightarrow \begin{matrix} x = -\frac{3}{2} \\ y = \frac{19}{2} \end{matrix} \quad D\left(-\frac{3}{2};\frac{19}{2}\right)$

b. M milieu du segment AC: $M\left(\frac{-2+\frac{3}{2}}{2};\frac{5+\frac{3}{2}}{2}\right) =$

$M\left(-\frac{1}{4};\frac{13}{4}\right)$

EXERCICE 4.10

a) $A(4;9)$ $\quad B(-2;5)$ $\quad M_{AB}\left(\frac{4-2}{2};\frac{9+5}{2}\right) = (1;7)$ $\quad \vec{AB} = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$

b) $A(1/2;-2)$ $\quad B(x_B,y_B)$ $\quad M\left(3;\frac{7}{2}\right)$ $\quad \vec{AB} = (5;11)$

$3 = \frac{1/2 + x_B}{2} \Rightarrow x_B + \frac{1}{2} = 6 \Rightarrow x_B = 5.5$

$\frac{7}{2} = \frac{-2+y_B}{2} \Rightarrow y_B = 9 \quad B(5,5;9)$

c) $A(x_A,y_A)$ $\quad B(0;7)$ $\quad M(-6;2)$ $\quad \frac{x_A+0}{2} = -6 \Rightarrow x_A = -12$

$A(-12;-3)$ $\quad \vec{AB} = \begin{pmatrix} 12 \\ 10 \end{pmatrix}$ $\quad \frac{y_A+7}{2} = 2 \Rightarrow y_A = -3$

d) $A(2;7)$ $\quad B(x_B,y_B)$ $\quad M(x_M,y_M)$ $\quad \vec{AB} = \begin{pmatrix} -3/2 \\ 11 \end{pmatrix}$

$\vec{AB} = \begin{pmatrix} x_B - 2 \\ y_B - 7 \end{pmatrix} = \begin{pmatrix} -3/2 \\ 11 \end{pmatrix} \Rightarrow \begin{matrix} x_B = 1/2 \\ y_B = 18 \end{matrix} \quad B(1/2;18)$

$M\left(\frac{2+1/2}{2};\frac{7+18}{2}\right) = \left(\frac{5}{4};\frac{25}{2}\right)$

c) $A(x_A, y_A)$ $B(x_B, y_B)$ $M(1;-5)$ $\vec{AB} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

$1 = \dfrac{x_A + x_B}{2} \Rightarrow x_A + x_B = 2$

$-5 = \dfrac{y_A + y_B}{2} \Rightarrow y_A + y_B = -10$

$\vec{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \Rightarrow \begin{matrix} x_B - x_A = 6 \\ y_B - y_A = -4 \end{matrix}$

$\begin{cases} x_A + x_B = 2 \\ -x_A + x_B = 6 \end{cases}$ (+) $2x_B = 8 \Rightarrow x_B = 4$
$\Rightarrow x_A = -2$

$\begin{cases} y_A + y_B = -10 \\ -y_A + y_B = -4 \end{cases}$ (+) $2y_B = -14 \Rightarrow y_B = -7$
$y_A = -3$

$A(-2;-3)$ $B(4;-7)$

EXERCICE 4.11

$A(-3;2)$ $M(-1;0)$ $\vec{AB} \parallel \begin{pmatrix} 1 \\ -3 \end{pmatrix}$
$\vec{BM} \parallel \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

a. $\vec{AM} = \vec{MC} \Rightarrow \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} x-1 \\ y-0 \end{pmatrix} \Rightarrow \begin{matrix} x = 1 \\ y = -2 \end{matrix}$ $C(1;-2)$

$\vec{AB} = \begin{pmatrix} x_B + 3 \\ y_B - 2 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ -3 \end{pmatrix} \Rightarrow \begin{vmatrix} x_B + 3 & 1 \\ y_B - 2 & -3 \end{vmatrix} = 0 \Rightarrow$

$\Rightarrow -3x_B - 9 - y_B + 2 = 0 \Rightarrow 3x_B - y_B = 7$ ①

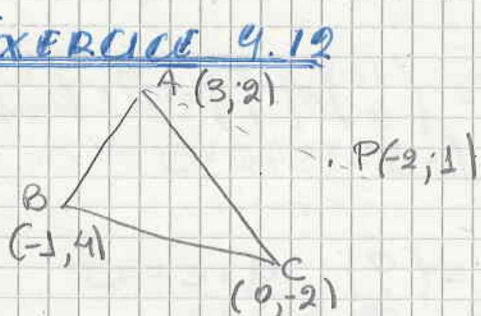
$\vec{BM} = \begin{pmatrix} -1 - x_B \\ 0 - y_B \end{pmatrix} \parallel \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{vmatrix} -1 - x_B & 0 \\ -y_B & 1 \end{vmatrix} = 0 \Rightarrow$

$\Rightarrow -1 - x_B = 0 \Rightarrow x_B = -1$

① $+3 - y_B = 7 \Rightarrow y_B = -4$ $B(-1;-4)$

$\vec{AB} = \vec{DC} \Rightarrow \begin{pmatrix} 2 \\ -6 \end{pmatrix} = \begin{pmatrix} 1 - x_D \\ -2 - y_D \end{pmatrix} \Rightarrow \begin{matrix} x_D = -1 \\ y_D = 4 \end{matrix}$ $D(-1;4)$

EXERCICE 4.12



$\vec{AA'} = 3\vec{AP} \Rightarrow \begin{pmatrix} x-3 \\ y-2 \end{pmatrix} = 3\begin{pmatrix} -5 \\ -1 \end{pmatrix} \Rightarrow \begin{matrix} x = -12 \\ y = -1 \end{matrix}$

$A'(-12;-1)$

$\vec{BB'} = 3\vec{BP} \Rightarrow \begin{pmatrix} x+1 \\ y-4 \end{pmatrix} = 3\begin{pmatrix} -1 \\ -3 \end{pmatrix} \Rightarrow \begin{matrix} x = -4 \\ y = -5 \end{matrix}$

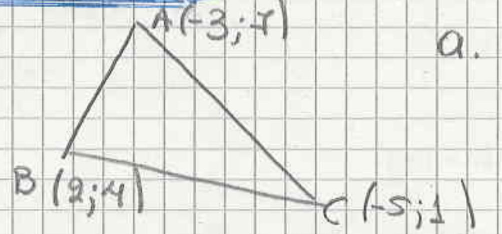
$B'(-4;-5)$

$\vec{CC'} = 3\vec{CP} \Rightarrow \begin{pmatrix} x-0 \\ y+2 \end{pmatrix} = 3\begin{pmatrix} -2 \\ 3 \end{pmatrix} \Rightarrow \begin{matrix} x = -6 \\ y = 7 \end{matrix}$

$C'(-6;7)$

EXERCICE 4.13

a.



a. $G\left(\frac{-3+2-5}{3};\frac{7+4+1}{3}\right)=(-2;4)$

b. $x_G=\frac{x_A+x_B+x_D}{3} \Rightarrow$

$5=\frac{-3+2+x_D}{3} \Rightarrow x_D=16$

$y_G=\frac{y_A+y_B+y_D}{3} \Rightarrow 2=\frac{7+4+y_D}{3} \Rightarrow y_D=-5$

$D(16;-5$

EXERCICE 4.14

$P(-2+3\lambda, 5-4\lambda) \quad \lambda \in \mathbb{R}$

a) $\lambda=0 \quad A(-2;5)$ b) $\lambda=1 \quad B(1,1)$

c) $x=0 \Rightarrow 3\lambda=2 \Rightarrow \lambda=2/3 \quad y=5-\frac{8}{3}=\frac{7}{3}$

$C(0;\frac{7}{3})$

d) $y=0 \Rightarrow 5=4\lambda \Rightarrow \lambda=5/4 \quad x=-2+\frac{15}{4}=\frac{7}{4}$

$D(\frac{7}{4};0)$

e) $y=2x \Rightarrow 5-4\lambda=2(-2+3\lambda) \Rightarrow$

$\Rightarrow 5-4\lambda=-4+6\lambda \Rightarrow 10\lambda=9 \Rightarrow \lambda=9/10$

$x=-2+\frac{27}{10}=\frac{7}{10} \quad y=\frac{14}{10}=\frac{7}{5} \quad E(\frac{7}{10};\frac{7}{5})$

f) $y=7 \Rightarrow 5-4\lambda=7 \Leftrightarrow 4\lambda=-2 \Rightarrow \lambda=-1/2$

$x=-2-\frac{3}{2}=-\frac{7}{2} \quad F(-\frac{7}{2};7)$

$\vec{AB}=\begin{pmatrix}3\\-4\end{pmatrix} \quad \vec{CD}=\begin{pmatrix}7/4\\-7/3\end{pmatrix} \parallel \begin{pmatrix}1/4\\-1/3\end{pmatrix} \parallel \begin{pmatrix}3\\-4\end{pmatrix}$

$\vec{EF}=\begin{pmatrix}-21/5\\28/5\end{pmatrix} \parallel \begin{pmatrix}-21\\28\end{pmatrix} \parallel \begin{pmatrix}-3\\4\end{pmatrix} \parallel \begin{pmatrix}3\\-4\end{pmatrix} \Rightarrow$ sont parallels

EXERCICE 4.15

a. d_1: $A(-2;3)$ $B(8;5)$ $\vec{AB}=\begin{pmatrix}10\\2\end{pmatrix} \parallel \begin{pmatrix}5\\1\end{pmatrix} \Rightarrow \vec{n}=\begin{pmatrix}1\\-5\end{pmatrix}$

