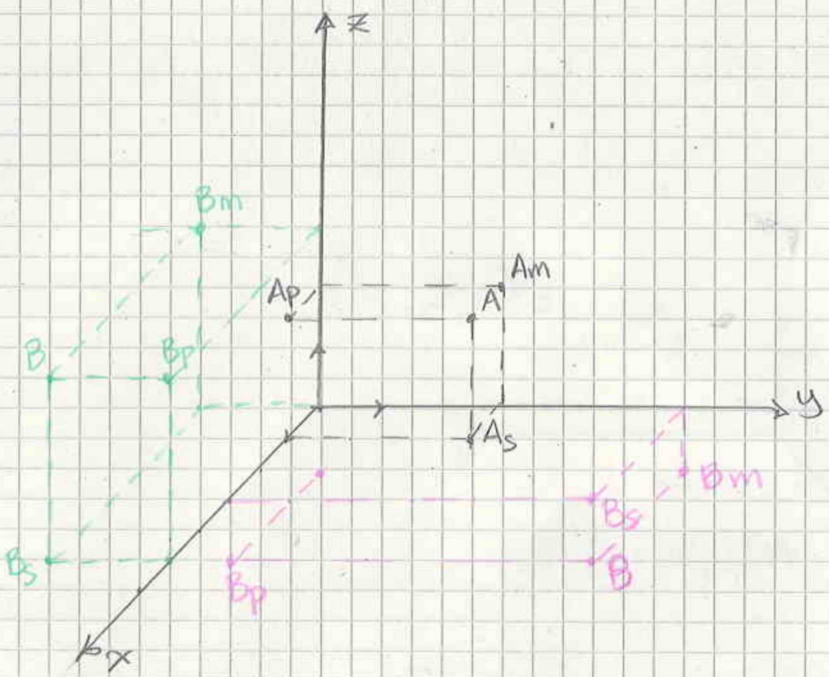
LDDR - Niveau 1

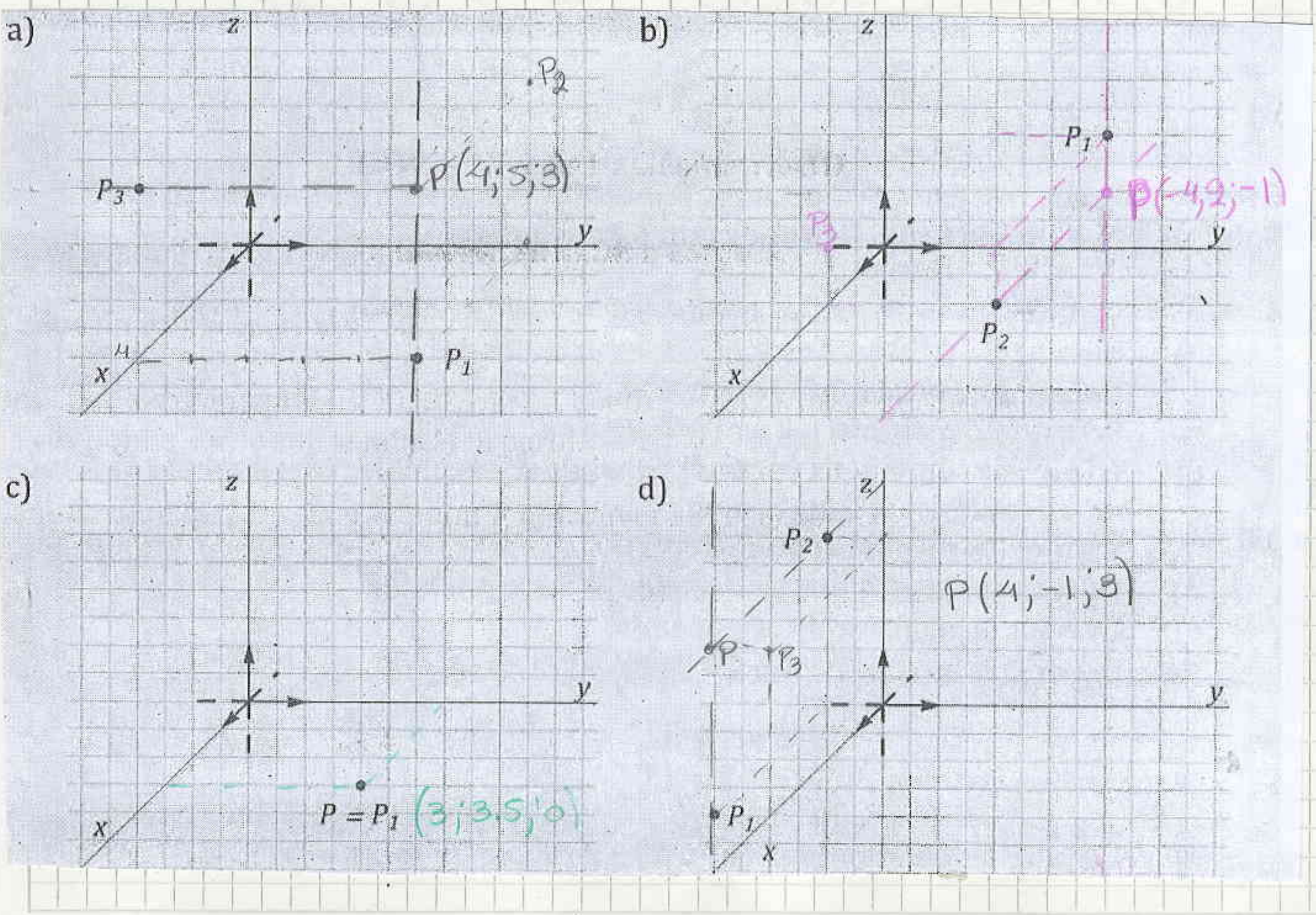
GEOMETRIE DE L'ESPACE

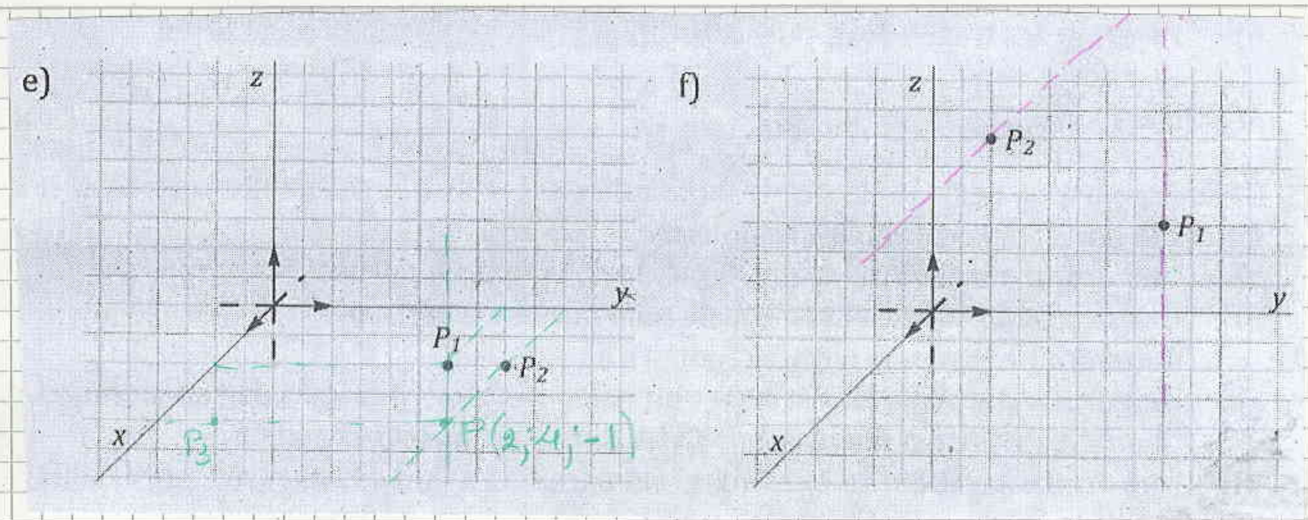
EXERCICE 1

A (1; 3; 2)   B (3; 6; -1)   C (5; -2; 3)

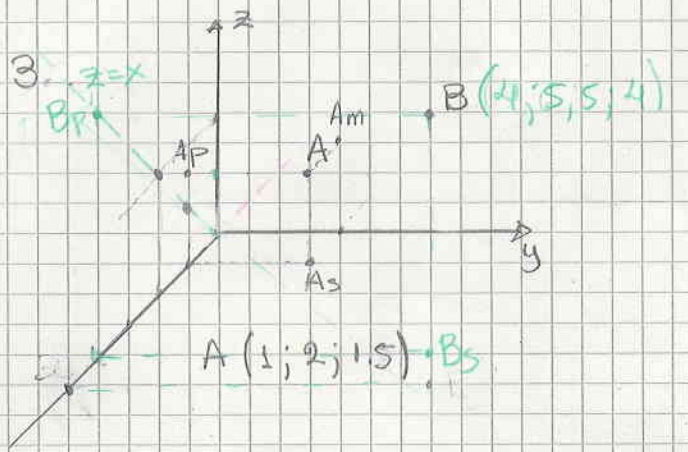


EXERCICE 2





3.



EXERCICE 4

d: $A(4;6;1)$ $B(2;9;2)$ $\vec{AB} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ $d: \begin{cases} x = 4 - 2\lambda \\ y = 6 + 3\lambda \\ z = 1 + \lambda \end{cases}$

a)

T_s: $z = 0 \Rightarrow \lambda = -1$ $T_s(6;3;0)$

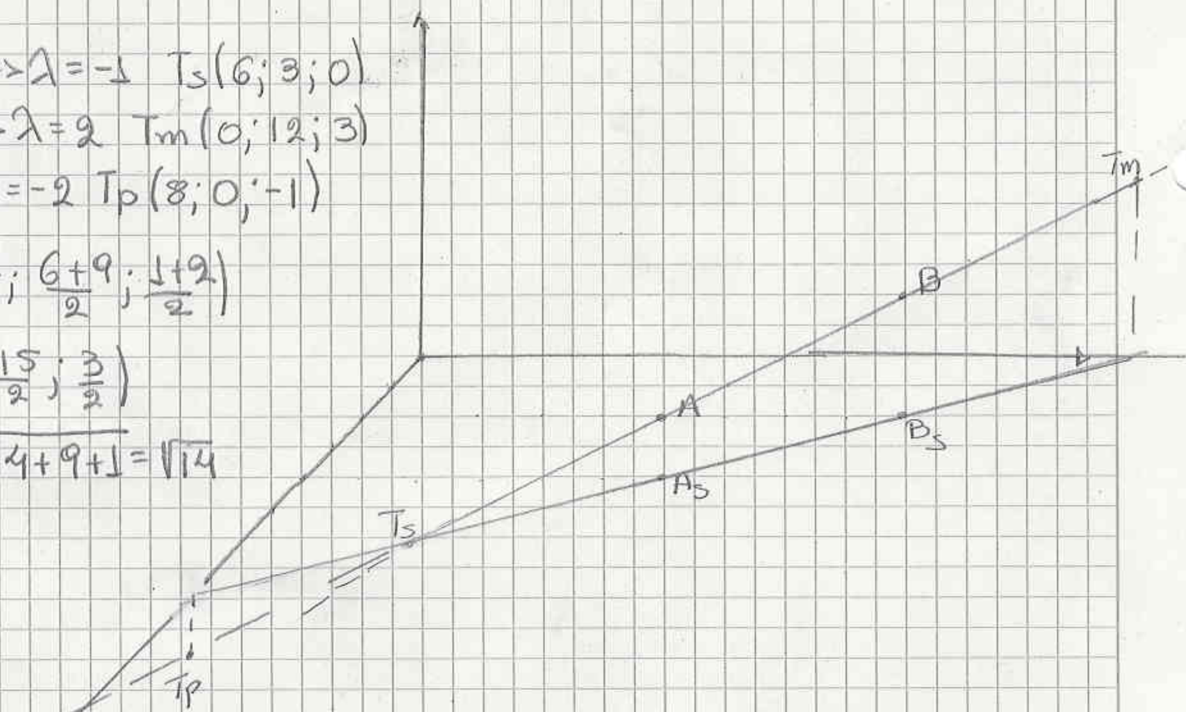
T_m: $x = 0 \Rightarrow \lambda = 2$ $T_m(0;12;3)$