$x-5y+c=0 \quad A: -2-15+c=0 \Rightarrow c=17$

$d_1: x-5y+17=0$

b: d_2: $A(-4;1)$ $d_2 \parallel O_x \Rightarrow \vec{n}=\begin{pmatrix}0\\1\end{pmatrix} \quad y+c=0$

$A: 1+c=0 \Rightarrow c=-1 \quad d_2: y-1=0$

c. $d_3 \parallel \vec{d}=\begin{pmatrix}2\\-5\end{pmatrix} \Rightarrow \vec{n}=\begin{pmatrix}5\\2\end{pmatrix} \quad 5x+2y+c=0$

$(0,0) \Rightarrow c=0 \qquad d_3: 5x+2y=0$

EXERCICE 4.16

a) $d_1 \begin{cases} x = 1 - \lambda \\ y = 2 + 2\lambda \end{cases}$ $d_2 : \begin{cases} x = 3 + \mu \\ y = 6 + 2\mu \end{cases}$

$A(1;2)$ $\vec{d_1} = \begin{pmatrix} -1 \\ +2 \end{pmatrix}$ $B(3;6)$ $\vec{d_2} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

b) $1 - \lambda = 3 + \mu \Rightarrow \mu + \lambda = -2$

$2 + 2\lambda = 6 + 2\mu \Rightarrow 2\mu - 2\lambda = -4 \Rightarrow \begin{cases} \mu + \lambda = -2 \\ \mu - \lambda = -2 \end{cases}$

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$2\mu = -4 \Rightarrow \mu = -2$

$\lambda = 0$

$I(1,2)$

c) $\vec{d_1} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \Rightarrow \vec{n_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $2x + y + c = 0$

$A : 2 + 2 + c = 0 \Rightarrow c = -4 : 2x + y - 4 = 0$

$\vec{d_2} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \Rightarrow \vec{n_2} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $2x - y + c = 0$

$B : 6 - 6 + c = 0 \Rightarrow c = 0$ $d_2 : 2x - y = 0$

EXERCICE 4.17.

$d_1 : -x + 2y + 3 = 0$ $d_2 : 3x - 4y - 12 = 0$

a : $x = 0$: $d : 2y = -3 \Rightarrow y = -\frac{3}{2}$ $(0; -\frac{3}{2})$

$d_2 : -4y = 12 \Rightarrow y = -3$ $(0, -3)$

$y = 0$ d_1 $x = 3$ $(3;0)$

$d_2 . \; x = 4$ $(4;0)$

b. $\vec{n_1} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \Rightarrow \vec{d_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\vec{n_2} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \Rightarrow d_2 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

c. $\vec{n_1} \not\parallel n_2 \Rightarrow$ sécantes

e. $\begin{cases} -x + 2y = -3 & \times 3 \\ 3x - 4y = 12 \end{cases}$ $\begin{cases} -3x + 6y = -9 \\ 3x - 4y = 12 \end{cases}$

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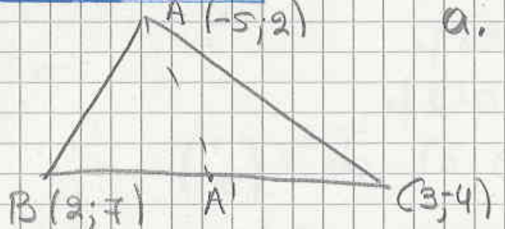
$2y = 3 \Rightarrow y = \frac{3}{2}$

$-x + 3 = -3 \Rightarrow x = 6$ $I(6; \frac{3}{2})$

f) $d_1 : \begin{cases} x = 3 + 2\lambda \\ y = \lambda \end{cases}$ $d_2 \begin{cases} x = 4\lambda \\ y = -3 + 3\lambda \end{cases}$

Exercice 4.18



a. $A'\left(\frac{2+3}{2};\frac{7-4}{2}\right)=\left(\frac{5}{2};\frac{3}{2}\right)$

m_A : $\overrightarrow{AA'}=\begin{pmatrix}-\frac{5}{2}\\-\frac{1}{2}\end{pmatrix} /\!/ \begin{pmatrix}-5\\-1\end{pmatrix} /\!/ \begin{pmatrix}5\\1\end{pmatrix}$

$\Rightarrow \vec{n}_A=\begin{pmatrix}1\\-5\end{pmatrix}$ $\quad x-5y+c=0$

$A : -5-10+c=0 \Rightarrow c=15$

$m_A : x-5y+15=0$

b. $\begin{cases} x=-5+5\lambda \\ y=2+\lambda \end{cases}$

c. • $D(10,1) \overset{?}{\in} m_A \quad 10-5+15=20\neq 0$ Non

• $10=-5+5\lambda \Rightarrow 5\lambda=15 \Rightarrow \lambda=3$
$\quad 1=2+\lambda \Rightarrow \lambda=-1$ $\quad \neq \Rightarrow$ Non

Exercice 4.19

a. $\begin{cases} x=5-2\lambda \\ y=-1+3\lambda \end{cases}$ $\quad \vec{d}=\begin{pmatrix}-2\\3\end{pmatrix} \Rightarrow \vec{n}=\begin{pmatrix}3\\2\end{pmatrix}$

$3x+2y+c=0$

$(5,-1) : 15-2+c=0 \Rightarrow c=-13$

eq cart $3x+2y-13=0$

b. eq cart $x+4y-10=0$ $\quad \vec{n}=\begin{pmatrix}1\\4\end{pmatrix} \Rightarrow \vec{d}=\begin{pmatrix}4\\-1\end{pmatrix}$

1 point $y=0 \quad x=10 \quad (10,0)$

$\begin{cases} x=10+4\lambda \\ y=-\lambda \end{cases}$

c. point $(-7;1)$ $\vec{d}=\begin{pmatrix}6\\5\end{pmatrix}$ eq par $\begin{cases} x=-7+6\lambda \\ y=1+5\lambda \end{cases}$

$\vec{n}=\begin{pmatrix}5\\-6\end{pmatrix}$ $\quad 5x-6y+c=0$

$(-7,1) : -35-6+c=0 \Rightarrow c=41$

eq cart $5x-6y+41=0$

d. point $(0;9)$ $\vec{d}=\begin{pmatrix}-2\\1\end{pmatrix}$ eq par $\begin{cases} x=-2\lambda \\ y=9+\lambda \end{cases}$

$\vec{n}=\begin{pmatrix}1\\2\end{pmatrix}$ $\quad x+2y+c=0$

$(0,9) \quad 18+c=0 \Rightarrow c=-18$

eq cart $x+2y-18=0$

Exercice 4.20

a. $d_1 : \begin{cases} x = 7 + 3\lambda \\ y = -1 + 2\lambda \end{cases}$ $\quad d_2 : \begin{cases} x = -4 - \mu \\ y = 5 + 7\mu \end{cases}$

$\vec{d_1} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \nparallel \vec{d_2} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} \Rightarrow$ sécantes

$\begin{matrix} 7 + 3\lambda = -4 - \mu \\ -1 + 2\lambda = 5 + 7\mu \end{matrix} \Rightarrow \begin{matrix} 3\lambda + \mu = -11 \quad \cdot 7 \\ 2\lambda - 7\mu = 6 \end{matrix} \Rightarrow$

$\Rightarrow \begin{cases} 21\lambda + 7\mu = -77 \\ 2\lambda - 7\mu = 6 \end{cases}$ $(+)$ $23\lambda = -71 \Rightarrow \lambda = -\frac{71}{23}$

$I\left(x = -\frac{52}{23} ; -\frac{165}{23}\right)$

b) $d_1 : \begin{cases} x = 7 + 3\lambda \\ y = -1 + 2\lambda \end{cases}$ $\quad d_2 : x + 8y - 5 = 0$

$7 + 3\lambda + 8(-1 + 2\lambda) - 5 = 0 \Rightarrow$

$\Rightarrow 7 + 3\lambda - 8 + 16\lambda - 5 = 0 \Rightarrow 19\lambda = 6 \Rightarrow \lambda = \frac{6}{19}$

$I\left(\frac{151}{19} ; -\frac{7}{19}\right)$

c) $\begin{cases} 3x - 2y = -6 \quad \cdot 4 \\ x + 8y = 5 \end{cases}$ $\quad \begin{cases} 12x - 8y = -24 \\ x + 8y = 5 \end{cases}$

$13x = -19 \Rightarrow x = -\frac{19}{13}$

$I\left(-\frac{19}{13} ; \frac{21}{26}\right)$ $\quad y = \frac{21}{26}$

Exercice 4.21

$\vec{BC} \parallel \vec{v} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ $\quad G\left(\frac{7}{3} ; \frac{1}{3}\right)$

$d_{AB} : 5x + 3y - 8 = 0$

$\vec{AG} = 2\vec{GA'} \Rightarrow \begin{pmatrix} 4/3 \\ -2/3 \end{pmatrix} = 2\begin{pmatrix} x - \frac{7}{3} \\ y - 1/3 \end{pmatrix} \Rightarrow \begin{matrix} \frac{4}{3} = 2x - \frac{14}{3} \Rightarrow x = 3 \\ -\frac{2}{3} = 2y - \frac{2}{3} \Rightarrow y = 0 \end{matrix}$

$A'(3 ; 0)$

$d_{BC} : \begin{cases} x = 3 - \lambda \\ y = 4\lambda \end{cases}$ $\quad B = d_{BC} \cap d_{AB}$

$5(3 - \lambda) + 3 \cdot 4\lambda - 8 = 0 \Rightarrow$

$\Rightarrow 15 - 5\lambda + 12\lambda - 8 = 0 \Rightarrow 7\lambda = -7 \Rightarrow \lambda = -1$

$B(4 ; -4)$

$\frac{7}{\cancel{3}} = \frac{1 + 4 + x_C}{\cancel{3}} \Rightarrow x_C = 2$ $\quad \frac{1}{\cancel{3}} = \frac{1 - 4 + y_C}{\cancel{3}} \Rightarrow y_C = 4$

$C(2 ; 4)$

Exercice 4.22.

$D(-7;2)$ $C \in d_1 : 3x+y+2=0$ $C \in d_2 : \begin{cases} x = 7+3\lambda \\ y = -2-2\lambda \end{cases}$

$C = d_1 \cap d_2$

$3(7+3\lambda) - 2 - 2\lambda + 2 = 0 \Leftrightarrow 21 + 9\lambda - \cancel{2} - 2\lambda + \cancel{2} = 0 \Leftrightarrow$

$\Leftrightarrow 7\lambda = -21 \Rightarrow \lambda = -3.$ $C(-2;4)$

d_{DC} : $\vec{DC} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \rightarrow \vec{n} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ $2x - 5y + c = 0$

C ; $-4 - 20 + c = 0 \Rightarrow c = 24$

$d_{DC} : 2x - 5y + 24 = 0$

d_{AD} : $\vec{AD} \,/\!/ \begin{pmatrix} 2 \\ -5 \end{pmatrix} \rightarrow AD : \begin{cases} x = -7 + 2\lambda \\ y = 2 - 5\lambda \end{cases}$

$A \in d_{AD}$ $A(-7+2\lambda ; 2-5\lambda)$

$\| \vec{DC} \| = \sqrt{25+4} = \sqrt{29}$

$\vec{AD} = \begin{pmatrix} -2\lambda \\ 5\lambda \end{pmatrix},$ $\| \vec{AD} \| = \sqrt{4\lambda^2 + 25\lambda^2} = \sqrt{29\lambda^2}$

$\| \vec{AD} \| = \| \vec{DC} \| \Rightarrow \sqrt{29} = \sqrt{29\lambda^2} \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$

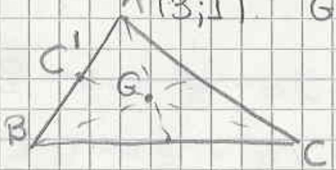
$\lambda = 1$ $A(-5;-3)$ $\lambda = -1$ $A(-9;7)$

$\vec{AB} = \begin{pmatrix} x+5 \\ y+3 \end{pmatrix} = \vec{DC} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ $\vec{AB} = \begin{pmatrix} x+9 \\ y-7 \end{pmatrix} = \vec{DC} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \Rightarrow$

$\Rightarrow$ $x = 0$, $y = -1$ $B(0;-1)$ $x = -4$, $y = 9$ $B(-4;9)$

Exercice 4.23

$A(3;1)$ $G(2;3)$ $C'(4;4)$



a. $\vec{CG} = 2\vec{GC'} \rightarrow \begin{pmatrix} 2-x \\ 3-y \end{pmatrix} = 2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} \Rightarrow$

$\Rightarrow 2 - x = 4$, $3 - y = 2$ $\Rightarrow x = -2$, $y = 1$ $C(-2;1)$

• $\vec{AC'} = \vec{C'B} \Rightarrow \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} x-4 \\ y-4 \end{pmatrix} \Rightarrow x = 5$, $y = 7$ $B(5;7)$

b. $\vec{CC'} = \begin{pmatrix} 4+2 \\ 4-1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} /\!/ \begin{pmatrix} 2 \\ 1 \end{pmatrix} \Rightarrow \vec{n} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ $x - 2y + c = 0$

C' : $4 - 8 + c = 0 \Rightarrow c = 4$: $m_c : x - 2y + 4 = 0$

c. $\vec{BC} = \begin{pmatrix} -7 \\ -6 \end{pmatrix}$ $d_{BC} : \begin{cases} x = -2 - 7\lambda \\ y = 1 - 6\lambda \end{cases}$

Exercice 4.24

$d_m : 4x - my + 2 = 0$

a. $A(2;-3)$ $\quad 8 + 3m + 2 = 0 \Rightarrow 3m = -10 \Rightarrow m = -\frac{10}{3}$

b. $// Oy$ $\quad -m = 0 \Rightarrow m = 0 \quad 4x = -2 \Rightarrow x = -\frac{1}{2} \; // Oy$

c. $\vec{d_m} = \begin{pmatrix} m \\ 4 \end{pmatrix} // \begin{pmatrix} 1 \\ 3 \end{pmatrix} \Rightarrow \begin{vmatrix} m & 1 \\ 4 & 3 \end{vmatrix} = 0 \Rightarrow 3m - 4 = 0 \Rightarrow$
$m = 4/3$

d. $\vec{BC} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \perp d_m \Rightarrow \vec{n_m} = \begin{pmatrix} 4 \\ -m \end{pmatrix} // \begin{pmatrix} 2 \\ -5 \end{pmatrix} \Rightarrow$
$\begin{vmatrix} 4 & 2 \\ -m & -5 \end{vmatrix} = 0 \Rightarrow -20 + 2m = 0 \Rightarrow 2m = 20 \Rightarrow m = 10$

Exercice 4.25

$d_{AB} : 3x - 5y + 1 = 0$
$d_{AC} : x - 9y - 29 = 0$
$C(11, y) \quad \vec{BC} // \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

- $A = d_{AB} \cap d_{AC}$ $\begin{cases} 3x - 5y + 1 = 0 \\ x - 9y - 29 = 0 \quad \times(-3) \end{cases}$ $\quad \begin{array}{r} 3x - 5y + 1 = 0 \\ + \; -3x + 27y + 87 = 0 \\ \hline 22y + 88 = 0 \end{array}$
$\Rightarrow y = -4$

$y = -4 \quad x + 36 - 29 = 0 \Rightarrow$
$x = -7$

$A(-7;-4)$

- $C \in d_{AC} : 11 - 9y = 29 \Rightarrow 9y = -18 \Rightarrow y = -2$
$C(11;-2)$

- $\vec{BC} = \begin{pmatrix} 11-x \\ -2-y \end{pmatrix} // \begin{pmatrix} 2 \\ -1 \end{pmatrix} \Rightarrow \begin{vmatrix} 11-x & 2 \\ -2-y & -1 \end{vmatrix} = 0 \Rightarrow -11 + x + 4 + 2y = 0$
$\Rightarrow x + 2y - 7 = 0$

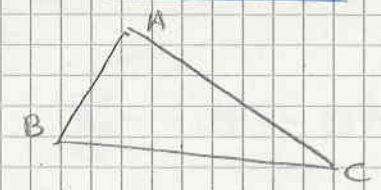
$B = \begin{cases} x + 2y = 7 \quad \times(-3) \\ 3x - 5y = -1 \end{cases}$ $\quad \begin{cases} -3x - 6y = -21 \\ 3x - 5y = -1 \end{cases}$
$-11y = -22 \Rightarrow y = 2$

$y = 2 \quad x + 4 = 7 \Rightarrow$
$x = 3$

$B(3;2)$

Exercice 4.26 → page 13

Exercice 4.27



$A(2;8)\quad B(-4;3)\quad C(4;6)$

$\vec{AB} = \begin{pmatrix}-6\\-5\end{pmatrix} \quad \vec{AC} = \begin{pmatrix}2\\-2\end{pmatrix} \quad \vec{BC} = \begin{pmatrix}8\\3\end{pmatrix}$

$\vec{AB}\cdot\vec{AC} = -12+10 = -2 < 0 \Rightarrow A$ obtus

- $\|\vec{AB}\| = \sqrt{36+25} = \sqrt{61} \quad \|\vec{AC}\| = \sqrt{4+4} = \sqrt{8} \quad \|\vec{BC}\| = \sqrt{64+9} = \sqrt{73}$

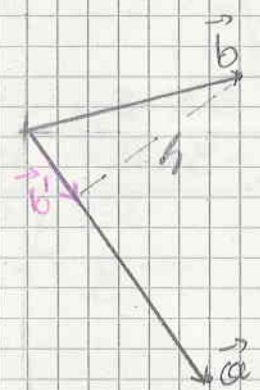
  $P = \sqrt{61} + \sqrt{8} + \sqrt{73}$

- $\hat{A}: \quad \cos A = \dfrac{\vec{AB}\cdot\vec{AC}}{\|\vec{AB}\|\cdot\|\vec{AC}\|} = \dfrac{-2}{\sqrt{61}\cdot\sqrt{8}} \Rightarrow \hat{A} = 95.2°$

  $\hat{B}: \quad \cos B = \dfrac{\vec{BA}\cdot\vec{BC}}{\|\vec{BA}\|\cdot\|\vec{BC}\|} = \dfrac{48+15}{\sqrt{61}\cdot\sqrt{73}} \Rightarrow \hat{B} = 19.25$