T_p: $y = 0 \Rightarrow \lambda = -2$ $T_p(8;0;-1)$

d) $M\left(\frac{4+2}{2}; \frac{6+9}{2}; \frac{1+2}{2}\right)$

$M\left(3; \frac{15}{2}; \frac{3}{2}\right)$

e) $\|\vec{AB}\| = \sqrt{4+9+1} = \sqrt{14}$

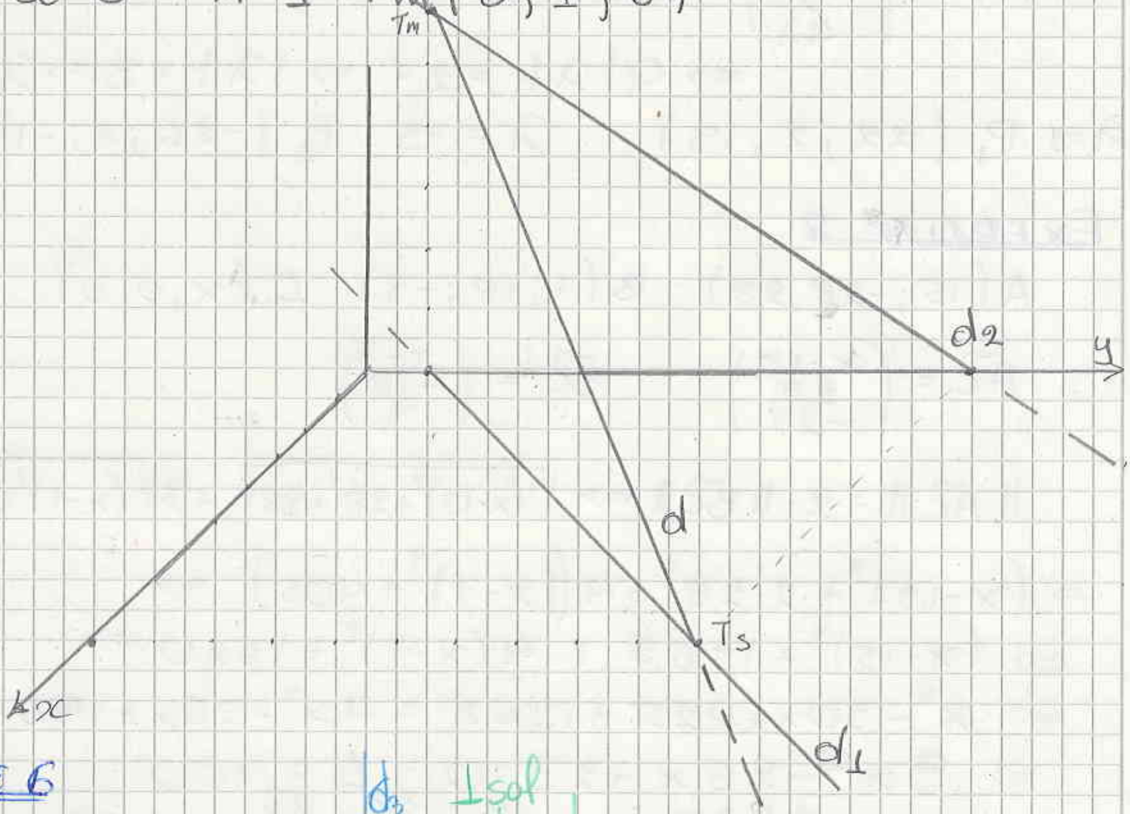


EXERCICE 5

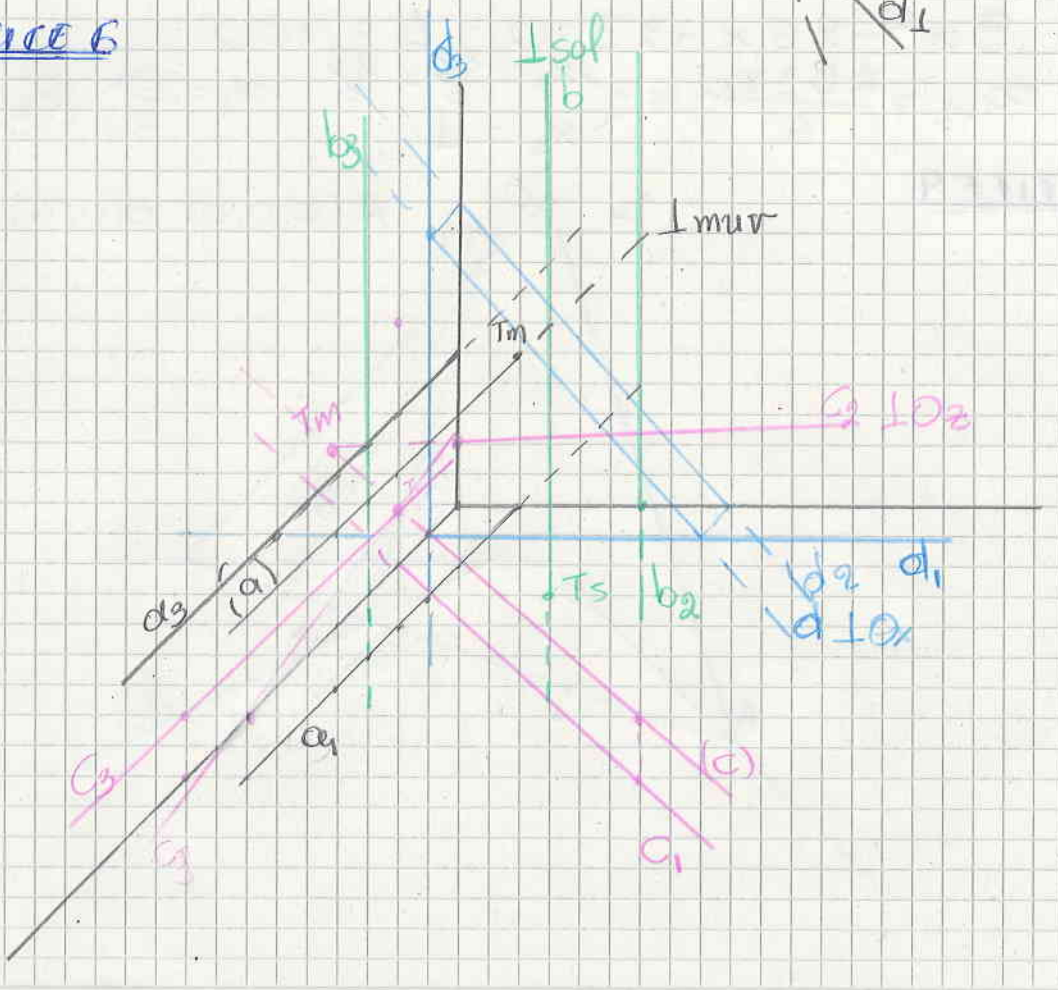
$$d : \begin{cases} x = -3 + 3\lambda \\ y = -2 + 3\lambda \\ z = 8 - 2\lambda \end{cases}$$

a) T_s : $z = 0 \Rightarrow \lambda = 4$ $\quad T_s(9;;10;0)$

T_m : $x = 0 \Rightarrow \lambda = 1$ $\quad T_m(0;1;6)$



EXERCICE 6



EXERCICE 7

$A(-2;5;1)$ $B(6;4;5)$ $\vec{AB}=\begin{pmatrix}8\\-1\\4\end{pmatrix}$

$d:\begin{cases}x=-2+8\lambda\\y=5-\lambda\\z=1+4\lambda\end{cases}$ $P\in d \Rightarrow P(-2+8\lambda;5-\lambda;1+4\lambda)$

$\vec{AP}=\begin{pmatrix}8\lambda\\-\lambda\\4\lambda\end{pmatrix}$ $\|\vec{AP}\|=27 \Rightarrow \sqrt{64\lambda^2+\lambda^2+16\lambda^2}=27 \Rightarrow$

$\Rightarrow 9|\lambda|=27 \Rightarrow |\lambda|=3 \Rightarrow \lambda=\pm 3$

$\lambda=3$ $P_1(22;2;13)$ $\lambda=-3$ $P_2(-26;8;-11)$

EXERCICE 8

$A(15;-28,28)$ $B(7,18;-9)$ $C(x;0;0)$

$\vec{AC}=\begin{pmatrix}x-15\\28\\-28\end{pmatrix}$ $\vec{BC}=\begin{pmatrix}x-7\\-18\\9\end{pmatrix}$

$\|\vec{AC}\|=2\cdot\|\vec{BC}\| \Rightarrow \sqrt{(x-15)^2+28^2+28^2}=2\sqrt{(x-7)^2+18^2+9^2} \Rightarrow$

$\Rightarrow (x-15)^2+2\cdot 28^2=4\left((x-7)^2+405\right) \Leftrightarrow$

$\Leftrightarrow (x-15)^2+1568=4(x-7)^2+1620 \Rightarrow$

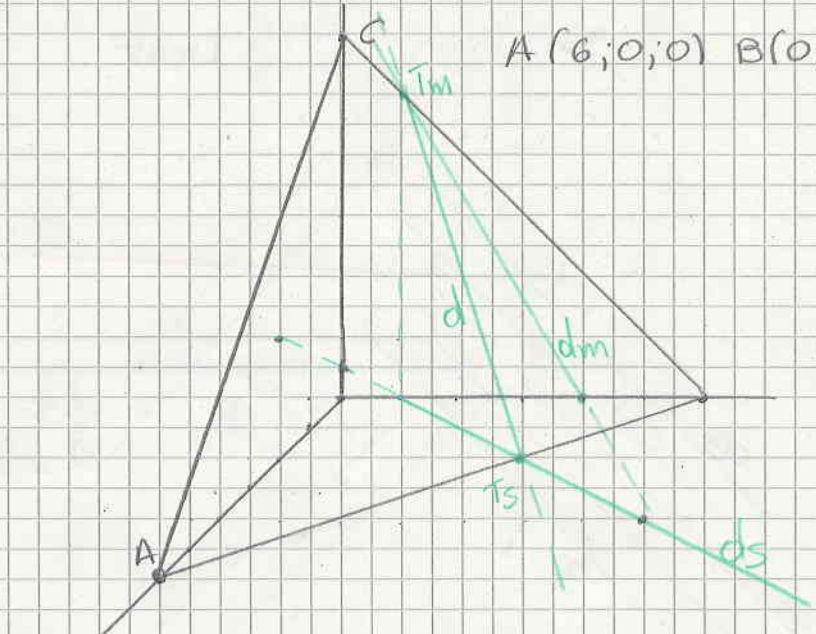
$\Leftrightarrow x^2-30x+225+1568=4x^2-56x+196+1620 \Leftrightarrow$

$\Leftrightarrow 3x^2-26x+23=0$ $\Delta=400$

$x_{1,2}=\dfrac{26\pm 20}{6}=$ $x_1=\frac{46}{6}=\frac{23}{3}$, $x_2=1$ $x_1=\frac{23}{3}$ $x_2=1$

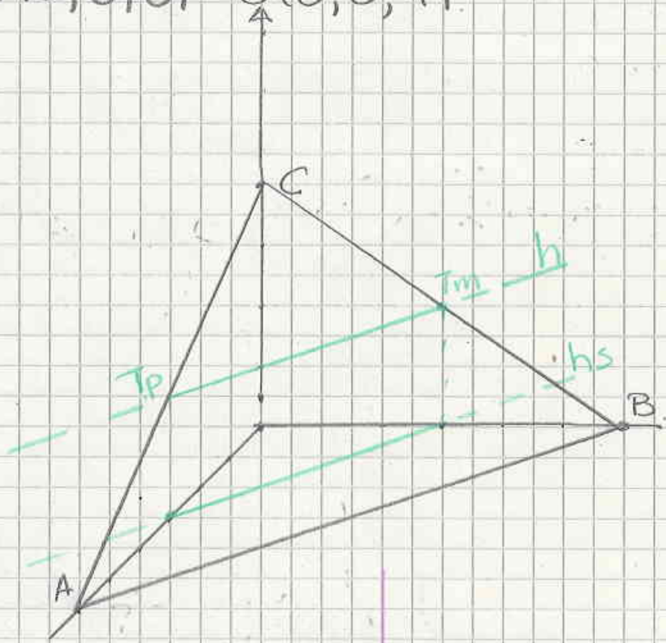
EXERCICE 9

$A(6;0;0)$ $B(0;6;0)$ $C(0;0;6)$

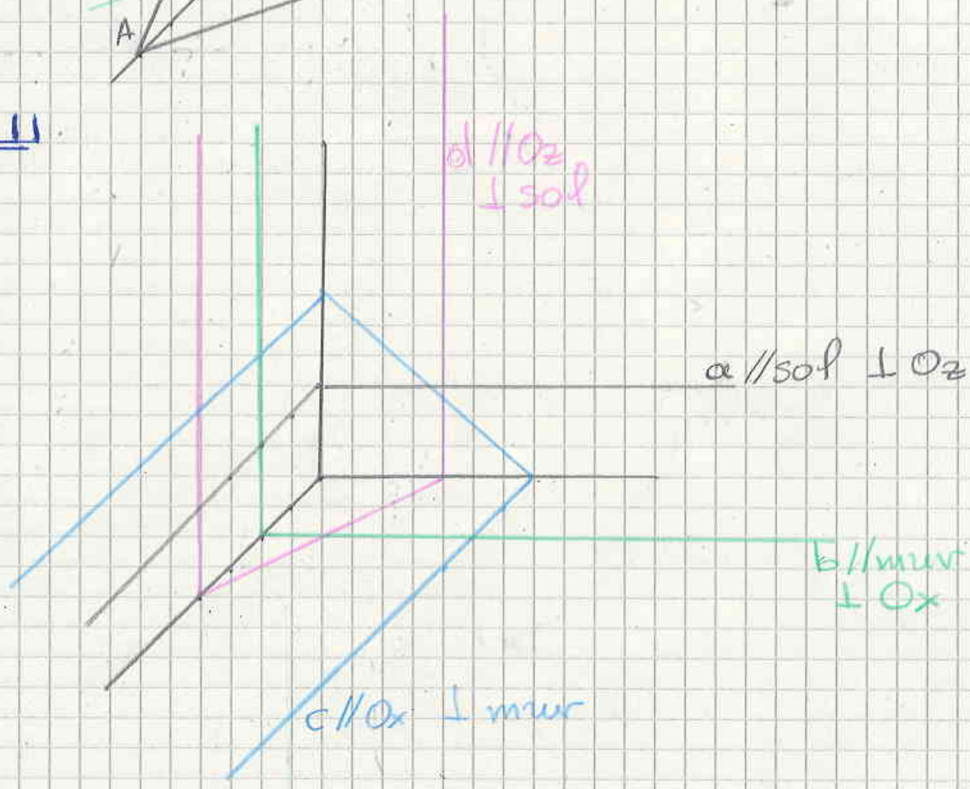


EXERCICE 10

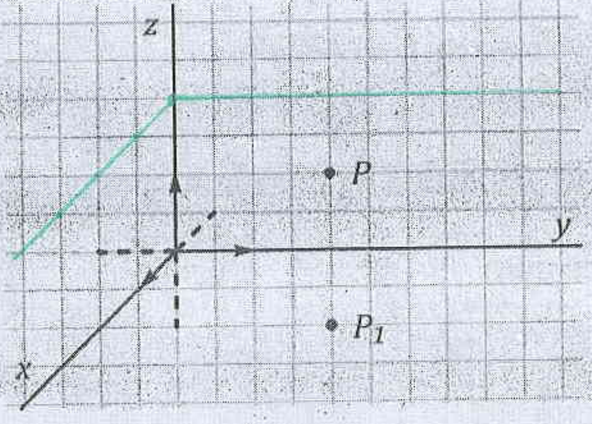
A (6;0;0) B (0;6;0) C (0;0;4)