$\Rightarrow \hat{C} = 180 - 95.2° - 19.25 \Rightarrow \hat{C} = 65.55°$

Exercice 4.28



b. $\vec{a}\cdot\vec{b} = 42 - 16 = 26$

$\|\vec{a}\| = \sqrt{36+64} = 10 \quad \|\vec{b}\| = \sqrt{49+4} = \sqrt{53}$

$\vec{a}\cdot\vec{b} = \|\vec{a}\|\cdot b' \Rightarrow b' = \dfrac{26}{10} = 2.6$

c. $A = \frac{1}{2}\, b\cdot h = \frac{1}{2}\|\vec{a}\|\cdot h$

$h = \sqrt{\|b\|^2 - b'^2} = \sqrt{53 - 2.6^2} = 6.8$

$A = \frac{1}{2}\, 10\cdot 6.8 = 34$

$A = \frac{1}{2}\left|\begin{vmatrix}6 & 7\\ -8 & 2\end{vmatrix}\right| = \frac{1}{2}|12+56| = 34$

d. $\left.\begin{array}{l}\vec{b'} \,/\!/\, \vec{a} \qquad \|\vec{b'}\| = 2.6\\ \vec{u}_a = \begin{pmatrix}6/10\\-8/10\end{pmatrix} = \begin{pmatrix}0.6\\-0.8\end{pmatrix}\end{array}\right\} \vec{b'} = 2.6\begin{pmatrix}0.6\\-0.8\end{pmatrix} \Rightarrow \vec{b'} = \begin{pmatrix}1.56\\-2.08\end{pmatrix}$

Exercice 4.29

a. $\vec{a} = \begin{pmatrix}-7\\4\end{pmatrix} \quad \vec{b} = \begin{pmatrix}x\\-3\end{pmatrix} \quad \vec{a}\perp\vec{b} \Rightarrow \vec{a}\cdot\vec{b} = 0 \Rightarrow -7x - 12 = 0 \Rightarrow$

$7x = -12 \Rightarrow x = -\dfrac{12}{7}$

b. $\vec{c} = \begin{pmatrix}x\\2x-3\end{pmatrix} \quad \vec{d} = \begin{pmatrix}x+1\\-4\end{pmatrix} \quad \vec{c}\perp\vec{d} \Rightarrow \vec{c}\cdot\vec{d} = 0 \Rightarrow$

$\Rightarrow x(x+1) - 4(2x-3) = 0 \Rightarrow$

$\Rightarrow x^2 + x - 8x + 12 = 0 \Rightarrow x^2 - 7x + 12 = 0$

$\Delta = 1 \quad x_{1,2} = \dfrac{7\pm 1}{2} = \begin{cases} x_1 = 4\\ x_2 = 3\end{cases}$

Exercice 4.26

$\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ $\vec{c} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ $\vec{d} = \begin{pmatrix} -5 \\ -2 \end{pmatrix}$

a. $\vec{a} \cdot \vec{b} = 10 - 21 = -11$ $\quad \vec{b} \cdot \vec{c} = -20 + 21 = 1$ $\quad \vec{b} \cdot \vec{d} = -25 - 14 = -39$

b. $\|\vec{a}\| = \sqrt{4+9} = \sqrt{13}$ $\quad \|\vec{b}\| = \sqrt{25+49} = \sqrt{74}$ $\quad \|\vec{c}\| = \sqrt{16+9} = 5$ $\quad \|\vec{d}\| = \sqrt{25+4} = \sqrt{29}$

c. $\vec{e} \perp \vec{a} \Rightarrow \vec{e} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

d. $\vec{u} = \begin{pmatrix} -4/5 \\ 3/5 \end{pmatrix}$

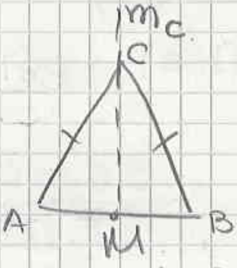
e. les vecteurs $\vec{a}, \vec{d}$

f. $\vec{f} = \begin{pmatrix} f_1 \\ -5 \end{pmatrix} \perp \vec{c} \Rightarrow \vec{f} \cdot \vec{c} = 0 \Rightarrow -4f_1 - 15 = 0 \Rightarrow f_1 = -\frac{15}{4}$

Exercice 4.30

le même que 4.28

Exercice 4.31



$A(-1;-2)$ $\quad B(7;4)$

Le point C appartient à la médiatrice du segment AB.

$M\left(\frac{-1+7}{2} ; \frac{-2+4}{2}\right) = (3;1)$

$\vec{AB} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} \parallel \begin{pmatrix} 4 \\ 3 \end{pmatrix} \Rightarrow \vec{d_m} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ $\quad m_c = \begin{cases} x = 3 + 3\lambda \\ y = 1 - 4\lambda \end{cases}$

$C(3+3\lambda ; 1-4\lambda)$

$\vec{CB} = \begin{pmatrix} 7-3-3\lambda \\ 4-1+4\lambda \end{pmatrix} = \begin{pmatrix} 4-3\lambda \\ 3+4\lambda \end{pmatrix}$ $\quad \vec{CA} = \begin{pmatrix} -1-3-3\lambda \\ -2-1+4\lambda \end{pmatrix} = \begin{pmatrix} -4-3\lambda \\ -3+4\lambda \end{pmatrix}$

$\det(\vec{CB}, \vec{CA}) = \begin{vmatrix} 4-3\lambda & -4-3\lambda \\ 3+4\lambda & -3+4\lambda \end{vmatrix} = (4-3\lambda)(-3+4\lambda) - (-4-3\lambda)(3+4\lambda)$

$= 50\lambda$

$\text{Aire} = \frac{1}{2} 50 |\lambda| = 75 \Rightarrow |\lambda| = 3 \Rightarrow \lambda = \pm 3$

$\lambda = 3$ $\quad C(12;-11)$ $\quad \lambda = -3$ $\quad C(-6;13)$

EXERCICE 4.32

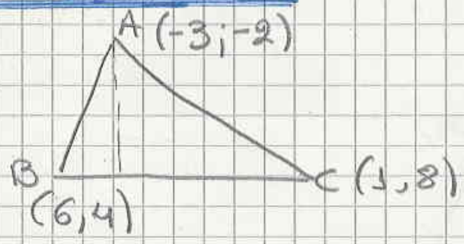
$\vec{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ $\vec{c} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ $\|\vec{a}\| = \sqrt{5}$ $\|\vec{b}\| = 5$ $\|\vec{c}\| = \sqrt{74}$

a. $\cos\gamma = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\cdot\|\vec{b}\|} = \dfrac{-8-3}{\sqrt{5}\cdot 5} \Rightarrow \gamma = 169.7^\circ$

b. $\cos\beta = \dfrac{\vec{a}\cdot\vec{c}}{\|\vec{a}\|\cdot\|\vec{c}\|} = \dfrac{10-7}{\sqrt{5}\cdot\sqrt{74}} \Rightarrow \beta = 81.027^\circ$

c. $\cos\alpha = \dfrac{\vec{b}\cdot\vec{c}}{\|\vec{b}\|\cdot\|\vec{c}\|} = \dfrac{-20+21}{5\cdot\sqrt{75}} \Rightarrow \alpha = 88.68^\circ$

EXERCICE 4.33



- $\vec{AB} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$ $\|\vec{AB}\| = \sqrt{81+36} = \sqrt{117}$

$\vec{AC} = \begin{pmatrix} 4 \\ 10 \end{pmatrix}$ $\|\vec{AC}\| = \sqrt{16+100} = \sqrt{116}$

$\vec{BC} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$ $\|\vec{BC}\| = \sqrt{25+16} = \sqrt{41}$

- $\cos\alpha = \dfrac{\vec{AB}\cdot\vec{AC}}{\|\vec{AB}\|\cdot\|\vec{AC}\|} = \dfrac{36+60}{\sqrt{117}\cdot\sqrt{116}} \Rightarrow \alpha = 34.5^\circ$

$\cos\beta = \dfrac{\vec{BA}\cdot\vec{BC}}{\|\vec{BA}\|\cdot\|\vec{BC}\|} = \dfrac{45-24}{\sqrt{117}\cdot\sqrt{41}} \Rightarrow \beta = 72.35^\circ$

$\cos\gamma = \dfrac{\vec{CA}\cdot\vec{CB}}{\|\vec{CA}\|\cdot\|\vec{CB}\|} = \dfrac{-20+40}{\sqrt{116}\cdot\sqrt{41}} \Rightarrow \gamma = 73.14^\circ$

- $\vec{BC} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} \Rightarrow \vec{n}_{BC} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $4x+5y+c=0$

$B: 24+20+c=0 \Rightarrow c=-44$

$d_{BC}: 4x+5y-44=0$

$h_A = \text{dist}(A, d_{BC}) = \dfrac{|-12-10-44|}{\sqrt{16+25}} \simeq 10.31$

$\vec{AC} = \begin{pmatrix} 4 \\ 10 \end{pmatrix} // \begin{pmatrix} 2 \\ 5 \end{pmatrix} \Rightarrow \vec{n}_{AC} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ $5x-2y+c=0$

$C: 5-16+c=0 \Rightarrow c=11$

$d_{AC}: 5x-2y+11=0$

$h_B = \text{dist}(B, d_{AC}) = \dfrac{|30-8+11|}{\sqrt{25+4}} \approx 6.13$

$\vec{AB} = \begin{pmatrix} 9 \\ 6 \end{pmatrix} // \begin{pmatrix} 3 \\ 2 \end{pmatrix} \Rightarrow \vec{n}_{AB} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ $2x-3y+c=0$

$B: 12-12+c=0 \Rightarrow c=0$ $d_{AB}: 2x-3y=0$

$h_C = \text{dist}(C, d_{AB}) = \dfrac{|2-24|}{\sqrt{4+9}} \approx 6.1$

Exercice 4.34

$\vec{a} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 1 \\ y \end{pmatrix}$ $\vec{a} \cdot \vec{b} = 12 - 5y$

$\|\vec{a}\| = \sqrt{144+25} = 13$ $\|\vec{b}\| = \sqrt{1+y^2}$

$\cos\varphi = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} = \cos 60° = \dfrac{1}{2} \Rightarrow \dfrac{1}{2} = \dfrac{12-5y}{13 \cdot \sqrt{1+y^2}} \Leftrightarrow$