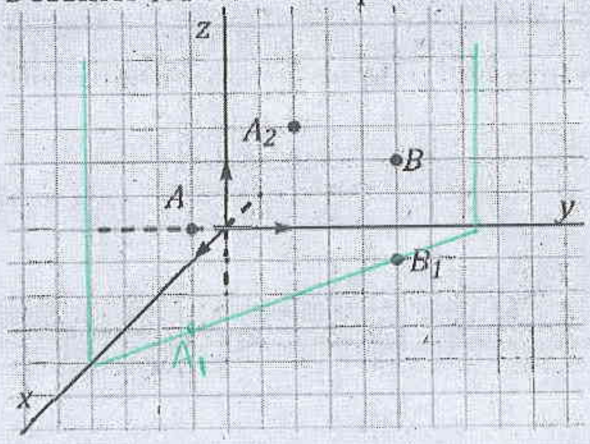
EXERCICE 11



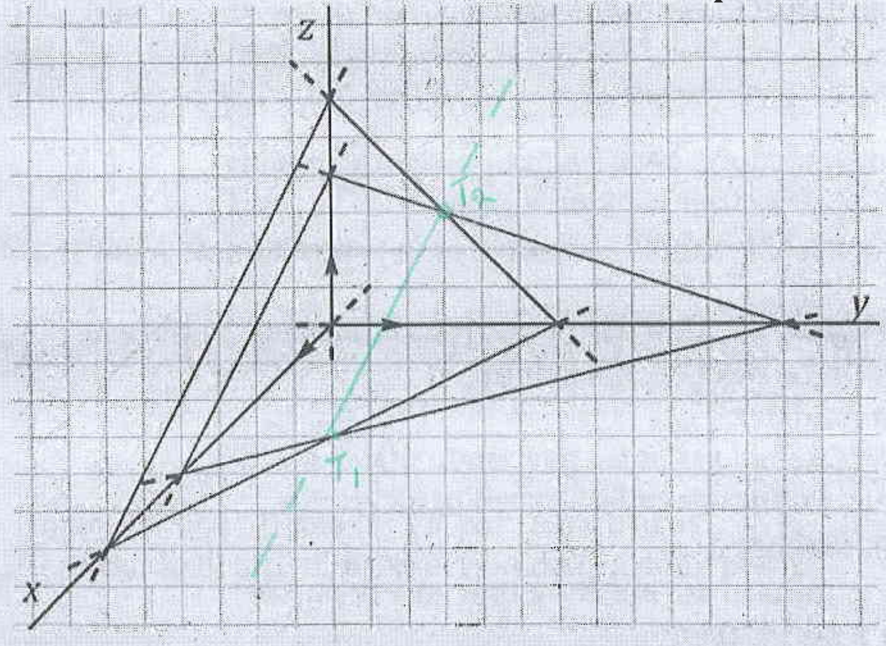
12. Dessiner les traces du plan horizontal contenant le point P donné.



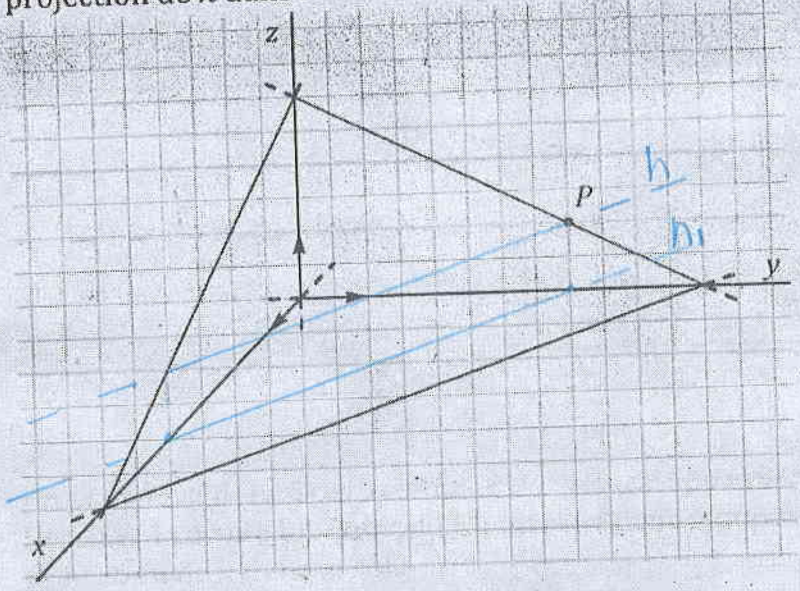
13. Dessiner les traces du plan vertical contenant les points A et B.



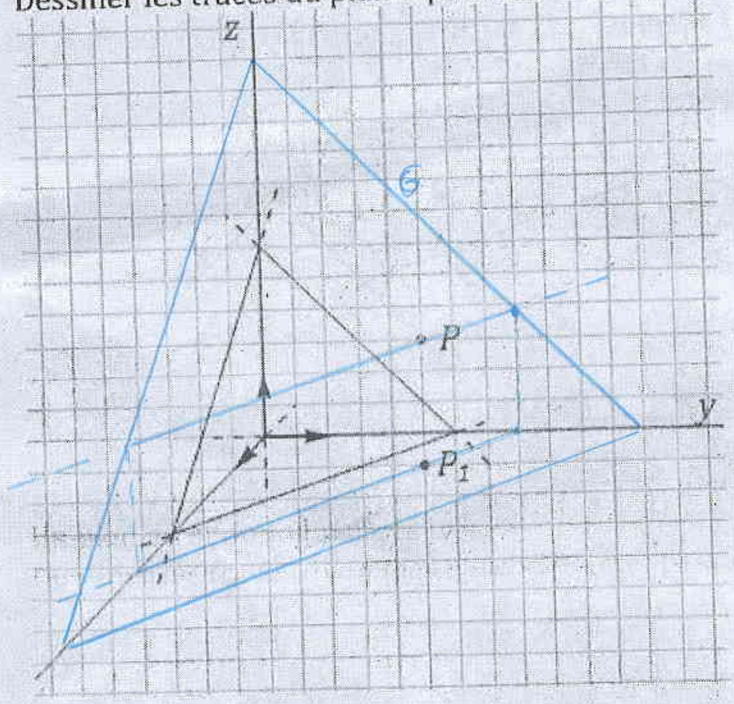
14. On donne deux plans α et β par leurs traces.
Dessiner la droite d'intersection de ces deux plans.



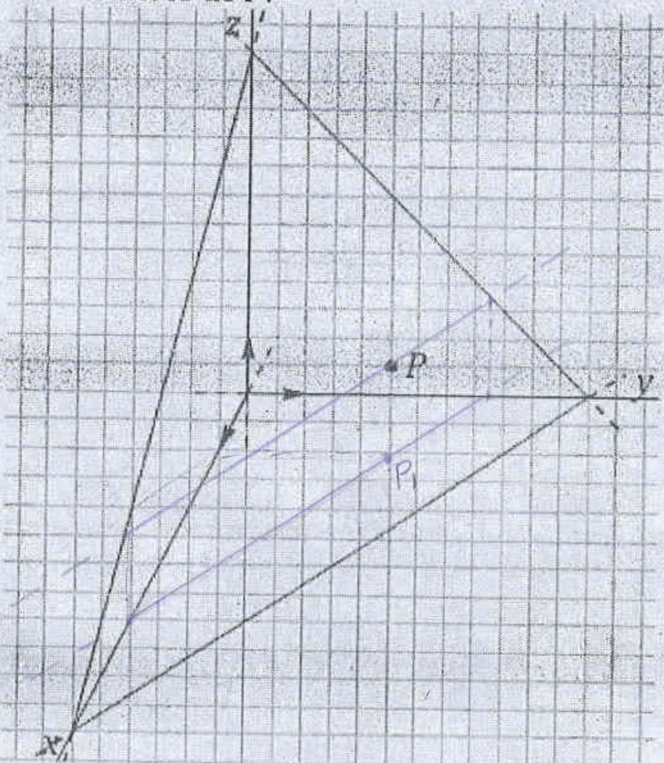
15. Un plan π est donné par ses traces. Le point P appartient à la trace de π dans le mur. Dessiner la droite horizontale h contenue dans π et passant par P. Dessiner aussi la projection de h dans le sol.



16. Un plan π est donné par ses traces.
Dessiner les traces du plan σ parallèle à π et passant par le point P.

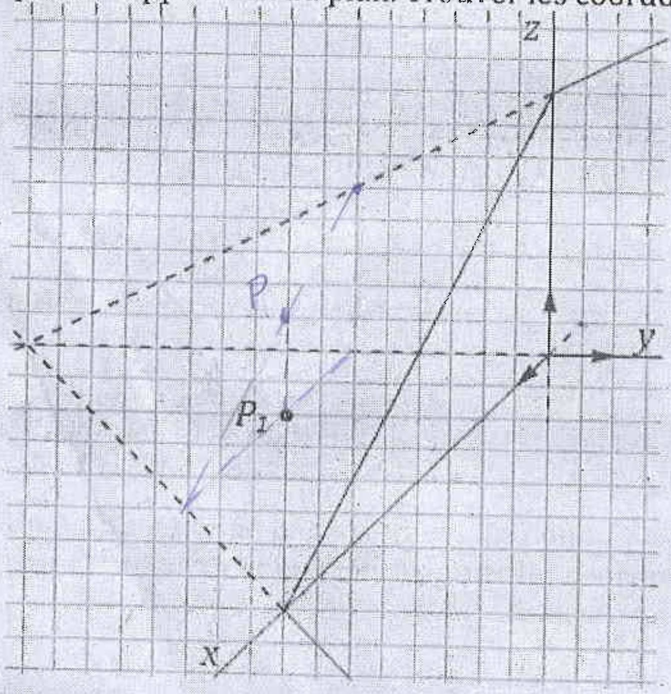


17. Un plan est donné par ses traces. Le point P appartient au plan. Trouver les coordonnées de P.



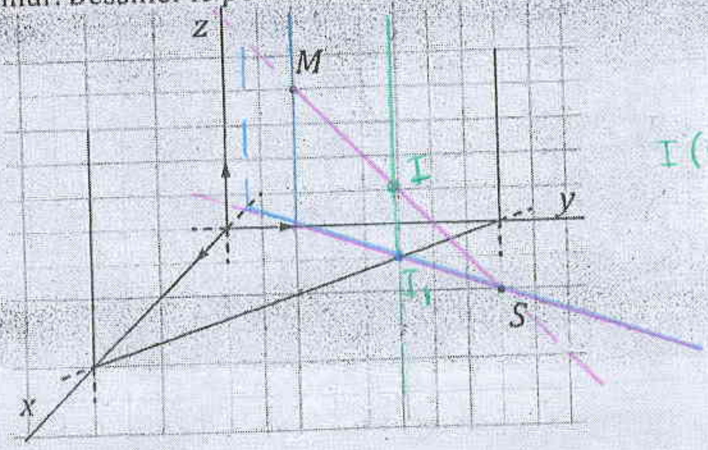
P(1; 3; 1.5)

18. Un plan est donné par ses traces. Le point P_1 est la projection sur le sol du point P. Le point P appartient au plan. Trouver les coordonnées de P.

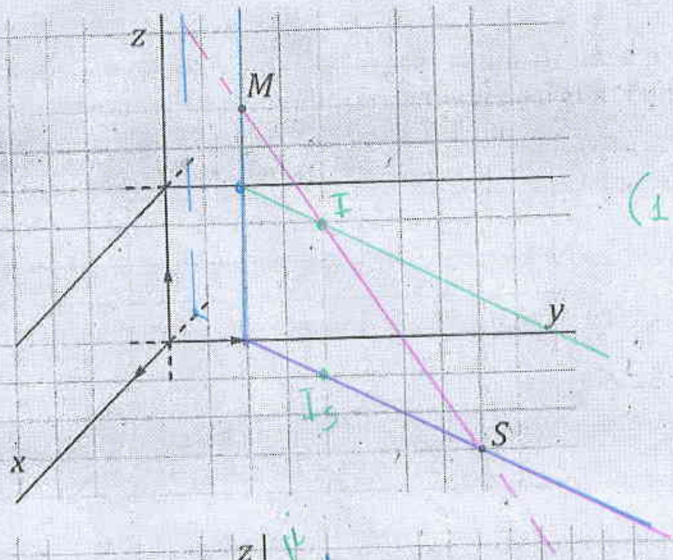


P(2; -3; 1.5)

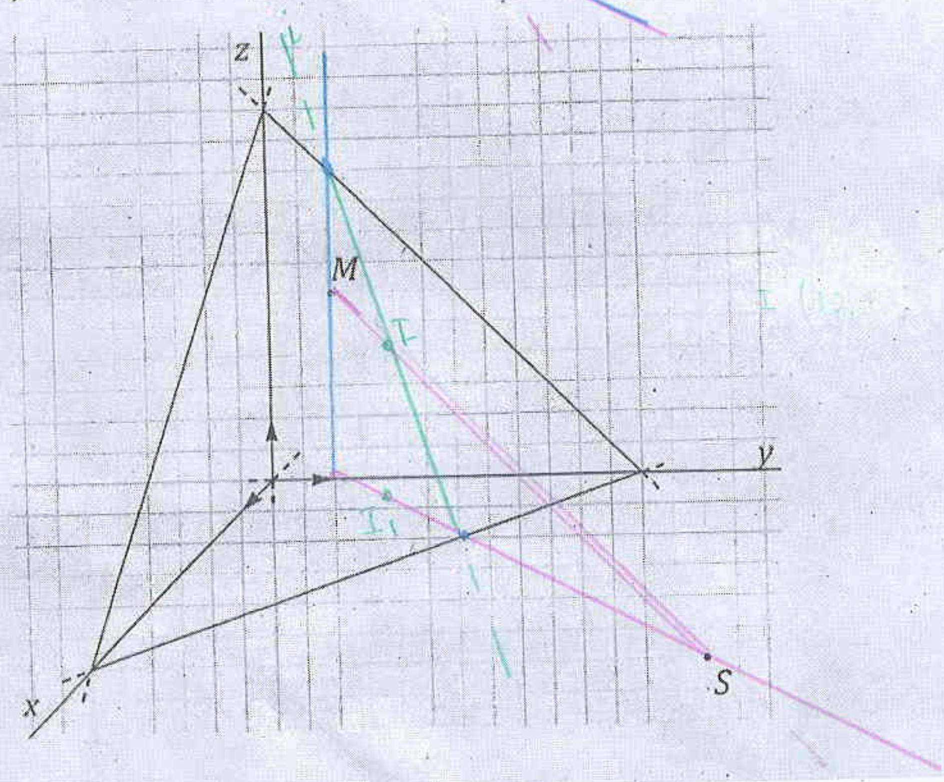
19. On donne un plan π par ses traces et une droite d par ses traces S et M dans le sol et le mur. Dessiner le point d'intersection I de d et π. Puis déterminer les coordonnées de I.