$\Leftrightarrow 2(12-5y) = 13\sqrt{1+y^2} \Leftrightarrow 4(12-5y)^2 = 169(1+y^2) \Leftrightarrow$

$\Leftrightarrow 4(144 - 120y + 25y^2) = 169 + 169y^2 \Leftrightarrow$

$\Leftrightarrow 69y^2 + 480y - 407 = 0$ $\Delta = 342732$

$y_{1,2} = \dfrac{-480 \pm \sqrt{342732}}{138}$ $\begin{cases} y_1 = 0{,}76 \\ y_2 = -7{,}72 \end{cases}$

Exercice 4.35

$d_1 : 5x - 2y + 8 = 0$ $d_2 : 3y = 4x - 1 \Rightarrow 4x - 3y - 1 = 0$

$\vec{d_1} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ $\vec{d_2} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\cos\varphi = \dfrac{\vec{d_1} \cdot \vec{d_2}}{\|\vec{d_1}\| \cdot \|\vec{d_2}\|} = \dfrac{6+20}{\sqrt{4+25}\sqrt{9+16}} \Rightarrow \varphi = 15°$

Exercice 4.36 $\vec{AP} \perp \vec{n}$

a. $\vec{AP} = \begin{pmatrix} x-3 \\ y-1 \end{pmatrix} \perp \vec{n} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \Rightarrow \vec{AP} \cdot \vec{n} = 0 \Rightarrow$

$\Rightarrow 2(x-3) - 3(y-1) = 0 \Rightarrow 2x - 3y - 3 = 0$ droite

b. $\vec{AP} = \begin{pmatrix} x+5 \\ y-2 \end{pmatrix} \perp \vec{n} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \Rightarrow \vec{AP} \cdot \vec{n} = 0 \Rightarrow$

$\Rightarrow -4(x+5) - (y-2) = 0 \Rightarrow -4x - 20 - y + 2 = 0 \Rightarrow$

$\Rightarrow -4x - y - 18 = 0 \Rightarrow 4x + y + 18 = 0$

Exercice 4.37

a. $\vec{d} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} \Rightarrow \vec{n} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ $5x + 2y + c = 0$

$A(-2;3)$ $-10 + 6 + c = 0 \Rightarrow c = 4$

$a : 5x + 2y + 4 = 0$

b. $\vec{n} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ $-4x + 3y + c = 0$ $B : (3;-1)$ $-12 - 3 + c = 0 \Rightarrow$
$c = 15$

$b : -4x + 3y + 15 = 0$

c. $\vec{n}_c = \vec{d}_a = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ $-2x+5y+c=0$

$C(-6;0)$ $12+c=0 \Rightarrow c=-12$ $c: -2x+5y-12=0$

d. $d // b \Rightarrow \vec{n}_d = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ $-4x+3y+c=0$

$D(5;2)$ $-20+6+c=0 \Rightarrow c=14$

$d: -4x+3y+14=0$

EXERCICE 4.38

a. $d_1: 3x-4y-12=0$ $\vec{n} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \Rightarrow \vec{d} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$x=0$ $y=-3$ $A(0;-3)$

$y=0$ $x=4$ $B(4;0)$

b. $d_2: 5x+3y+9=0$ $\vec{n} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

$x=0$ $y=-3$ $A(0;-3)$

c. $A(-2;5)$ $B(4;1)$ $\vec{d} = \vec{AB} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \Rightarrow \vec{n} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

EXERCICE 4.39

$d: x+4y-5=0$ $A(-1;4)$ $B(5;2)$ $\vec{AB} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

a. $C \in d$ $x+4y-5=0 \Rightarrow x=-4y+5$

$C(-4y+5, y)$ $\vec{CA} = \begin{pmatrix} -6+4y \\ 4-y \end{pmatrix}$ $\vec{CB} = \begin{pmatrix} 4y \\ 2-y \end{pmatrix}$

- $\vec{CA} \perp \vec{CB} \Rightarrow \vec{CA} \cdot \vec{CB} = 0 \Rightarrow$

$(-6+4y) \cdot 4y + (4-y)(2-y) = 0 \Rightarrow$

$\Rightarrow -24y+16y^2+8-2y-4y+y^2=0 \Leftrightarrow$

$\Leftrightarrow 17y^2-30y+8=0$ $\Delta = 356$

$y_{1,2} = \dfrac{30 \pm \sqrt{356}}{34}$ $y_1 = 1.43$ $C_1(-0.72; 1.43)$; $y_2 = 0.33$ $C_2(3.7; 0.33)$

- $\vec{AB} \perp \vec{CB} \Rightarrow \vec{AB} \cdot \vec{CB} = 0 \Rightarrow 24y - 2(2-y) = 0 \Rightarrow$

$\Rightarrow 24y - 4 + 2y = 0 \Rightarrow 26y = 4 \Rightarrow y = \dfrac{2}{13}$

$C_3\left(\dfrac{57}{13}; \dfrac{2}{13}\right)$

- $\vec{AB} \perp \vec{CA} \Rightarrow \vec{AB} \cdot \vec{CA} \Rightarrow 6(-6+4y) - 2(4-y) = 0 \Leftrightarrow$

$\Rightarrow -36+24y-8+2y=0 \Rightarrow 26y = 44 \Rightarrow y = \dfrac{22}{13}$

$C_4\left(-\dfrac{23}{13}; \dfrac{22}{13}\right)$

b. $C \in d \quad C(-4y+5, y)$

$\|\vec{CA}\| = \|\vec{CB}\| \Rightarrow \sqrt{(4y-6)^2+(4-y)^2} = \sqrt{16y^2+(2-y)^2} \Rightarrow$

$\Rightarrow \cancel{16y^2} - 48y + 36 + 16 - 8y + \cancel{y^2} = \cancel{16y^2} + 4 - 4y + \cancel{y^2} \Rightarrow$

$\Rightarrow -56y + 52 = -4y + 4 \Rightarrow$

$\Rightarrow 52y = 48 \Rightarrow y = \frac{24}{26} = \frac{12}{13}$

$C\left(\frac{17}{13}; \frac{12}{13}\right)$

c. $\vec{AB} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} 6-4y \\ y-4 \end{pmatrix}$

$\begin{vmatrix} 6 & 6-4y \\ -2 & y-4 \end{vmatrix} = 6(y-4) + 2(6-4y) =$

$= 6y - 24 + 12 - 8y = -2y - 12$

$A = 6 = \frac{1}{2}|-2y-12| \Rightarrow |2y+12| = 12 \Rightarrow$

$\Rightarrow |y+6| = 6 \Rightarrow \begin{cases} y+6 = 6 \Rightarrow y_1 = 0 \\ y+6 = -6 \Rightarrow y_2 = -12 \end{cases}$

$y_1 = 0 \quad C(5,0) \qquad y_2 = -12 \quad C(53;-12)$

EXERCICE 4.40

$a: 3x - 4y - 17 = 0$

a. $b \perp a \quad \vec{n}_a = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \Rightarrow \vec{n}_b = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad 4x + 3y + c = 0$

$B(-3;6) \quad -12 + 18 + c = 0 \Rightarrow c = -6$

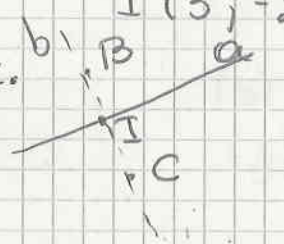
$b: 4x + 3y - 6 = 0$

b. $I = a \cap b \quad \begin{cases} 3x - 4y = 17 & \cdot 3 \\ 4x + 3y = 6 & \cdot 4 \end{cases} \Rightarrow \begin{cases} 9x - 12y = 51 \\ 16x + 12y = 24 \end{cases} \quad +$

$25x = 75 \Rightarrow x = 3$

$x = 3 \quad 9 - 4y = 17 \Rightarrow y = -2$

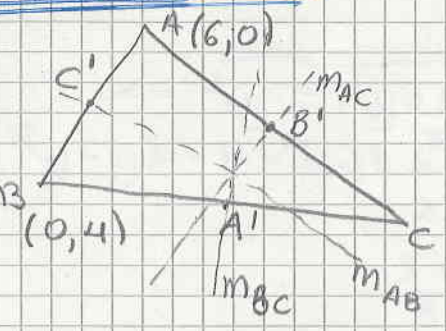
$I(3;-2)$

c.