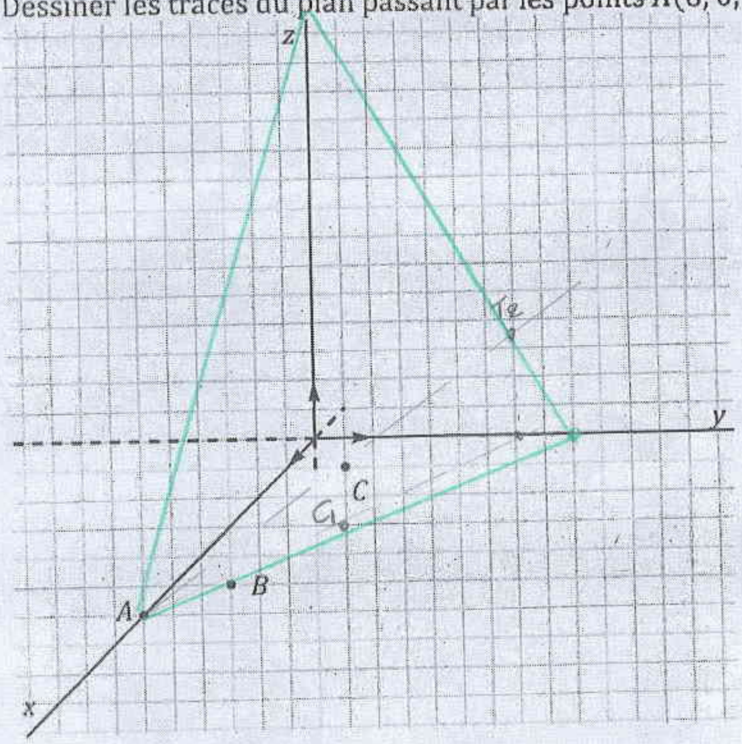
I(1;3;1)



(1;2,5;2)



20. Dessiner les traces du plan passant par les points $A(6;0;0)$, $B(5;1;0)$ et $C(3;2;1)$.



EXERCICE 21

$A(5;-3;8)$ $B(1;7;2)$ $C(-3;2;5)$

$\vec{BA} = \begin{pmatrix} 4 \\ -10 \\ 6 \end{pmatrix}$ $\vec{BC} = \begin{pmatrix} -4 \\ -5 \\ 3 \end{pmatrix}$ $\vec{BA} \cdot \vec{BC} = -16 + 50 + 18 = 52$

$\|\vec{BA}\| = \sqrt{16+100+36} = \sqrt{152}$

$\cos\varphi = \dfrac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\| \cdot \|\vec{BC}\|} =$ $\|\vec{BC}\| = \sqrt{16+25+9} = \sqrt{50}$

$= \dfrac{52}{\sqrt{152} \cdot \sqrt{50}} \Rightarrow \varphi = 53,38^\circ$

EXERCICE 22

$A(2;5;0)$ $B(-2;2;k)$

$\vec{AO} = \begin{pmatrix} -2 \\ -5 \\ 0 \end{pmatrix}$ $\vec{AB} = \begin{pmatrix} -4 \\ -3 \\ k \end{pmatrix}$ $\vec{AO} \cdot \vec{AB} = 8 + 15 = 23$

$\|\vec{AO}\| = \sqrt{4+25} = \sqrt{29}$ $\|\vec{AB}\| = \sqrt{16+9+k^2} = \sqrt{k^2+25}$

$\cos\varphi = \cos 60^\circ = \dfrac{1}{2} = \dfrac{\vec{AO} \cdot \vec{AB}}{\|\vec{AO}\| \cdot \|\vec{AB}\|} \Rightarrow \dfrac{1}{2} = \dfrac{23}{\sqrt{29}\sqrt{k^2+25}} \Leftrightarrow$

$\Leftrightarrow \sqrt{29}\sqrt{k^2+25} = 46 \Rightarrow 29(k^2+25) = 46^2 = 2116 \Rightarrow$

$\Rightarrow k^2 + 25 = 72.965 \Rightarrow k^2 = 47.965 \Rightarrow$

$k = \pm 6.926$

EXERCICE 23

a) $\Pi \perp \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \Rightarrow \Pi : 2x - 3y + 4z + c = 0$

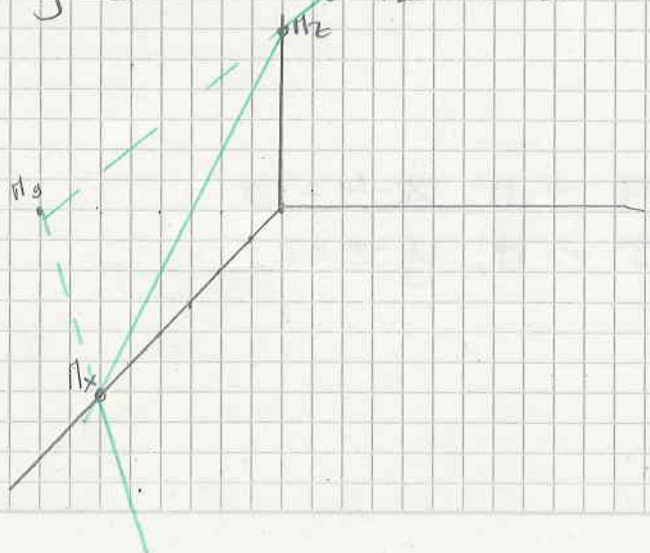
$(1;2;4) : 2 - 6 + 16 + c = 0 \Rightarrow c = -12$

$\Pi : 2x - 3y + 4z - 12 = 0$

$\Pi_x : y = z = 0 \Rightarrow 2x = 12 \Rightarrow x = 6$ $\Pi_x(6;0;0)$

$\Pi_y : x = 0 = z \Rightarrow 3y = -12 \Rightarrow y = -4$ $\Pi_y(0;-4;0)$

$\Pi_z : x = y = 0 \Rightarrow 4z = 12 \Rightarrow z = 3$ $\Pi_z(0;0;3)$



b) $\pi: 2x+y-6=0$ $\pi_x: y=z=0 \Rightarrow \pi_x(3;0;0)$

$\pi_y: x=z=0 \Rightarrow \pi_y(0,6,0)$

$\pi_z: x=y=0$ impossible $\Rightarrow \pi \parallel Oz$, $\pi \perp$ sol

c) $A(1;-3;4)$ $B(5;2;4)$ $C(3;-5;4)$

$\vec{AB} = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$ $\vec{n} = \vec{AB} \wedge \vec{AC}$

$\vec{n} = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 5\cdot 0+2\cdot 0 \\ 0\cdot 2-0\cdot 4 \\ 4(-2)-2\cdot 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -18 \end{pmatrix} \parallel \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\begin{matrix} 4 & 2 \\ 5 & -2 \end{matrix}$

$\pi: z+d=0$ $A: 4+d=0 \Rightarrow d=-4$

$\pi: z-4=0$

d) $\pi: 2x+5z-10=0$ π_y $x=z=0$ impossible $\Rightarrow$

$\pi \parallel Oy \Rightarrow \pi \perp$ paroi $\Rightarrow \pi \perp Ox$

Exercice 24

$P(3;1;1) \in \pi \perp d: A(1;0;5)$ $B(3;-3;8)$

$\vec{n}_\pi = \vec{AB} = \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix}$ $\pi: 2x-3y+3z+d=0$

$P: 6-3+3+d=0 \Rightarrow d=-6$

$\pi: 2x-3y+3z-6=0$

Exercice 25

$A(2;5;4)$

a) $\pi \parallel$ sol $z=4 \Rightarrow \pi: z-4=0$

b) $\pi \parallel$ mur $y=5 \Rightarrow \pi: y-5=0$

EXERCICE 26

$\Pi : x - 2y + 3z - 36 = 0 \quad A(10; y; 4) \quad \vec{n}_\Pi = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

a) $A \in \Pi \quad 10 - 2y + 12 - 36 = 0 \Rightarrow 2y = -14 \Rightarrow y = -7$

b) $A(10; -7; 4)$

Soit $P(x, y, z) \in \Pi : \quad x - 2y + 3z = 36 \quad (1)$

$\vec{AP} = \begin{pmatrix} x - 10 \\ y + 7 \\ z - 4 \end{pmatrix}$ // mur $\Rightarrow \vec{AP} \perp Ox \Rightarrow \vec{AP} \perp \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$

$\vec{AP} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0 \Rightarrow x - 10 = 0 \Rightarrow x = 10$

$\|\vec{AP}\| = \sqrt{13} \Rightarrow \sqrt{(y+7)^2 + (z-4)^2} = \sqrt{13} \Rightarrow$

$\Rightarrow (y+7)^2 + (z-4)^2 = 13 \quad (2)$

(1) pour $x = 10 \quad 10 - 2y + 3z = 36 \Rightarrow$

$-2y + 3z = 26 \Rightarrow y = \frac{3}{2}z - 13$

(2) $\left(\frac{3}{2}z - 13 + 7\right)^2 + (z-4)^2 = 13 \Rightarrow$

$\Rightarrow \left(\frac{3}{2}z - 6\right)^2 + (z-4)^2 = 13 \Rightarrow$

$\Rightarrow \frac{9}{4}z^2 - 18z + 36 + z^2 - 8z + 16 - 13 = 0 \Rightarrow$

$\Rightarrow \frac{13z^2}{4} - 26z + 39 = 0 \Rightarrow 13z^2 - 104z + 156 = 0$

$\Delta = 2704 \quad z_{1,2} = \frac{104 \pm 52}{26} \begin{cases} + & z_1 = 6 \\ - & z_2 = 2 \end{cases}$

$z_1 = 6 \quad y_1 = -4 \quad P_1(10; -4; 6)$

$z_2 = 2 \quad y_2 = -10 \quad P_2(10; -10; 2)$

EXERCICE 27

$\Pi : 2x + 3y + 4z - 22 = 0 \quad d : A(2; 0; -3) \quad B(0; 5; -2)$

a) $\vec{AB} = \vec{d} = \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix} \quad d : \begin{cases} x = 2 - 2\lambda \\ y = 5\lambda \\ z = -3 + \lambda \end{cases}$

$d \to \Pi : \quad 2(2 - 2\lambda) + 3 \cdot 5\lambda + 4(-3 + \lambda) - 22 = 0$

$\Leftrightarrow 4 - \cancel{4\lambda} + 15\lambda - 12 + \cancel{4\lambda} - 22 = 0 \Rightarrow$

$\Leftrightarrow 15\lambda = 30 \Rightarrow \lambda = 2 \Rightarrow I(-2; 10; -1)$

b) $\cos\varphi = \frac{\vec{d} \cdot \vec{n}_\Pi}{\|\vec{d}\| \cdot \|\vec{n}_\Pi\|} = \frac{|-4 + 15 + 4|}{\sqrt{4+25+1}\ \sqrt{4+9+16}} = \frac{15}{\sqrt{30}\ \sqrt{29}} \Rightarrow$

$\varphi = 59{,}43 \quad$ Donc $\alpha = 90 - \varphi = 30{,}567 = \alpha$

Exercice 28

$\Pi: 2x - y + 3z + d = 0 \ni D(2;-2;0)$

$d: A(1;20;15) \quad B(6;5;-5)$

a) $D \in \Pi: \quad 4 + 2 + 3\cdot 0 + d = 0 \Rightarrow d = -6.$

$\Pi: 2x - y + 3z - 6 = 0$

$\vec{AB} = \begin{pmatrix} 5 \\ -15 \\ -20 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ -3 \\ -4 \end{pmatrix} \quad d: \begin{cases} x = 6 + \lambda \\ y = 5 - 3\lambda \\ z = -5 - 4\lambda \end{cases}$

$I = \Pi \cap d: \quad 2(6+\lambda) - (5-3\lambda) + 3(-5-4\lambda) - 6 = 0$

$\Rightarrow 12 + 2\lambda - 5 + 3\lambda - 15 - 12\lambda - 6 = 0 \Rightarrow$

$\Rightarrow -7\lambda - 14 = 0 \Rightarrow \lambda = -2 \qquad I(4;11;3)$

b) $\vec{d} = \begin{pmatrix} 1 \\ -3 \\ -4 \end{pmatrix} \quad \vec{n}_\Pi = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \quad \vec{n}_\Pi \cdot \vec{d} = 2 + 3 - 12 = -7$

$\cos\alpha = \dfrac{|\vec{d} \cdot \vec{n}_\Pi|}{\|\vec{d}\| \cdot \|\vec{n}_\Pi\|} = \dfrac{7}{\sqrt{26}\cdot\sqrt{14}} \Rightarrow \alpha = 68.47 \qquad \|\vec{d}\| = \sqrt{1+9+16} = \sqrt{26} \quad \|\vec{n}_\Pi\| = \sqrt{4+1+9} = \sqrt{14}$

$\Rightarrow \varphi = 90 - 68.47 \Rightarrow \underline{\varphi = 21.5245^\circ}$

Exercice 29

$\alpha: 2x + y - z + 2 = 0 \qquad d: A(3;k;3) \quad B(4;4;4)$

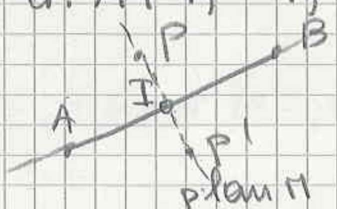
$n_\alpha = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \qquad \vec{d} = \vec{AB} = \begin{pmatrix} 1 \\ 4-k \\ 1 \end{pmatrix}$