$\vec{BI} = \vec{IC} \Rightarrow \begin{pmatrix} 6 \\ -8 \end{pmatrix} = \begin{pmatrix} x-3 \\ y+2 \end{pmatrix} \Rightarrow \begin{matrix} x = 9 \\ y = -10 \end{matrix}$

$C(9;-10)$

EXERCICE 4.41



a. $A' : (-1;2)$ $B'(2;0)$ $C'(3;2)$

$\vec{AB} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} -8 \\ 0 \end{pmatrix}$ $\vec{BC} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

• $m_{AB} : -6x + 4y + c = 0$

$C' : -18 + 8 + c = 0 \Rightarrow c = 10$

$-3x + 2y + 5 = 0$

• $m_{AC} : -8x + c = 0$ $B' : -16 + c = 0 \Rightarrow c = 16$

$-8x + 16 = 0 \Rightarrow x - 2 = 0$

• $m_{BC} : -2x - 4y + c = 0$ $A' : 2 - 8 + c = 0 \Rightarrow c = 6$

$-2x - 4y + 6 = 0 \Rightarrow x + 2y - 3 = 0$

b. $M = m_{AC} \cap m_{BC}$ $\begin{cases} x = 2 \\ x + 2y = 3 \end{cases} \Rightarrow \begin{matrix} x = 2 \\ y = \frac{1}{2} \end{matrix}$ $M(2; \frac{1}{2})$

c. le centre $= M$ $r = \|\vec{MA}\|$

$\vec{MA} = \begin{pmatrix} 4 \\ -\frac{1}{2} \end{pmatrix}$ $r = \sqrt{16 + \frac{1}{4}} = \frac{\sqrt{65}}{2}$

EXERCICE 4.42

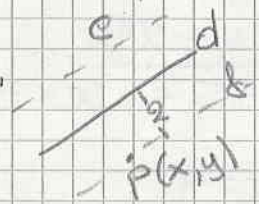
$d : 4x - 3y - 24 = 0$

a. $\text{dist}(O, d) = \frac{|-24|}{\sqrt{16+9}} = \frac{24}{5}$

$B(11; -10)$ $\text{dist}(B, d) = \frac{|44 + 30 - 24|}{\sqrt{16+9}} = \frac{50}{5} = 10$

$C(9; 4)$ $\text{dist}(C, d) = \frac{|36 - 12 - 24|}{5} = 0 \Rightarrow C \in d$

b.



$\text{dist}(P, d) = 2 \Rightarrow \frac{|4x - 3y - 24|}{5} = 2 \Rightarrow$

$\Rightarrow |4x - 3y - 24| = 10 \Rightarrow$

$\begin{cases} 4x - 3y - 24 = 10 \Rightarrow 4x - 3y - 34 = 0 \quad (e) \\ 4x - 3y - 24 = -10 \Rightarrow 4x - 3y - 14 = 0 \quad (f) \end{cases}$

Exercice 4.43

$a: 4x+3y-12=0 \quad b: 7x-y-46=0 \Rightarrow y = 7x-46$

$B \in b \quad B(x, 7x-46)$

$$\text{dist}(B,a) = 5 \Rightarrow \frac{|4x+3(7x-46)-12|}{\sqrt{16+9}} = 5 \Rightarrow$$

$$\Rightarrow \frac{|4x+21x-138-12|}{5} = 5 \Rightarrow |25x-150| = 25 \Rightarrow$$

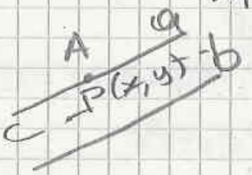
$$\begin{cases} 25x-150 = 25 \Rightarrow 25x = 175 \Rightarrow x_1 = 7 & B_1(7;3) \\ 25x-150 = -25 \Rightarrow 25x = 125 \Rightarrow x_2 = 5 & B_2(5;-11) \end{cases}$$

Exercice 4.44

$a.\ 4x+3y-24=0 \quad b \parallel a \quad B(0;13)$

a. $b: 4x+3y+c=0 \quad B: 39+c=0 \Rightarrow c=-39$

$4x+3y-39=0$



Soit $A \in a: x=0 \quad 3y=24 \Rightarrow y=8$

$A(0;8) \quad \text{dist}(A,b) = \text{dist}(a,b) =$

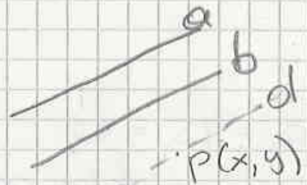
$$= \frac{|24-39|}{5} = \frac{15}{5} = 3$$

b. Soit $P(x,y) \in c$

$$\text{dist}(P,a) = \text{dist}(P,b) \Rightarrow \frac{|4x+3y-24|}{\cancel{5}} = \frac{|4x+3y-39|}{\cancel{5}} \Rightarrow$$

$$\Rightarrow \begin{cases} \cancel{4x}+\cancel{3y}-24 = \cancel{4x}+\cancel{3y}-39 & \text{impossible} \\ 4x+3y-24 = -4x-3y+39 \Rightarrow 8x+6y-63=0 \ (c) \end{cases}$$

c.



$\text{dist}(P,a) = 2\,\text{dist}(P,b) \Rightarrow$

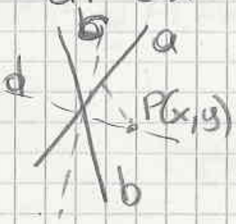
$$\frac{|4x+3y-24|}{\cancel{5}} = 2\,\frac{|4x+3y-39|}{\cancel{5}} \Rightarrow$$

$$\begin{cases} 4x+3y-24 = 8x+6y-78 \Rightarrow -4x-3y+54=0 \ (d_1) \\ 4x+3y-24 = -8x-6y+78 \Rightarrow 12x+9y-102=0 \ (d_2) \end{cases}$$

$$\Rightarrow 4x+3y-34=0$$

Exercice 4.45

$a: 3x-4y+12=0 \quad b: 12x+5y-15=0$



Soit $P \in$ bissectrice.

$$\text{dist}(P,a) = \text{dist}(P,b) \Rightarrow \frac{|3x-4y+12|}{5} = \frac{|12x+5y-15|}{13}$$

$$\Rightarrow \begin{cases} 13(3x-4y+12) = 5(12x+5y-15) \\ 13(3x-4y+12) = -5(12x+5y-15) \end{cases} \Rightarrow$$

$$\begin{cases} 39x-52y+156 = 60x+25y-75 \\ 39x-52y+156 = -60x-25y+75 \end{cases} \Rightarrow \begin{cases} 21x+77y-231=0 \ (c) \\ 99x-27y+81=0 \ (d) \end{cases}$$

Exercice 4.46

$M(5;3)$

a. $P(x,y)$ $\vec{PM} = \begin{pmatrix} x-5 \\ y-3 \end{pmatrix}$ $\|\vec{PM}\| = \sqrt{(x-5)^2+(y-3)^2} = 5 \Rightarrow$

$\Rightarrow (x-5)^2+(y-3)^2 = 25$

b. $y=0$ $(x-5)^2+(-3)^2 = 25 \Rightarrow (x-5)^2 = 16 \Rightarrow$

$x-5=4 \Rightarrow x_1 = 9$ $(9;0)$

$x-5=-4 \Rightarrow x_2 = 1$ $(1;0)$

Exercice 4.47

a. $x^2+y^2-14x-2y-126=0 \Rightarrow (x-7)^2-49+(y-1)^2-1-126=0$

$\Leftrightarrow (x-7)^2+(y-1)^2 = 176$ $K(7;1)$ $r=\sqrt{176}$

b. $x^2+y^2+10x+14y+123=0 \Rightarrow (x+5)^2-25+(y+7)^2-49+123=0$

$\Leftrightarrow (x+5)^2+(y+7)^2 = -49 < 0$ impossible

c. $x^2+y^2+8x-16y+80=0 \Leftrightarrow (x+4)^2-16+(y-8)^2-64+80=0 \Rightarrow$

$\Leftrightarrow (x+4)^2+(y-4)^2 = 0$ C'est pas un cercle

d. $3x^2+3y^2+7x-10=0 \Rightarrow x^2+y^2+\frac{7}{3}x-\frac{10}{3}=0$

$\Rightarrow \left(x+\frac{7}{6}\right)^2-\frac{49}{36}+y^2-\frac{10}{3}=0 \Leftrightarrow \left(x+\frac{7}{6}\right)^2+y^2 = \frac{169}{36}$

$K\left(-\frac{7}{6};0\right)$ $r=\frac{13}{6}$

Exercice 4.48

a. $C(-7;4)$ $r=13$ $(x+7)^2+(y-4)^2=169$

b: $A(a_1,9) \in C$ $(a_1+7)^2+(9-4)^2=169 \Rightarrow$

$\Rightarrow a_1^2+14a_1+49+25=169 \Rightarrow a_1^2+14a_1-95=0$

$\Delta = 576$ $a_{1,2} = \frac{-14\pm 24}{2}$ $\begin{cases} a_1 = 5 > 0 \\ a_2 = -19 \end{cases}$ $A(5;9)$

$B(-2,b_2) \in C$:

$(-2+7)^2+(b_2-4)^2=169 \Rightarrow 25+(b_2-4)^2=169 \Leftrightarrow$

$\Leftrightarrow (b_2-4)^2 = 144 \Rightarrow \begin{cases} b_2-4=12 \Rightarrow b_2=14 \\ b_2-4=-12 \Rightarrow b_2=-8<0 \end{cases}$ $B(-2;-8)$

c:

$M\left(\frac{3}{2};\frac{1}{2}\right)$ $\vec{AB} = \begin{pmatrix} -7 \\ -17 \end{pmatrix} \parallel \begin{pmatrix} 7 \\ 17 \end{pmatrix} = \vec{n}_m$

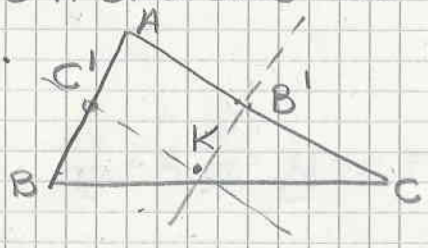
$\left.\begin{array}{l} 7x+17y+c=0 \\ M: \frac{21}{2}+\frac{17}{2}+c=0 \Rightarrow c=-19 \end{array}\right\} 7x+17y-19=0$

d. $C(-7,4) \in m$ $-49+68-19=0$ ✓ $\Rightarrow C \in m$

EXERCICE 4.49

$A(-3,3)$ $B(-1,-3)$ $C(5;3)$

- On cherche la médiatrice du segment AB



$C'(-2;0)$

$\vec{AB} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ -3 \end{pmatrix} \perp m_A$