$\alpha \parallel d \Rightarrow \vec{n}_\alpha \perp \vec{d} \Rightarrow \vec{n}_\alpha \cdot \vec{d} = 0 \Rightarrow 2 + 4 - k - 1 = 0 \Rightarrow$

$\underline{k = 5}$

Exercice 30

$d: A(4;-4;7) \quad B(-5;-1;-8) \quad P(5;4;1)$



On cherche le plan $\Pi \perp AB$

$P \in \Pi$

$\Pi \perp \vec{AB} \Rightarrow \vec{n}_\Pi = \vec{AB} = \begin{pmatrix} -9 \\ 3 \\ -15 \end{pmatrix} \parallel \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$

$\Pi: 3x - y + 5z + d = 0.$

$P: 15 - 4 + 5 + d = 0 \Rightarrow d = -16$

$\Pi: 3x - y + 5z - 16 = 0.$

On cherche $I = \Pi \cap d \qquad d: \begin{cases} x = 4 + 3\lambda \\ y = -4 - \lambda \\ z = 7 + 5\lambda \end{cases}$

$3(4+3\lambda) - (-4-\lambda) + 5(7+5\lambda) - 16 = 0$

$\Rightarrow 12 + 9\lambda + 4 + \lambda + 35 + 25\lambda - 16 = 0 \Rightarrow$

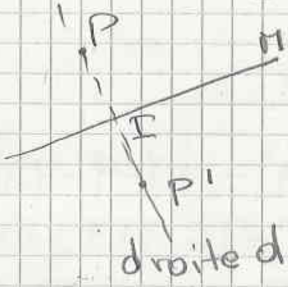
$\Rightarrow 35\lambda = -35 \Rightarrow \lambda = -1 \qquad I(1;-3;2)$

Donc $\vec{PI} = \vec{IP'} \Rightarrow \begin{pmatrix}-4\\-7\\+1\end{pmatrix} = \begin{pmatrix}x-1\\y+3\\z-2\end{pmatrix} \Rightarrow \begin{cases}x=-3\\y=-10\\z=3\end{cases}$

$P'(-3;-10;3)$

Exercice 31

$\Pi : x-2y+3z-8=0 \qquad P(2;-5;8)$



On cherche la droite $d \perp \Pi$
$P \in d$.

$d \perp \Pi \Rightarrow \vec{d} = \vec{n}_\Pi = \begin{pmatrix}1\\-2\\3\end{pmatrix}$

$d : \begin{cases}x = 2+\lambda\\ y = -5-2\lambda\\ z = 8+3\lambda\end{cases}$

On cherche $I = \Pi \cap d$

$2+\lambda-2(-5-2\lambda)+3(8+3\lambda)-8=0 \Rightarrow$

$\Rightarrow 2+\lambda+10+4\lambda+24+9\lambda-8=0 \Rightarrow$

$\Rightarrow 14\lambda = -28 \Rightarrow \lambda = -2 \qquad I(0;-1;2)$

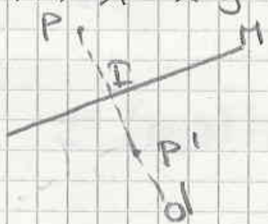
Alors $\vec{PI} = \vec{IP'} \Rightarrow \begin{pmatrix}-2\\4\\-6\end{pmatrix} = \begin{pmatrix}x\\y+1\\z-2\end{pmatrix} \Rightarrow$

$\Rightarrow x=-2 \quad y=3 \quad z=-4$

$P'(-2;3;-4)$

Exercice 32

$\Pi : x-2y+3z-8=0 \qquad P(1;0;0) \quad P'(5;8;k)$



$d \perp \Pi \Rightarrow \vec{d} = \vec{n}_\Pi = \begin{pmatrix}1\\-2\\3\end{pmatrix} \qquad d = \begin{cases}x = 1+\lambda\\ y = -2\lambda\\ z = 3\lambda\end{cases}$

$I = d \cap \Pi = \left(\frac{3}{2}; -1; \frac{3}{2}\right)$

$1+\lambda-2(-2\lambda)+3\cdot 3\lambda-8=0 \Rightarrow$

$\Rightarrow 1+\lambda+4\lambda+9\lambda-8=0 \Rightarrow 14\lambda = 7 \Rightarrow \lambda = 1/2$

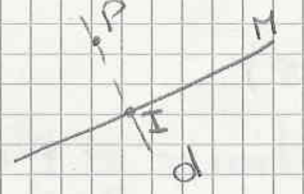
$\vec{PI} = \vec{IP'} \Rightarrow \begin{pmatrix}1/2\\-1\\3/2\end{pmatrix} = \begin{pmatrix}5-3/2\\8+1\\k-3/2\end{pmatrix} \Rightarrow 1/2 = 7/2$ impossible.

EXERCICE 33

$\pi : 13x+16y-4z+7=0 \quad P(-4;-27;-9)$

a) $\text{dist}(P,\pi) = \dfrac{|13(-4)+16(-27)-4\cdot(-9)+7|}{\sqrt{13^2+16^2+4^2}} = 21$

b)



On trouve la droite $d \perp \pi \Rightarrow \vec{d} = \vec{n}_\pi$

$d: \begin{cases} x = -4+13\lambda \\ y = -27+16\lambda \\ z = -9-4\lambda \end{cases}$

$I = d \cap \pi \quad 13(-4+13\lambda)+16(-27+16\lambda)-4(-9-4\lambda)+7=0$

$\Rightarrow -52+169\lambda-432+256\lambda+36+16\lambda+7=0 \Rightarrow$

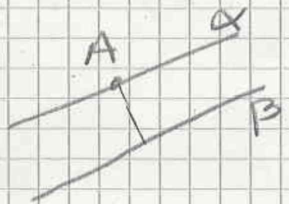
$\Rightarrow 441\lambda = 441 \Rightarrow \lambda = 1 \quad I(9;-11;-13)$

c) $\text{dist}(I;P) = \|\vec{IP}\| = \sqrt{13^2+16^2+4^2} = 21$

$\vec{IP} = \begin{pmatrix} -13 \\ -16 \\ 4 \end{pmatrix}$

EXERCICE 34

$\alpha : 3x+12y-4z-18=0 \quad \beta : 3x+12y-4z+73=0$



On trouve un point $A \in \alpha$

$y=z=0 \quad 3x=18 \Rightarrow x=6$

$A(6;0;0)$

$\text{dist}(A,\beta) = \text{dist}(\alpha,\beta)$

Donc $\text{dist}(\alpha,\beta) = \dfrac{|18+12\cdot 0-4\cdot 0+73|}{\sqrt{3^2+12^2+4^2}} = \dfrac{91}{13} = 7$

EXERCICE 35

$\pi : 4x-3y+12z-36=0 \quad \sigma // \pi \quad \text{dist}(\sigma,\pi)=5$

$\sigma // \pi \Rightarrow \sigma : 4x-3y+12z+d=0$

Soit $A \in \pi \quad y=z=0 \quad 4x=36 \Rightarrow x=9$

$A(9;0;0) \quad \text{dist}(A,\sigma)=5$

Donc $\dfrac{|36+d|}{\sqrt{4^2+9+144}} = 5 \Rightarrow \dfrac{|36+d|}{13} = 5 \Rightarrow |d+36| = 65$

$\Rightarrow \begin{cases} d+36=65 \Rightarrow d_1 = 29 \\ d+36=-65 \Rightarrow d_2 = -101 \end{cases}$

$\sigma_1 : 4x-3y+12z+29=0 \quad \sigma_2 : 4x-3y+12z-101=0$

EXERCICE 36

$\Pi : 12x - 3y - 4z + 12 = 0 \quad d : A(-11;8;3)\ B(10;-4;-3)$

Soit $A \in d$: $\text{dist}(A, \Pi) = 4$

$\vec{d} = \vec{AB} = \begin{pmatrix} 21 \\ -12 \\ -6 \end{pmatrix} // \begin{pmatrix} 7 \\ -4 \\ -2 \end{pmatrix} \quad \vec{d} = \begin{cases} x = -11 + 7\lambda \\ y = 8 - 4\lambda \\ z = 3 - 2\lambda \end{cases}$

$A \in d \Rightarrow A(-11+7\lambda ; 8-4\lambda ; 3-2\lambda)$

$\text{dist}(A;\Pi) = 4 \Rightarrow \dfrac{|12(-11+7\lambda) - 3(8-4\lambda) - 4(3-2\lambda) + 12|}{\sqrt{12^2+9+16}} = 4$

$\Rightarrow \dfrac{|-132+84\lambda-24+12\lambda-12+8\lambda+12|}{13} = 4 \Rightarrow$

$\Rightarrow |-156+104\lambda| = 52 \Rightarrow$

$\Rightarrow \begin{cases} 104\lambda - 156 = 52 \Rightarrow \lambda_1 = 2 & A_1(3;0;-1) \\ 104\lambda - 156 = -52 \Rightarrow \lambda_2 = 1 & A_2(-4;4,1) \end{cases}$

EXERCICE 37

$\vec{a} = \begin{pmatrix} 2 \\ 4 \\ -3 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 7 \\ -2 \\ 5 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$

$\vec{a} \wedge \vec{b} = \begin{pmatrix} 2 \\ 4 \\ -3 \end{pmatrix} \wedge \begin{pmatrix} 7 \\ -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 20-6 \\ -21-10 \\ -4-28 \end{pmatrix} = \begin{pmatrix} 14 \\ -31 \\ -32 \end{pmatrix}$

$\begin{matrix} 2 & & 7 \\ 4 & & -2 \end{matrix}$

$\vec{a} \wedge \vec{c} = \begin{pmatrix} 2 \\ 4 \\ -3 \end{pmatrix} \wedge \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -8-0 \\ -3+4 \\ 0-4 \end{pmatrix} = \begin{pmatrix} -8 \\ 1 \\ -4 \end{pmatrix}$

$\begin{matrix} 2 & & 1 \\ 4 & & 0 \end{matrix}$

$\vec{b} \wedge \vec{a} = -\vec{a} \wedge \vec{b} = \begin{pmatrix} -14 \\ 31 \\ 32 \end{pmatrix} \quad \vec{c} \wedge \vec{b} = \begin{pmatrix} -4 \\ -19 \\ -2 \end{pmatrix}$

EXERCICE 38

$A(3;0;6) \quad B(6;-6;-4) \quad C(-2;-4;4)$

a) $\vec{AB} = \begin{pmatrix} 3 \\ -6 \\ -10 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} -5 \\ -4 \\ -2 \end{pmatrix}$

$\vec{n} = \vec{AB} \wedge \vec{AC} = \begin{pmatrix} 3 \\ -6 \\ -10 \end{pmatrix} \wedge \begin{pmatrix} -5 \\ -4 \\ -2 \end{pmatrix} = \begin{pmatrix} 12-40 \\ 50+6 \\ -12-30 \end{pmatrix} = \begin{pmatrix} -28 \\ 56 \\ -42 \end{pmatrix} // \begin{pmatrix} 2 \\ -4 \\ +3 \end{pmatrix}$

$\begin{matrix} 3 & & -5 \\ -6 & & -4 \end{matrix}$

$\Pi : 2x - 4y + 3z + d = 0$

$A : 6 + 18 + d = 0 \Rightarrow d = -24$

$\Pi \quad 2x - 4y + 3z - 24 = 0$

c) h horisontale $\Rightarrow h \perp Oz \Rightarrow h \perp \begin{pmatrix}0\\0\\1\end{pmatrix}$

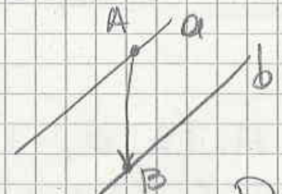
$h \in \Pi \Rightarrow \vec{h} \perp \vec{n}_\Pi$

Donc $\vec{h} = \begin{array}{c}\begin{pmatrix}0\\0\\1\end{pmatrix}\\0\\0\end{array} \wedge \begin{array}{c}\begin{pmatrix}2\\-4\\3\end{pmatrix}\\2\\-4\end{array} = \begin{pmatrix}4\\2\\0\end{pmatrix} \parallel \begin{pmatrix}2\\1\\0\end{pmatrix}$

$h: \begin{cases} x = 3+2\lambda \\ y = \lambda \\ z = 6 \end{cases}$

EXERCICE 39

$a: A(5;1;2) \quad b: B(3;9;7) \parallel \begin{pmatrix}1\\-1\\5\end{pmatrix}$



$\vec{n}_\Pi \perp \begin{pmatrix}1\\-1\\5\end{pmatrix}$

$\vec{n}_\Pi \perp \vec{AB} = \begin{pmatrix}-2\\8\\5\end{pmatrix}$

Donc $\vec{n}_\Pi = \begin{array}{c}\begin{pmatrix}1\\-1\\5\end{pmatrix}\\1\\-1\end{array} \wedge \begin{array}{c}\begin{pmatrix}-2\\8\\5\end{pmatrix}\\-2\\8\end{array} = \begin{pmatrix}-5-40\\-10-5\\8-2\end{pmatrix} = \begin{pmatrix}-45\\-15\\6\end{pmatrix}$

$\vec{n}_\Pi \parallel \begin{pmatrix}15\\5\\-2\end{pmatrix}$

$\Pi: 15x+5y-2z+d=0$

$A: 75+5-4+d=0 \Rightarrow d=-76$

$\Pi: 15x+5y-2z-76=0$

EXERCICE 40

$d: A(4;y;4) \quad \parallel$ mur $\quad d \in \Pi: 3x+2y+z=0$

$A \in d \in \Pi \Rightarrow A \in \Pi: 12+2y+4=0 \Rightarrow 2y=-16 \Rightarrow y=-8$

$A(4;-8;4)$

$d \parallel$ mur $\Rightarrow \vec{d} \perp Ox \Rightarrow \vec{d} \perp \begin{pmatrix}1\\0\\0\end{pmatrix}$

$\vec{d} \in \Pi \Rightarrow \vec{d} \perp \vec{n}_\Pi \Rightarrow \vec{d} \perp \begin{pmatrix}3\\2\\1\end{pmatrix}$

$\vec{d} = \begin{array}{c}\begin{pmatrix}1\\0\\0\end{pmatrix}\\1\\0\end{array} \wedge \begin{array}{c}\begin{pmatrix}3\\2\\1\end{pmatrix}\\3\\2\end{array} = \begin{pmatrix}0\\-1\\2\end{pmatrix} \quad d: \begin{cases} x = 4 \\ y = -8-\lambda \\ z = 4+2\lambda \end{cases}$

Exercice 41

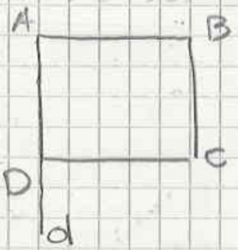
$\pi: 3x - 12y + 4z - 24 = 0$

$A(8;1;z_A) \quad B(x_B;5;6) \in \pi$

$A \in \pi: \; 24 - 12 + 4z_A - 24 = 0 \Rightarrow 4z_A = 12 \Rightarrow z_A = 3$

$B \in \pi: \; 3x_B - 60 + 24 - 24 = 0 \Rightarrow x_B = 20$

$A(8;1;3) \quad B(20;5;6)$



On trouve la droite $d_{AD} = d$

$\left.\begin{array}{l}\vec{d} \perp \vec{AB} \\ \vec{d} \perp \vec{n}_\pi\end{array}\right\} \vec{d} = \vec{AB} \wedge \vec{n}_\pi = \begin{pmatrix}12\\4\\3\end{pmatrix} \wedge \begin{pmatrix}3\\-12\\4\end{pmatrix} = \begin{pmatrix}52\\-39\\-156\end{pmatrix}$

$\Rightarrow \vec{d} = \begin{pmatrix}4\\-3\\-12\end{pmatrix} \quad d: \begin{cases}x = 8 + 4\lambda \\ y = 1 - 3\lambda \\ z = 3 - 12\lambda\end{cases}$

$D \in d \quad D(8+4\lambda; 1-3\lambda; 3-12\lambda) \quad \vec{AD} = \begin{pmatrix}4\lambda\\-3\lambda\\-12\lambda\end{pmatrix}$

$\|\vec{AB}\| = \|\vec{AD}\| \Rightarrow \sqrt{12^2+4^2+9} = \sqrt{16\lambda^2+9\lambda^2+144\lambda^2}$

$169 = 169\lambda^2 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$

$\lambda_1 = 1 \quad D(12;-2;-9) \quad C(24;2;-6)$

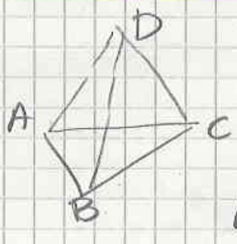
$\vec{AB} = \vec{DC} \Rightarrow \begin{pmatrix}12\\4\\3\end{pmatrix} = \begin{pmatrix}x-12\\y+2\\z+9\end{pmatrix} \Rightarrow \begin{array}{l}x = 24\\y = 2\\z = -6\end{array}$

$\lambda = -1 \quad D(4;4;15) \quad C(16;8;18)$

$\vec{AB} = \vec{DC} \Rightarrow \begin{pmatrix}12\\4\\3\end{pmatrix} = \begin{pmatrix}x-4\\y-4\\z-15\end{pmatrix} \Rightarrow \begin{array}{l}x = 16\\y = 8\\z = 18\end{array}$

Exercice 42

$A(0;2;3) \quad B(-2;2;-1) \quad C(4;-2;2)$



$V = \frac{1}{3} A_{base} \cdot \text{hauteur}$

$A_{base} = \frac{1}{2}\|\vec{AB} \wedge \vec{AC}\| = \frac{1}{2}\sqrt{16^2+18^2+8^2} = \frac{\sqrt{644}}{2} = \sqrt{161}$

$\vec{AB} = \begin{pmatrix}-2\\0\\-4\end{pmatrix} \quad \vec{AC} = \begin{pmatrix}4\\-4\\-1\end{pmatrix}$

$\vec{AB} \wedge \vec{AC} = \begin{pmatrix}-2\\0\\-4\end{pmatrix} \wedge \begin{pmatrix}4\\-4\\-1\end{pmatrix} = \begin{pmatrix}-16\\-18\\8\end{pmatrix}$

hauteur $= \text{dist}(D;\pi)$

On trouve le plan π contenant les points A, B, C

$\vec{n}_\Pi = \vec{AB} \wedge \vec{AC} = \begin{pmatrix} -16 \\ -18 \\ 8 \end{pmatrix} \parallel \begin{pmatrix} 8 \\ 9 \\ -4 \end{pmatrix}$

$\Pi : 8x + 9y - 4z + d = 0$

$A : 18 - 12 + d = 0 \Rightarrow d = -6.$

$\Pi : 8x + 9y - 4z - 6 = 0.$

$\text{dist}(\Pi, D) = \dfrac{|24 + 54 - 6|}{\sqrt{8^2 + 9^2 + 4^2}} = \dfrac{72}{\sqrt{161}} = h$

$V = \frac{1}{3} \cancel{\sqrt{161}} \cdot \dfrac{72}{\cancel{\sqrt{161}}} \Rightarrow V = 24$

EXERCICE 43

$a : \begin{cases} x = 17 + 3\lambda \\ y = -1 + \lambda \\ z = -12 - 4\lambda \end{cases} \qquad b : \begin{cases} x = -6 + 4\lambda \\ y = 4 - 5\lambda \\ z = 4 + 2\lambda \end{cases}$

a) $\vec{a} = \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} \not\parallel \vec{b} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} \Rightarrow$ sécantes / gauches

$17 + 3\lambda = -6 + 4k \Rightarrow 3\lambda - 4k = -23$

$-1 + \lambda = 4 - 5k \Rightarrow \lambda + 5k = 5$

$-12 - 4\lambda = 4 + 2k$ (3)

$\begin{cases} 3\lambda - 4k = -23 \\ \lambda + 5k = 5 \quad \cdot(-3) \end{cases} \qquad \begin{cases} 3\lambda - 4k = -23 \\ -3\lambda - 15k = -15 \end{cases}$

$+ \quad -19k = -38 \Rightarrow k = 2$

$\lambda = 5 - 10 \Rightarrow \lambda = -5$

(3) $-12 + 20 \stackrel{?}{=} 4 + 4$ oui $\Rightarrow$ sécantes

$I(2; -6; 8)$

b) $\vec{a} = \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} \Rightarrow \|\vec{a}\| = \sqrt{9 + 1 + 16} = \sqrt{26}.$

$\vec{b} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} \Rightarrow \|\vec{b}\| = \sqrt{16 + 25 + 4} = \sqrt{45}$

$\vec{a} \cdot \vec{b} = 12 - 5 - 8 \Rightarrow \vec{a} \cdot \vec{b} = -1$

$\cos\varphi = \dfrac{|\vec{a} \cdot \vec{b}|}{\|\vec{a}\| \|\vec{b}\|} = \dfrac{1}{\sqrt{26} \cdot \sqrt{45}} \Rightarrow \varphi = 88.32°$

c) $\vec{n}_\Pi = \vec{a} \wedge \vec{b} = \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} \wedge \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} -18 \\ -22 \\ -19 \end{pmatrix} \parallel \begin{pmatrix} 18 \\ 22 \\ 19 \end{pmatrix}$

$\begin{matrix} 3 & 4 \\ 1 & -5 \end{matrix}$

$\Pi : 18x + 22y + 19z + d = 0$

$A(17; -1; -12) : d = -56$ $\qquad \Pi : 18x + 22y + 19z - 56 = 0$

Exercice 44

$$a : \begin{cases} x = -5-\lambda \\ y = 8+4\lambda \\ z = 3+\lambda \end{cases} \quad // \quad b \begin{cases} x = k + m\lambda \\ y = 4-8\lambda \\ z = 2-2\lambda \end{cases}$$

a) $\vec{a} = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} \xrightarrow{\cdot(-2)} // \begin{pmatrix} m \\ -8 \\ -2 \end{pmatrix} \Rightarrow m = -2$

b) $B(k, 4; 2) \in b$ $\quad \begin{aligned} k &= k+2\lambda \rightarrow \lambda = 0 \\ 4 &= 4-8\lambda \Rightarrow \lambda = 0 \\ 2 &= 2-2\lambda \rightarrow \lambda = 0 \end{aligned} \Rightarrow B \in b \ \forall k \in \mathbb{R}$

c) $B \in a$ $\quad \begin{aligned} k &= -5-\lambda \\ 4 &= 8+4\lambda \\ 2 &= 3+\lambda \Rightarrow \lambda = -1 \end{aligned} \quad \begin{aligned} k &= -5+1 \Rightarrow \underline{k = -4} \\ \lambda &= -1 \end{aligned}$

Donc si $k = -4$ $B \in a$ $B \in b$

d) Si $B \in a \Rightarrow a, b$ sont confondues

Si $B \notin a \Rightarrow a // b \Rightarrow$

Pour $k \neq -4$ $a // b$

Exercice 45

$P(5; -2; 1)$ $\quad d : A(-3; 8; 16)$ $\quad B(-2; 14; 18)$

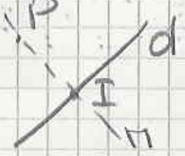
$\vec{d} = \vec{AB} = \begin{pmatrix} 1 \\ 6 \\ 2 \end{pmatrix}$ $\quad d : \begin{cases} x = -3+\lambda \\ y = 8+6\lambda \\ z = 16+2\lambda \end{cases}$

a) $\operatorname{dist}(P, d) = \dfrac{\| \vec{d} \wedge \vec{AP} \|}{\| \vec{d} \|} = \dfrac{\sqrt{9225}}{\sqrt{41}} = \sqrt{225} = 15$

$\vec{AP} = \begin{pmatrix} 8 \\ -10 \\ -15 \end{pmatrix}$ $\quad \vec{d} \wedge \vec{AP} = \begin{vmatrix} 1 \\ 6 \\ 2 \\ 1 \\ 6 \end{vmatrix} \wedge \begin{vmatrix} 8 \\ -10 \\ -15 \\ 8 \\ -10 \end{vmatrix} = \begin{pmatrix} -70 \\ 31 \\ -58 \end{pmatrix}$

$\| \vec{d} \wedge \vec{AP} \| = \sqrt{70^2 + 31^2 + 58^2} = \sqrt{9225}$

$\| \vec{d} \| = \sqrt{1+36+4} = \sqrt{41}$

b) 

$M \perp d \Rightarrow \vec{n}_M = \vec{d} = \begin{pmatrix} 1 \\ 6 \\ 2 \end{pmatrix}$

$M : x + 6y + 2z + d = 0$ $\quad P : 5 - 12 + 2 + d = 0 \Rightarrow d = 5$

$M : x + 6y + 2z + 5 = 0$

$I = d \cap M$ $\quad -3 + \lambda + 6(8+6\lambda) + 2(16+2\lambda) + 5 = 0 \Rightarrow$

$\Rightarrow -3 + \lambda + 48 + 36\lambda + 32 + 4\lambda + 5 = 0 \Rightarrow 41\lambda = -82 \Rightarrow \lambda = -2$

$I(-5; -4; 12)$

Exercice 46

$\alpha : x - 2y + z - 8 = 0 \qquad \beta : x + y - 3z + 1 = 0$

a) $\vec{n}_\alpha = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \not\parallel \vec{n}_\beta = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} \Rightarrow$ sécants

b) $\cos\varphi = \dfrac{|\vec{n}_\alpha \cdot \vec{n}_\beta|}{\|\vec{n}_\alpha\| \cdot \|\vec{n}_\beta\|} = \dfrac{|1-2-3|}{\sqrt{6}\,\sqrt{11}} \Rightarrow \varphi = 60.5^\circ$

c) $\left.\begin{array}{l} \vec{i} \perp \vec{n_\alpha} \\ \vec{i} \perp \vec{n_\beta} \end{array}\right\} \quad \vec{i} = \vec{n}_\alpha \wedge \vec{n}_\beta = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$

d) sol $z = 0$

$$\begin{array}{rl} \alpha : & x - 2y = 8 \\ (-)\ \beta : & x + y = -1 \\ \hline & -3y = 9 \Rightarrow y = -3 \end{array} \qquad x = 2$$

$A \in \alpha \cap \beta : \ A(2; -3; 0)$

$\vec{i} = \begin{cases} x = 2 + 5\lambda \\ y = -3 + 4\lambda \\ z = 3\lambda \end{cases}$

Exercice 47

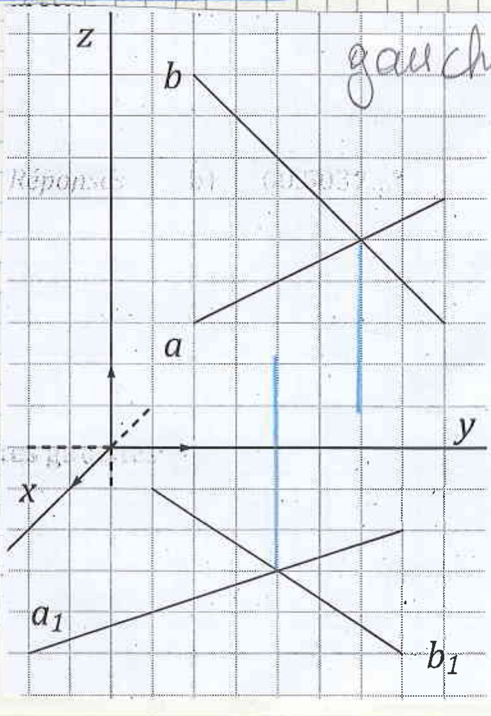
$\alpha : x + y - 2z + 4 = 0 \qquad \beta : x + y + mz + 5 = 0$