$x - 3y + c = 0$ C': $-2 + c = 0 \Rightarrow c = 2$

$m_A : x - 3y + 2 = 0$

- La médiatrice du segment AC

$B'(1;3)$ $\vec{AC} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ 0 \end{pmatrix} \perp m_B$

$\left.\begin{array}{l} x + c = 0 \quad B': 1 + c = 0 \Rightarrow c = -1 \end{array}\right\} \Rightarrow m_B : x - 1 = 0$

- Le centre du cercle est le point d'intersection de m_A et m_B

$\begin{cases} x - 3y = -2 \\ x = 1 \end{cases} \Rightarrow \begin{cases} y = +1 \\ x = 1 \end{cases}$ $K(1;1)$

- Le $r = \|\vec{KA}\|$ $\vec{KA} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

$\Rightarrow r = \sqrt{16+4} = \sqrt{20}$

$C : (x-1)^2 + (y-1)^2 = 20$

EXERCICE 4.50

$\Gamma : x^2 + y^2 - 10x - 8y - 8 = 0 \Rightarrow (x-5)^2 - 25 + (y-4)^2 - 16 - 8 = 0$

$\Leftrightarrow (x-5)^2 + (y-4)^2 = 49$ $K(5;4)$ $r = 7$

a. $P_1(3;-3)$ $(3-5)^2 + (-3-4)^2 = 4 + 49 = 53 > 49$

$\Rightarrow P_1$ est à l'extérieure du cercle

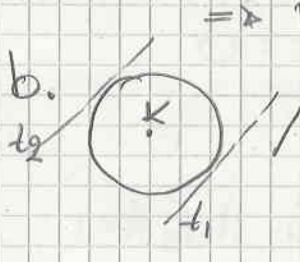
$P_2(0;6)$ $(0-5)^2 + (6-4)^2 = 25 + 4 = 29 < 49$

$\Rightarrow P_2$ est à l'intérieure du cercle

$P_3 : (5 - \sqrt{13}; 10) : (5 - \sqrt{13} - 5)^2 + (10-4)^2 = 13 + 36 = 49$

$\Rightarrow P_3$ est sur le cercle

b.



$\vec{d} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \Rightarrow \vec{n}_t = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ $x - 2y + c = 0$

$dist(K, t) = r = 7 \Rightarrow$

$\dfrac{|5 - 8 + c|}{\sqrt{1+4}} = 7 \Rightarrow |c - 3| = 7\sqrt{5} \Rightarrow$

$\begin{cases} c - 3 = 7\sqrt{5} \Rightarrow c_1 = 3 + 7\sqrt{5} & t_1 : x - 2y + 3 + 7\sqrt{5} = 0 \\ c - 3 = -7\sqrt{5} \Rightarrow c_2 = 3 - 7\sqrt{5} & t_2 : x - 2y + 3 - 7\sqrt{5} = 0 \end{cases}$

c. $d: 3x-2y+17=0 \quad (x-5)^2+(y-4)^2=49 \quad K(5;4)$

$\text{dist}(K;d) = \dfrac{|15-8+17|}{\sqrt{9+4}} = \dfrac{24}{\sqrt{13}} \approx 6.65 < r=7 \Rightarrow$ sécants

On cherche $I = d \cap C$

$3x-2y+17=0 \Rightarrow 2y = 3x+17 \Rightarrow y = \frac{3}{2}x+\frac{17}{2}$

$(x-5)^2+\left(\frac{3}{2}x+\frac{17}{2}-4\right)^2 = 49 \Rightarrow$

$\Rightarrow (x-5)^2+\left(\frac{3}{2}x+\frac{9}{2}\right)^2 = 49 \Rightarrow$

$\Rightarrow x^2-10x+25+\frac{9}{4}x^2+\frac{27}{2}x+\frac{81}{4}-49=0 \Leftrightarrow$

$\Leftrightarrow \frac{13}{4}x^2+\frac{7}{2}x-\frac{15}{4}=0 \Rightarrow 13x^2+14x-15=0$

$\Delta = 976 \quad x_{1,2} = \dfrac{-14 \pm \sqrt{976}}{26} \begin{cases} x_1 = 0.66 \\ x_2 = -1.74 \end{cases}$

$x_1 = 0.66 \Rightarrow y_1 = 9.49 \quad (0.66;9.49)$

$x_2 = -1.74 \Rightarrow y_2 = 5.89 \quad (-1.74;5.89)$

Exercice 4.51

a. $K \in Oy \quad K(0;y_0) \quad A(1;2) \quad B(7;4)$

$\vec{KA} = \begin{pmatrix} 1 \\ 2-y_0 \end{pmatrix} \quad \vec{KB} = \begin{pmatrix} 7 \\ 4-y_0 \end{pmatrix}$

$\|\vec{KA}\| = \|\vec{KB}\| \Rightarrow \sqrt{1+(2-y_0)^2} = \sqrt{49+(4-y_0)^2} \Rightarrow$

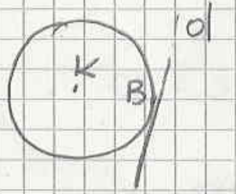
$\Rightarrow 1+4-4y_0+\cancel{y_0^2} = 49+16-8y_0+\cancel{y_0^2} \Leftrightarrow$

$\Rightarrow 4y_0 = 60 \Rightarrow y_0 = 15 \quad K(0;15)$

$r = \|\vec{KA}\| = \sqrt{1+13^2} = \sqrt{170}$

$x^2+(y-15)^2 = 170$

b. $B \in \Pi$? $\quad B(7;4) \quad 49+11^2 = 170$ ✓



$\vec{n}_d = \vec{KB} = \begin{pmatrix} 7 \\ -11 \end{pmatrix} \Rightarrow 7x-11y+c=0$

$B: 49-44+c=0 \Rightarrow c=-5$

$d: 7x-11y-5=0$

c. $\vec{d} = \begin{pmatrix} 11 \\ 7 \end{pmatrix} \quad \vec{d_y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \|\vec{d}\| = \sqrt{11^2+49} = \sqrt{170} \quad \|\vec{d_y}\| = 1$

$\cos\varphi = \dfrac{\vec{d}\cdot\vec{d_y}}{\|\vec{d}\|\cdot\|\vec{d_y}\|} = \dfrac{7}{\sqrt{170}\cdot 1} \Rightarrow \varphi = 57.53°$

Exercice 4.52

$\Gamma: x^2+y^2+10x+2y+13=0 \Rightarrow (x+5)^2-25+(y+1)^2-1+13=0$

$\Leftrightarrow (x+5)^2+(y+1)^2=13 \quad K(-5;-1) \quad r=\sqrt{13}$

$T \in \Gamma ? \quad T(-3;2) \quad (-3+5)^2+(2+1)^2 = 2^2+3^2=13$ ✓

$\vec{n}_t = \vec{KT} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad 2x+3y+c=0$

$T: -6+6+c=0 \Rightarrow c=0$

$t: 2x+3y=0$

Exercice 4.53

$\begin{cases} d: x+y-4=0 \Rightarrow y=-x+4 \\ \Gamma: (x-1)^2+(y+3)^2=20 \end{cases}$

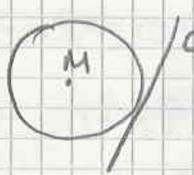
$(x-1)^2+(-x+7)^2=20 \Rightarrow x^2-2x+1+x^2-14x+49-20=0$

$\Leftrightarrow 2x^2-16x+30=0 \Rightarrow x^2-8x+15=0 \; \begin{cases} x_1=3 \\ x_2=5 \end{cases}$

$A(3;1) \quad B(5;-1)$

Exercice 4.54

$M(-2;3) \quad d: x+2y=0$



$d: x+2y=0$

$r = \text{dist}(M,d) = \dfrac{|-2+6|}{\sqrt{1+4}} = \dfrac{4}{\sqrt{5}}$

$(x+2)^2+(y-3)^2=\dfrac{16}{5}$

Exercice 4.55

$\Gamma: (x-2)^2+(y+5)^2-17=0 \qquad t \,//\, d: x-4y+10=0$

$K(2;-5) \quad r=\sqrt{17} \qquad t: x-4y+c=0$

$\text{dist}(K;t)=\sqrt{17} \Rightarrow$

$\dfrac{|2+20+c|}{\sqrt{1+16}}=\sqrt{17} \Rightarrow |c+22|=17 \Rightarrow \begin{cases} c+22=17 \Rightarrow c_1=-5 \\ c+22=-17 \Rightarrow c_2=-39 \end{cases}$

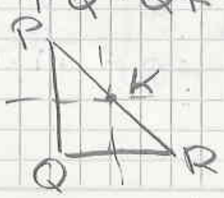
$t_1: x-4y-5=0 \qquad t_2: x-4y-39=0$

Exercice 4.56

$P(-2;7) \quad Q(2;3) \quad R(4;5)$

$\vec{PQ}=\begin{pmatrix} 4 \\ -4 \end{pmatrix} \quad \vec{PR}=\begin{pmatrix} 6 \\ -2 \end{pmatrix} \quad \vec{QR}=\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