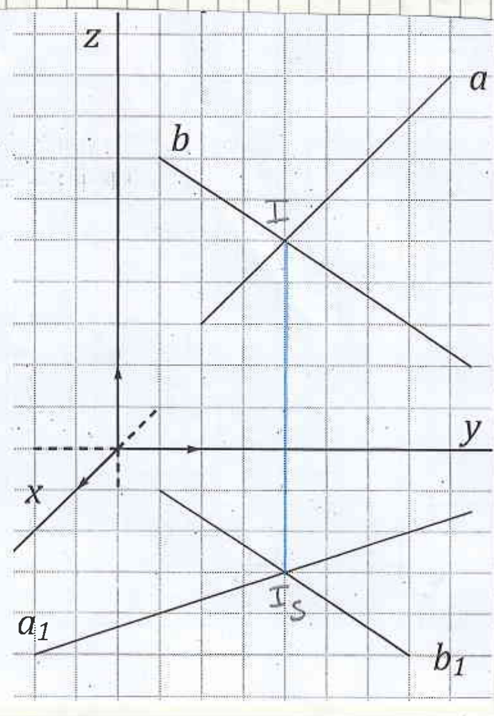
a) $\vec{n}_\alpha = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \parallel \vec{n}_\beta = \begin{pmatrix} 1 \\ 1 \\ m \end{pmatrix} \Rightarrow m = -2$

b) $\alpha \perp \beta \Rightarrow \vec{n}_\alpha \perp \vec{n}_\beta \Rightarrow \vec{n}_\alpha \cdot \vec{n}_\beta = 0 \Rightarrow$

$\Rightarrow 1 + 1 - 2m = 0 \Rightarrow 2m = 2 \Rightarrow m = 1$

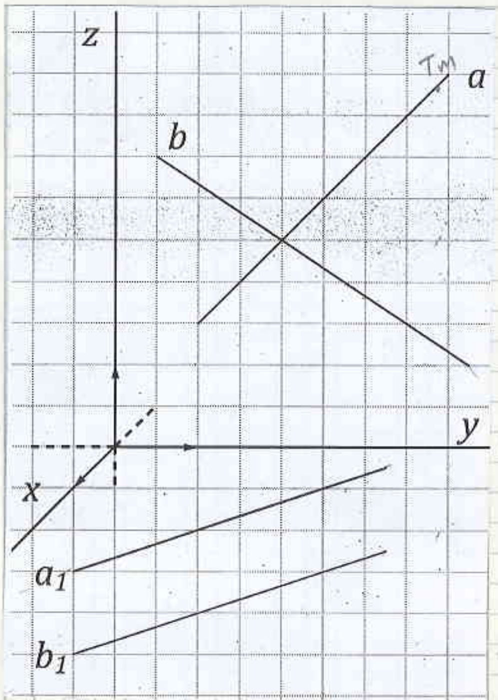
Exercice 48





sécantes

$I(3; 3{,}5; 4)$



gauches

EXERCICE 49

$a: \begin{cases} x = 10 + 6\lambda \\ y = 4 - 2\lambda \\ z = 10 - 5\lambda \end{cases}$ $b = \begin{cases} x = 2 + k \\ y = 4 + 2k \\ z = -27 - 2k \end{cases}$

$\vec{a} = \begin{pmatrix} 6 \\ -2 \\ -5 \end{pmatrix} \not\parallel \vec{b} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ < sécantes / gauches

$10 + 6\lambda = 2 + k$
$4 - 2\lambda = 4 + 2k$
$10 - 5\lambda = -27 - 2k$ ③

$\begin{cases} 6\lambda - k = -8 \\ -2\lambda - 2k = 0 \end{cases} \Rightarrow \begin{cases} 6\lambda - k = -8 \\ +\lambda + k = 0 \end{cases}$

$7\lambda = -8 \Rightarrow \lambda = -\frac{8}{7}$

$k = \frac{8}{7}$

③ $-5\lambda + 2k = -37$

$\frac{40}{7} + \frac{16}{7} \stackrel{?}{=} -37 \Rightarrow \frac{56}{7} = 8 \neq -37 \Rightarrow$ gauches

EXERCICE 50

$a: \begin{cases} x = 1 + 3\lambda \\ y = 2 - \lambda \\ z = 8 - 2\lambda \end{cases}$ $b \begin{cases} x = 7 - 3\lambda \\ y = 3 + 2\lambda \\ z = -5 + m \cdot \lambda \end{cases}$

$\vec{a} = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} \not\parallel \vec{b} = \begin{pmatrix} -3 \\ 2 \\ m \end{pmatrix} \Rightarrow$ < sécantes / gauches

$1 + 3\lambda = 7 - 3k$
$2 - \lambda = 3 + 2k$
$8 - 2\lambda = -5 + mk$

$\begin{cases} 3\lambda + 3k = 6 \\ -\lambda - 2k = 1 \end{cases} \Rightarrow \begin{cases} \lambda + k = 2 \\ -\lambda - 2k = 1 \end{cases}$

$-k = 3 \Rightarrow k = -3$

$\lambda = 5$

③ $8 - 10 = -5 - 3m \Rightarrow 3m = -3 \Rightarrow m = -1$

- Si $m = -1$ sécantes $I(16; -3; -2)$
- Si $m \neq -1$ gauches

- Si $m=-1$ (sécantes) $\vec{a}=\begin{pmatrix}3\\-1\\-2\end{pmatrix}$ $\vec{b}=\begin{pmatrix}-3\\2\\-1\end{pmatrix}$

$\cos\varphi = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\cdot\|\vec{b}\|} = \frac{|-9-2+2|}{\sqrt{9+1+4}\sqrt{9+4+1}} \Rightarrow \varphi = 49.9947°$

EXERCICE 51

a) $C(5;-3;7)$ $r=3$ $(x-5)^2+(y+3)^2+(z-7)^2=9$

paroi: $y=0$ $(x-5)^2+\cancel{9}+(z-7)^2=\cancel{9} \Leftrightarrow (x-5)^2+(z-9)^2=0 \Rightarrow$

$x=5$ $z=7$ $I(5;0;7)$

b) $C(4;-3;-2)$ $\pi: 16x-15y-12z+42=0$

S tangente au plane $\Rightarrow dist(C,\pi)=r \Rightarrow$

$r=\frac{|64+45+24+42|}{\sqrt{16^2+15^2+12^2}}=7$ $(x-4)^2+(y+3)^2+(z+2)^2=49$

$S\cap Ox$: $y=z=0$ $(x-4)^2+9+4=49 \Rightarrow (x-4)^2=36$

$x-4=6 \Rightarrow x_1=10$ $x-4=-6 \Rightarrow x_2=-2$

$I_1(10;0;0)$ $I_2(-2;0;0)$

EXERCICE 52

a) $x^2+y^2+z^2-6x+2y-10z-46=0$

$(x-3)^2-9+(y+1)^2-1+(z-5)^2-25-46=0 \Leftrightarrow$

$\Leftrightarrow (x-3)^2+(y+1)^2+(z-5)^2=81$

$C(3;-1;5)$ $r=9$

b) $4x^2+4y^2+4z^2+8x-16y-4z=123$

$\Leftrightarrow x^2+y^2+z^2+2x-4y-z=\frac{123}{4} \Rightarrow$

$\Rightarrow (x+1)^2-1+(y-2)^2-4+(z-\frac{1}{2})^2-\frac{1}{4}=\frac{123}{4} \Leftrightarrow$

$\Leftrightarrow (x+1)^2+(y-2)^2+(z-\frac{1}{2})^2=36$

$C(-1;2;\frac{1}{2})$ $r=6$

EXERCICE 53

$d: \begin{cases} x=3+2\lambda \\ y=1-\lambda \\ z=4-\lambda \end{cases}$ $S: (x-1)^2+(y-4)^2+(z-2)^2=13$

$d\cap S$: $(3+2\lambda-1)^2+(1-\lambda-4)^2+(4-\lambda-2)^2=13 \Leftrightarrow$

$\Leftrightarrow (2+2\lambda)^2+(-3-\lambda)^2+(2-\lambda)^2=13 \Leftrightarrow$

$\Leftrightarrow 4+8\lambda+4\lambda^2+9+6\lambda+\lambda^2+4-4\lambda+\lambda^2=13 \Leftrightarrow$

$\Leftrightarrow 6\lambda^2+10\lambda+4=0 \Leftrightarrow 3\lambda^2+5\lambda+2=0$

$\Delta=25-24=1$ $\lambda_{1,2}=\frac{-5\pm1}{6}$ $\lambda_1=-1$, $\lambda_2=-\frac{2}{3}$

$\lambda_1=-1$ $I_1(1;2;5)$ $\lambda_2=-\frac{2}{3}$ $I_2(\frac{5}{3};\frac{5}{3};\frac{14}{3})$

EXERCICE 54

$$d:\begin{cases} x = 10-\lambda \\ y = 10-\lambda \\ z = m\lambda \end{cases} \qquad S: x^2+y^2+z^2=9$$

$S \cap d: (10-\lambda)^2+(10-\lambda)^2+m^2\lambda^2=9 \Rightarrow$

$\Rightarrow 200-40\lambda+2\lambda^2+m^2\lambda^2-9=0 \Leftrightarrow$

$(2+m^2)\lambda^2-40\lambda+191=0$

d tangente à la sphère => 1 point commun

$\Rightarrow \Delta=0 \Rightarrow 40^2-4(2+m^2)\cdot 191=0 \Rightarrow$

$\Rightarrow 1600-1528-764m^2=0 \Rightarrow$

$\Rightarrow 764m^2=72 \Rightarrow m^2=\frac{18}{191} \Rightarrow m=\pm\sqrt{\frac{18}{191}}$

EXERCICE 55

$$d:\begin{cases} x = 3+2\lambda \\ y = m+\lambda \\ z = 2\lambda \end{cases} \qquad S: x^2+y^2+z^2=9$$

$d \cap S: (3+2\lambda)^2+(m+\lambda)^2+4\lambda^2=9 \Leftrightarrow$

$\Rightarrow \cancel{9}+12\lambda+4\lambda^2+m^2+2m\lambda+\lambda^2+4\lambda^2-\cancel{9}=0$

$\Leftrightarrow 9\lambda^2+(12+2m)\lambda+m^2=0$

$\Delta=(12+2m)^2-36m^2=144+48m+4m^2-36m^2=$

$=-32m^2+48m+144=-16(2m^2-3m-9)$

- $\Delta=0 \Rightarrow 2m^2-3m-9=0 \Rightarrow \Delta=9+72=81$

$m_{1,2}=\frac{3\pm 9}{4} \begin{cases} 3=m_1 \\ -\frac{3}{2}=m_2 \end{cases}$

Dans ce cas on a 2 points communs => d est tangente

- $\Delta>0$

	$-\frac{3}{2}$		3	
$-$	0	$+$	0	$-$

$\Rightarrow -\frac{3}{2}<m<3$

on a 2 points communs => d est sécante

- $\Delta<0$ $\quad m<-\frac{3}{2}$ ou $m>3$ aucun point commun

Exercice 56

$S: (x-5)^2+(y+1)^2+(z-3)^2=49 \quad C(5;-1;3) \quad r=7$

a) $p>0 \quad P(7;p;9) \in S$

$(7-5)^2+(p+1)^2+(9-3)^2=49 \Rightarrow$

$\Rightarrow 4+p^2+2p+1+36=49 \Leftrightarrow p^2+2p-8=0$

$\Delta=4+32=36 \quad p_{1,2}=\frac{-2\pm 6}{2} \quad p_1=-4, \ p_2=2 \quad p>0 \Rightarrow \underline{p=2}$

b) $d \,//\, sol \Rightarrow \vec{d} \perp Oz \Rightarrow \vec{d} \perp \begin{pmatrix}0\\0\\1\end{pmatrix}$

d tangente à sphère $\Rightarrow \vec{CP} \perp \vec{d}$

$P(7;2;9)$ Donc $\vec{d} = \begin{pmatrix}0\\0\\1\end{pmatrix} \wedge \vec{CP} = \begin{pmatrix}0\\0\\1\end{pmatrix} \wedge \begin{pmatrix}2\\3\\6\end{pmatrix} = \begin{pmatrix}-3\\2\\0\end{pmatrix}$

$d: \begin{cases} x=7-3\lambda \\ y=2+2\lambda \\ z=9 \end{cases}$

Exercice 57

$C(3,1,-2) \quad r=\sqrt{24}$

$S: (x-3)^2+(y-1)^2+(z+2)^2=24 \quad P(-1,a;0)$

a) $a<0 \quad P\in S: (-1-3)^2+(a-1)^2+2^2=24 \Rightarrow (a-1)^2=4 \Rightarrow$

$a-1=2 \Rightarrow a_1=3 \qquad a-1=-2 \Rightarrow a=-1<0 \quad P(-1;-1;0)$

b) $\vec{n}_\Pi = \vec{CP} = \begin{pmatrix}-4\\-2\\+2\end{pmatrix} \,//\, \begin{pmatrix}2\\1\\-1\end{pmatrix} \quad \Pi: 2x+y-z+d=0$

$P: -2-1+d=0 \Rightarrow d=3 \quad \Pi: 2x+y-z+3=0$

c) $d \,//\, mur \Rightarrow \vec{d} \perp Ox \Rightarrow \vec{d} \perp \begin{pmatrix}1\\0\\0\end{pmatrix}$

d tangente à la sphère $\Rightarrow \vec{d} \perp \vec{CP} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$

$\Rightarrow \vec{d} = \begin{pmatrix}1\\0\\0\end{pmatrix} \wedge \begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}0\\1\\1\end{pmatrix} = \vec{d}$

Exercice 58

$\Pi: 12x+4y+3z-12=0 \quad C(3;1;0) \quad r=26$

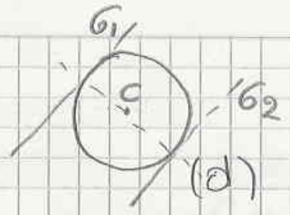
a) $\sigma \,//\, \Pi \Rightarrow \sigma: 12x+4y+3z+d=0$

$dist(\sigma, C)=r \Rightarrow \frac{|36+4+d|}{\sqrt{12^2+4^2+3^2}}=26 \Rightarrow$

$|40+d|=338 \Rightarrow \begin{cases} 40+d=338 \Rightarrow d_1=298 \\ 40+d=-338 \Rightarrow d_2=-378 \end{cases}$

$\sigma_1: 12x+4y+3z+298=0 \quad \sigma_2: 12x+4y+3z-378=0$

b)

b)  On trouve la droite d

$\vec{d} = \vec{n}_6 = \begin{pmatrix} 12 \\ 4 \\ 3 \end{pmatrix}$ $d = \begin{cases} x = 3 + 12\lambda \\ y = 1 + 4\lambda \\ z = 3\lambda \end{cases}$

$I_1 = G_1 \cap d$: $12(3+12\lambda) + 4(1+4\lambda) + 3 \cdot 3\lambda + 298 = 0$

$\Rightarrow 36 + 144\lambda + 4 + 16\lambda + 9\lambda + 298 = 0$

$\Rightarrow 169\lambda = -338 \Rightarrow \lambda_1 = -2$ $I_1(-21; -7; -6)$

$I_2 = G_2 \cap d$ $12(3+12\lambda) + 4(1+4\lambda) + 3 \cdot 3\lambda - 378 = 0$

$\Rightarrow 36 + 144\lambda + 4 + 16\lambda + 9\lambda - 378 = 0$

$\Rightarrow 169\lambda = 338 \Rightarrow \lambda = 2$ $I_2(27; 9; 6)$

Exercice 59

$C(1; 2, 4)$ $r = 3$

$S : (x-1)^2 + (y-2)^2 + (z-4)^2 = 9$ $\Pi : z - 12 = 0$

a)  $dist(C, \Pi) = \frac{|4 - 12|}{\sqrt{1}} = 8 > r$

Alors $x = 8 - 3 = 5$

b) On cherche le point A

On trouve la droite $d : \vec{d} = \vec{n}_\Pi = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $d \begin{cases} x = 1 \\ y = 2 \\ z = 4 + \lambda \end{cases}$

$A(1; 2; 4+\lambda) \in S$

$(1-1)^2 + (2-2)^2 + (\lambda)^2 = 9 \Rightarrow \lambda^2 = 9 \Rightarrow \lambda = \pm 3$

$\lambda = 3$ $A(1; 2, 7)$ $\lambda = -3$ $A(1; 2; 1)$

$dist(A, \Pi) = \frac{|7 - 12|}{\sqrt{1}} = 5$ $dist(A, \Pi) = \frac{|1 - 12|}{\sqrt{1}} = 11$

Donc le point cherché est $A(1; 2; 7)$

c) On cherche le point B

$B = d \cap \Pi$ $4 + \lambda - 12 = 0 \Rightarrow \lambda = 8$ $B(1; 2; 12)$

Exercice 60

$\Pi : 3x + 4y - 12 = 0$

a) $\Pi // Oz \Rightarrow \Pi \perp sol$

b) $C(0, 0, z_0)$ $z_0 > 0$

tangente au sol $\Rightarrow r = z_0$

tangent au Π : $dist(C, \Pi) = r \Rightarrow \frac{|-12|}{\sqrt{9+16}} = r \Rightarrow r = \frac{12}{5}$

$z_0 = \frac{12}{5} = 2.4$ $C(0, 0; \frac{12}{5})$

c) $O(0,0,0)$

d) 

On trouve la droite d

$\vec{d} = \vec{n}_\pi = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ $\quad d = \begin{cases} x = 3\lambda \\ y = 4\lambda \\ z = \frac{12}{5} \end{cases}$

$C(0,0,\frac{12}{5})$ $\quad S: x^2+y^2+(z-\frac{12}{5})^2 = \frac{144}{25}$

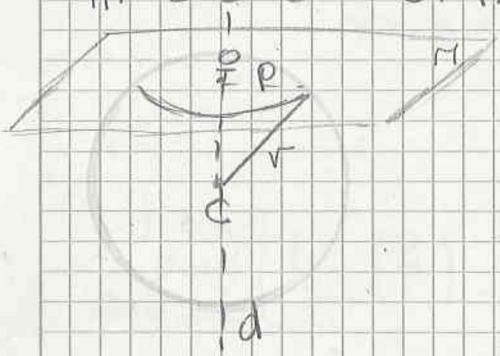
$S \cap d \quad 9\lambda^2 + 16\lambda^2 = \frac{144}{25} \Rightarrow 25\lambda^2 = \frac{144}{25} \Rightarrow \lambda^2 = \frac{144}{625}$

$\Rightarrow \lambda = \pm \frac{12}{25} \qquad (\frac{36}{25}; \frac{48}{25}; \frac{12}{5})$ ou $(-\frac{36}{25}; -\frac{48}{25}; \frac{12}{5})$

Le point le plus proche au plan est $(\frac{36}{25}; \frac{48}{25}; \frac{12}{5})$

EXERCICE 61

$\pi: z-8=0 \qquad S: (x-6)^2+(y-7)^2+(z-4)^2 = 25 \quad C(6;7;4) \quad r=5$



• $\text{dist}(\Omega, C) = \text{dist}(\pi; C) = \frac{|4-8|}{\sqrt{1}} = 4$

$r^2 = \rho^2 + \Omega C^2 \Rightarrow \rho^2 = 25 - 16 = 9 \Rightarrow \rho = 3$

• $d = \begin{cases} x = 6 \\ y = 7 \\ z = 4+\lambda \end{cases}$

$\Omega = d \cap \pi \quad 4+\lambda-8=0 \Rightarrow \lambda = 4 \quad \Omega(6;7;8)$

EXERCICE 62

$S: (x-3)^2+(y+2)^2+(z-1)^2 = 100 \qquad \pi: 2x-2y-z+9=0$

$C(3;-2;1) \quad r=10$

• $\text{dist}(\Omega, C) = \text{dist}(\pi, C) = \frac{|6+4-1+9|}{\sqrt{4+4+1}} = \frac{18}{3} = 6$

$r^2 = \rho^2 + \Omega C^2 \Rightarrow \rho^2 = 100 - 36 = 64 \Rightarrow \rho = 8$

• $d = \begin{cases} x = 3+2\lambda \\ y = -2-2\lambda \\ z = 1-\lambda \end{cases}$

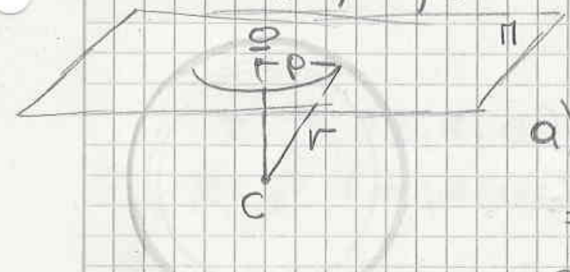
$\Omega = d \cap \pi: 2(3+2\lambda) - 2(-2-2\lambda) - (1-\lambda) + 9 = 0 \Leftrightarrow$

$\Leftrightarrow 6+4\lambda+4+4\lambda-1+\lambda+9 = 0 \Leftrightarrow 9\lambda = -18 \Rightarrow \lambda = -2$

$\Omega(-1; 2; 3)$

Exercice 63

$S : C(2;-2;3) \quad r = 10 \quad (x-2)^2+(y+2)^2+(z-3)^2=100$



$\Omega(6;-6;5)$

a) $\text{dist}(\Omega, C) = \|\overrightarrow{\Omega C}\| = \left\| \begin{pmatrix} -4 \\ 4 \\ -2 \end{pmatrix} \right\| =$

$= \sqrt{16+16+4} = 6$

$\rho^2 = r^2 - \Omega C^2 \Rightarrow \rho^2 = 100 - 36 = \underbrace{8 = \rho}$

b) $\vec{n}_\Pi = \begin{pmatrix} -4 \\ 4 \\ -2 \end{pmatrix} /\!/ \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \quad \Pi : 2x - 2y + z + d = 0$

$\Omega : 12 + 12 + 5 + d = 0 \Rightarrow d = -29$

$\Pi : 2x - 2y + z - 29 = 0$

Exercice 64

$\Pi : 6x + 2y + 3z - 17 = 0 \quad S : (x+13)^2 + y^2 + (z+1)^2 = 245$

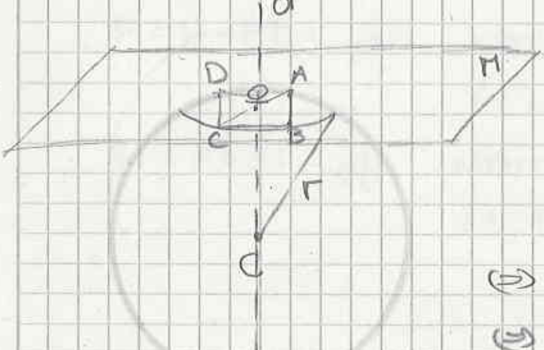
$A(-4;10;7) \quad C(-13;0;-1) \quad r = \sqrt{245}$

a) $\mathcal{C} = \Pi \cap S \Rightarrow$ on doit montrer que $A \in \Pi \cap S$

$A \in \Pi : -24 + 20 + 21 - 17 = 0$ ✓

$A \in S : (-4+13)^2 + 10^2 + (7+1)^2 = 9^2 + 10^2 + 8^2 = 245$ ✓

$\Rightarrow A \in$ au cercle



$\cdot \; d = \begin{cases} x = -13 + 6\lambda \\ y = \quad +2\lambda \\ z = -1 + 3\lambda \end{cases}$

$\Omega \quad d \cap \Pi$

$6(-13+6\lambda) + 4\lambda + 3(-1+3\lambda) - 17 = 0 \Rightarrow$

$\Leftrightarrow -78 + 36\lambda + 4\lambda - 3 + 9\lambda - 17 = 0 \Leftrightarrow$

$\Leftrightarrow 49\lambda = 98 \Rightarrow \lambda = 2 \quad \underline{\Omega(-1;4;5)}$

• $\overrightarrow{A\Omega} = \overrightarrow{\Omega C} \Rightarrow \begin{pmatrix} 3 \\ -6 \\ -2 \end{pmatrix} = \begin{pmatrix} x+1 \\ y-4 \\ z-5 \end{pmatrix} \Rightarrow \begin{matrix} x = 2 \\ y = -2 \\ z = 3 \end{matrix} \quad \underline{C(2;-2;3)}$

• $\overrightarrow{\Omega B} \perp \vec{n}_\Pi = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$

$\overrightarrow{\Omega B} \perp \overrightarrow{\Omega A} = \begin{pmatrix} 3 \\ -6 \\ -2 \end{pmatrix} \Rightarrow \overrightarrow{\Omega B} = \vec{n}_\Pi \wedge \overrightarrow{\Omega A} = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} \wedge \begin{pmatrix} 3 \\ -6 \\ -2 \end{pmatrix} = \begin{pmatrix} 14 \\ 21 \\ -42 \end{pmatrix} /\!/ \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$

$\begin{matrix} 6 & \quad 3 \\ 2 & \quad -6 \end{matrix}$

car ABCD carré

$d_{\Omega B} = \begin{cases} x = -1 + 2\lambda \\ y = 4 + 3\lambda \\ z = 5 - 6\lambda \end{cases} \quad B(-1+2\lambda; 4+3\lambda; 5-6\lambda) \quad \overrightarrow{\Omega B} = \begin{pmatrix} 2\lambda \\ 3\lambda \\ -6\lambda \end{pmatrix}$

$\|\overrightarrow{\Omega B}\| = \|\overrightarrow{\Omega A}\| \Rightarrow 4\lambda^2 + 9\lambda^2 + 36\lambda^2 = 9 + 36 + 4 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$

$\lambda = 1 \quad B(1;7;-1)$

$\lambda = -1 \quad D(-3;1;11)$

$\rho = \|\Omega A\| = \left\| \begin{pmatrix} 3 \\ -6 \\ -2 \end{pmatrix} \right\| = 7 = \rho$

Exercice 65

$S_1 : (x-10)^2 + (y-8)^2 + (z-1)^2 = 25 \quad C_1(10;8;1) \quad r_1 = 5$

$S_2 \quad (x-10)^2 + (y-8)^2 + (z-m)^2 = 4 \quad C_2(10;8;m) \quad r_2 = 2$

On veut $\|\vec{C_1C_2}\| < |r_1 - r_2| = 3$

$\vec{C_1C_2} = \begin{pmatrix} 0 \\ 0 \\ m-1 \end{pmatrix} \quad \|\vec{C_1C_2}\| = \sqrt{(m-1)^2} < 3 \Rightarrow (m-1)^2 < 9 \Rightarrow$

$\Rightarrow m^2 - 2m + 1 - 9 < 0 \Rightarrow m^2 - 2m - 8 < 0$

$\Delta = 4 + 32 = 36 \quad m_{1,2} = \frac{2 \pm 6}{2}$ $\begin{cases} m_1 = 4 \\ m_2 = -2 \end{cases}$

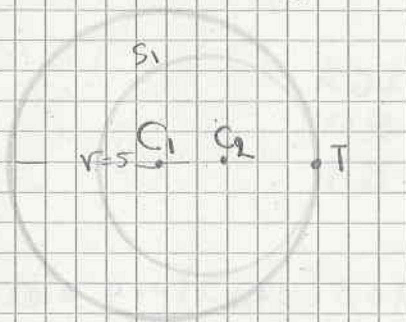
	$-\infty$		-2		4		$+\infty$
m^2-2m-8		$+$	0	$-$	0	$+$	

$\Rightarrow -2 < m < 4$

Exercice 66

$S_1 : (x-10)^2 + (y-8)^2 + (z-1)^2 = 25 \quad C_1(10;8;1) \quad r_1 = 5$

$S_2 : (x-10)^2 + (y-b)^2 + (z-1)^2 = 16 \quad C_2(10,b,1) \quad r_2 = 4$



- $y_T = 8 + 5 = 13 \quad T(10;13;1)$
- Si S_2 intérieur $\quad y_2 = 13 - 4 = 9$

  $C_2(10;9;1)$
- Si S_2 extérieur $\quad y_2 = 13 + 4 = 17$

  $C_2(10;17;1)$