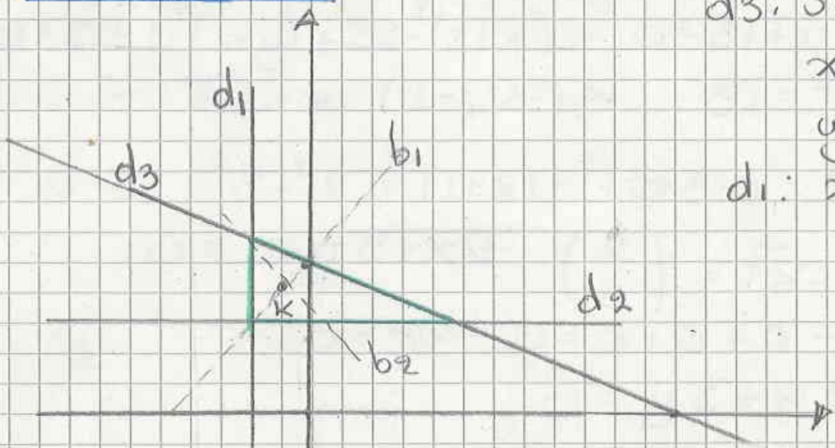
$\vec{PQ}\cdot\vec{QR}=8-8=0 \Rightarrow \vec{PQ}\perp\vec{QR} \Rightarrow$ triangle rectangle



Le centre du cercle est le milieu du PR $\quad K(1;6) \quad r=\|KR\|=\left\|\begin{pmatrix} 3 \\ -1 \end{pmatrix}\right\|=\sqrt{10}$

$(x-1)^2+(y-6)^2=10$

EXERCICE 4.57



d_3: $5x+12y=60$

$x=0 \quad y=5$

$y=0 \quad x=12$

d_1: $x+2=0$

le centre du cercle inscrit est le point d'intersection des bissectrices du triangle

- $d_1 - d_2$ b_1: $y = x+5 \Rightarrow x-y+5=0$
- d_1-d_3: b_2 Soit $P(x,y) \in b_2$

$dist(P,d_1) = dist(P,d_3) \Rightarrow \frac{|x+2|}{\sqrt{1}} = \frac{|5x+12y-60|}{\sqrt{25+144}} \Rightarrow$

$|x+2| = \frac{|5x+12y-60|}{13} \Rightarrow 13|x+2| = |5x+12y-60| \Leftrightarrow$

$$\begin{cases} 13x+26 = 5x+12y-60 \Leftrightarrow 8x-12y+86=0 \Rightarrow 2x-3y+21{,}5=0 \\ 13x+26 = -5x-12y+60 \Leftrightarrow 18x+12y-34=0 \Rightarrow 9x+6y-17=0 \ (b_2) \end{cases}$$

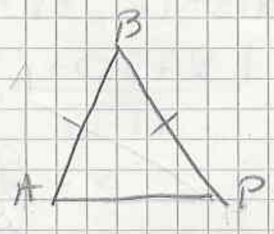
$K = b_2 \cap b_1$ $\begin{cases} x-y=-5 & \cdot 6 \\ 9x+6y=17 \end{cases}$ $+\begin{cases} 6x-6y=-30 \\ 9x+6y=17 \end{cases}$

$15x = -13 \Rightarrow x = -\frac{13}{15}$

$x = -\frac{13}{15} \quad y = -\frac{13}{15}+5 = \frac{62}{15} \quad K\left(-\frac{13}{15}; \frac{62}{15}\right)$

$r = dist(K; d_1) = \frac{\left|-\frac{13}{15}+2\right|}{1} = \frac{17}{15}$

EXERCICE 4.58



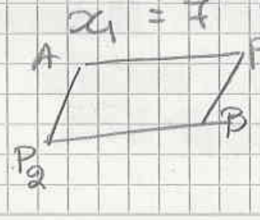
$A(-3;3) \quad B(4;0) \quad P_1, P_2 \in d: y = x$

$P_1(x,x) \quad \vec{BA} = \begin{pmatrix} -7 \\ 3 \end{pmatrix} \quad \vec{BP} = \begin{pmatrix} x-4 \\ x \end{pmatrix}$

$\|\vec{BA}\| = \|\vec{BP}\| \Rightarrow \sqrt{49+9} = \sqrt{(x-4)^2+x^2} \Leftrightarrow$

$58 = x^2-8x+16+x^2 \Leftrightarrow 2x^2-8x-42=0 \Rightarrow x^2-4x-21=0$

$x_1 = 7 \quad P_1(7,7) \quad x_2 = -3 \quad P_2(-3;-3)$



$\vec{AP_1} = \begin{pmatrix} 10 \\ 4 \end{pmatrix} \quad \vec{AP_2} = \begin{pmatrix} 0 \\ -6 \end{pmatrix} \quad \begin{vmatrix} 10 & 0 \\ 4 & -6 \end{vmatrix} = -60$

$A = |-60| = 60$

EXERCICE 4.59

d: $y = mx$

a. $x^2+y^2-10x+16=0$

dnc: $x^2+m^2x^2-10x+16=0 \Leftrightarrow (1+m^2)x^2-10x+16=0$

$\Delta = 100-4\cdot(1+m^2)\cdot 16 = 100-64(1+m^2) =$

$= 100-64-64m^2 = 36-64m^2 = 4(9-16m^2)$

$= 4(3-4m)(3+4m)$

m		$-3/4$		$3/4$	
Δ	$-$	0	$+$	0	$-$

• si $m \in]-3/4, 3/4[$ $\Delta>0 \Rightarrow$ 2 points d'intersection

• si $m = \pm 3/4$ $\Delta = 0$ 1 point intersection

b. $x^2+y^2-10x+16=0 \Leftrightarrow (x-5)^2-25+y^2+16=0 \Leftrightarrow$

$\Leftrightarrow (x-5)^2+y^2=9$ $K(5,0)$ $r=3$

d: $mx-y=0$ $\text{dist}(K,d)=r \Rightarrow$

$\Rightarrow \dfrac{|5m|}{\sqrt{m^2+1}} = 3 \Rightarrow |5m| = 3\sqrt{m^2+1} \Rightarrow$

$\Rightarrow 25m^2 = 9(m^2+1) \Rightarrow 16m^2 = 9 \Rightarrow m^2 = \dfrac{9}{16} \Leftrightarrow$

$\Rightarrow m = \pm\dfrac{3}{4}$

EXERCICE 4.60

$K \in d: 3x+7y-39=0$ $a: 3x-4y+12=0$ $b: x=0$

Un point de d: $y=0$ $x=13$ $(13;0)$

$\vec{n}_d = \begin{pmatrix}3\\7\end{pmatrix} \Rightarrow \vec{d} = \begin{pmatrix}7\\-3\end{pmatrix}$ d: $\begin{cases} x = 13+7\lambda \\ y = -3\lambda \end{cases}$

$K(13+7\lambda; -3\lambda)$

$\text{dist}(K;a) = \text{dist}(K,b) =$

$\Rightarrow \dfrac{|3(13+7\lambda)-4(-3\lambda)+12|}{\sqrt{9+16}} = \dfrac{|13+7\lambda|}{1} \Leftrightarrow$

$\Leftrightarrow |39+21\lambda+12\lambda+12| = 5|13+7\lambda| \Leftrightarrow$

$\Leftrightarrow |33\lambda+51| = |65+35\lambda| \Leftrightarrow \begin{cases} 33\lambda+51 = 65+35\lambda \\ 33\lambda+51 = -65-35\lambda \end{cases}$

$\Rightarrow \begin{cases} 2\lambda = -14 \Rightarrow \lambda = -7 \\ 68\lambda = -116 \Rightarrow \lambda = -\dfrac{29}{17} \end{cases}$ $K_1(-36;21)$ $K_2\left(\dfrac{18}{17}; \dfrac{87}{17}\right)$

$r_1 = \text{dist}(K_1,b) = \dfrac{|-36|}{1} = 36$ $r_2 = \text{dist}(K_2,b) = \dfrac{18/17}{1} = \dfrac{18}{17}$

$(x+36)^2+(y-21)^2=1226 \qquad \left(x-\frac{18}{17}\right)^2+\left(y-\frac{87}{17}\right)^2=\frac{324}{289}$

EXERCICE 4.61

a. $\Gamma_1 : x^2+y^2=25 \qquad \Gamma_2 : (x-1)^2+(y-1)^2=29$

$\cancel{x^2}+\cancel{y^2}-25=\cancel{x^2}-2x+1+\cancel{y^2}-2y+1-29 \Leftrightarrow$

$2x+2y=-2 \Rightarrow x+y=-1$ axe radical

$\Rightarrow y=-x-1 \ (d)$

$\Gamma_1 \cap d : x^2+(-x-1)^2=25 \Leftrightarrow x^2+x^2+2x+1-25=0$

$2x^2+2x-24=0 \Leftrightarrow x^2+x-12=0$ $\begin{cases} x_1=3 & I_1(3;-4) \\ x_2=-4 & I_2(-4;3) \end{cases}$

b. $\Gamma_1 \ (x+4)^2+(y+5)^2=194 \qquad \Gamma_2 : (x-3)^2+(y-2)^2=40$

$\cancel{x^2}+8x+16+\cancel{y^2}+10y+25-194=\cancel{x^2}-6x+9+\cancel{y^2}-4y+4-40 \Leftrightarrow$

$\Leftrightarrow 14x+14y=126 \Rightarrow x+y=9 \Rightarrow y=-x+9$

$\Gamma_1 \cap d \quad (x+4)^2+(-x+14)^2=194 \Leftrightarrow$

$\Leftrightarrow x^2+8x+16+x^2-28x+196-194=0 \Leftrightarrow$

$\Leftrightarrow 2x^2-20x+18=0 \Leftrightarrow x^2-10x+9=0$

$\Delta=100-36=64 \quad x_{1,2}=\frac{10\pm 8}{2}$ $\begin{cases} x_1=9 \\ x_2=1 \end{cases}$

$I_1(9;0) \quad I_2(1;8)$