

EQUATIONS DU 2^{ème} DEGRE

EXERCICE 1

1. $x^2 - 3x + 3 = 0$ $\Delta = 9 - 12 = -3 < 0$ Non
2. $2x^2 - 5x - 13 = 0$ $\Delta = 25 + 4 \cdot 2 \cdot 13 = 129 > 0$ Oui.
3. $7x^2 - 3x = 0$ $\Delta = 9 > 0$ Oui
4. $6x^2 + x - 61 = 0$ $\Delta = 1 + 4 \cdot 6 \cdot 61 = 1465$ Oui
5. $x^2 + 4 = 0$ $\Delta = -16 < 0$ Non
6. $x^2 - 3 = 0$ $\Delta = +12 > 0$ Oui

EXERCICE 2

1. $x^2 - 4x + 3 = 0$
 $S = 4$ $P = 3$ $x_1 = 3$ $x_2 = 1$
2. $x^2 - 3x = 0 \Leftrightarrow x(x - 3) = 0 \Leftrightarrow x_1 = 0$ $x_2 = 3$
3. $x^2 - 3(x + 3) + 3x + 5(x^2 + 1) = 0 \Leftrightarrow$
 $\Leftrightarrow x^2 - 3x - 9 + 3x + 5x^2 + 5 = 0 \Leftrightarrow 6x^2 - 4 = 0 \Leftrightarrow 3x^2 = 2 \Leftrightarrow$
 $\Leftrightarrow x^2 = \frac{2}{3} \Leftrightarrow x = \pm \sqrt{\frac{2}{3}}$
4. $(x - 3)(2x + 5) = 0 \Leftrightarrow x_1 = 3$ $x_2 = -\frac{5}{2}$
5. $3x^2 - 17x - 6 = 0$
 $\Delta = 17^2 + 4 \cdot 3 \cdot 6 = 361 = 19^2$
 $x_{1,2} = \frac{17 \pm 19}{6} = \begin{cases} x_1 = 6 \\ x_2 = -\frac{2}{6} = -\frac{1}{3} \end{cases}$
6. $9y^2 - 12y + 8 = 0$ $\Delta = 12^2 - 4 \cdot 9 \cdot 8 = -144 < 0$
 impossible.
7. $6t^2 - 11t = 4t \Leftrightarrow 6t^2 - 15t = 0 \Leftrightarrow 3t^2 - 5t = 0 \Leftrightarrow t(3t - 5) = 0$
 $\Leftrightarrow t_1 = 0$ $t_2 = \frac{5}{3}$
8. $t(t - 3) = 2 \Leftrightarrow t^2 - 3t - 2 = 0$
 $\Delta = 9 + 8 = 17$ $x_{1,2} = \frac{3 \pm \sqrt{17}}{2}$

EXERCICE 3

1. $S = 17$ $P = 30$ $x^2 - 17x + 30 = 0$ $\Delta = 169 > 0$ Oui
2. $S = \frac{24}{5}$ $P = -1$ $x^2 - \frac{24}{5}x - 1 = 0 \Leftrightarrow 5x^2 - 24x - 5 = 0$
 $\Delta = 676 > 0$ Oui
3. $S = 1$ $P = -56$ $x^2 - x + 56 = 0$ $\Delta = 1 - 4 \cdot 56 = -223 < 0$
 Non

$$4. S = \frac{4}{5} \quad P = \frac{2}{5} \quad x^2 - \frac{4}{5}x + \frac{2}{5} = 0 \Leftrightarrow 5x^2 - 4x + 2 = 0$$

$$\Delta = -24 < 0 \quad \text{NON}$$

$$5. S = 1 \quad P = -1 \quad x^2 - 1x + 1 = 0 \quad \Delta = 1 - 4 = -3 < 0 \quad \text{NON}$$

$$6. S = 2 \quad P = -48 \quad x^2 - 2x - 48 = 0 \quad \Delta = 196 > 0 \quad \text{OUI}$$

$$7. S = -3 \quad P = 5 \quad x^2 - 3x + 5 = 0 \quad \Delta = -11 \quad \text{NON}$$

$$8. S = 20 \quad P = 91 \quad x^2 - 20x + 91 = 0 \quad \Delta = 36 > 0 \quad \text{OUI}$$

EXERCICE 4

$$7x^2 - 3x + K = 0$$

$$P = \frac{c}{a} = \frac{K}{7} = -4 \Leftrightarrow K = -28$$

Donc l'équation devient: $7x^2 - 3x - 28 = 0$.

$$\Delta = 793 > 0 \Rightarrow \text{les solutions existent}$$

$$S = -\frac{b}{a} = \frac{3}{7}$$

EXERCICE 5

$$a) Kx^2 - (5-K)x + 11 = 0 \quad \text{1er degré} \Rightarrow K = 0$$

$$-5x + 11 = 0$$

$$b) 7x^2 - 3x + K = 0 \quad \Delta < 0 \Leftrightarrow 9 - 28K < 0 \Leftrightarrow$$

$$9 < 28K \Leftrightarrow \frac{9}{28} < K$$

$$c) (1-K)x^2 - (K+5)x + 2 = 0 \quad \Delta = 0$$

$$\Delta = 0 \Leftrightarrow (K+5)^2 - 8(1-K) = 0 \Leftrightarrow K^2 + 10K + 25 + 8K - 8 = 0$$

$$\Leftrightarrow K^2 + 18K + 17 = 0 \quad \Delta_1 = 256 = 16^2$$

$$K_{1,2} = \frac{-18 \pm 16}{2} = \begin{cases} K_1 = -17 \\ K_2 = -1 \end{cases}$$

$$d) (1-K)x^2 - (K+5)x + 2 = 0 \quad \text{degré zéro:}$$

$$1-K=0 \Leftrightarrow K=1 \quad \text{et} \quad K+5=0 \Leftrightarrow K=-5$$

Impossible

$$e) (2+3K)x^2 - 3x + 10 = 0 \quad \text{3 et une solution.}$$

$$(2+3K)9 - 3 \cdot 3 + 10 = 0 \Leftrightarrow 18 + 27K - 9 + 10 = 0 \Leftrightarrow$$

$$19 + 27K = 0 \Leftrightarrow 27K = -19 \Leftrightarrow K = -\frac{19}{27}$$

f) $(2+3k)x^2 - 3x + 10 = 0$ 0 est une solution.
 $10 = 0$ impossible

g) $(2+3k)x^2 - 3x + 7 - 3k = 0$ 0 est une solution.
 $7 - 3k = 0 \Leftrightarrow k = \frac{7}{3}$

EXERCICE 6

1. $\frac{3x^2 + 5x + 2}{x^2 - 1} = \frac{3(x+1)(x+\frac{2}{3})}{(x-1)(x+1)} = \frac{3x+2}{x-1}$

$\Delta = 25 - 24 = 1$ $x_{1,2} = \frac{-5 \pm 1}{6} = \begin{cases} x_1 = -1 \\ x_2 = -\frac{4}{6} = -\frac{2}{3} \end{cases}$

2. $\frac{3x^2 + 5x + 2}{x^2 + 2x + 1} = \frac{3(x+1)(x+\frac{2}{3})}{(x+1)^2} = \frac{3x+2}{x+1}$

3. $\frac{x^2 - 4x + 3}{x^2 - 6x + 5} = \frac{(x-1)(x-3)}{(x-5)(x-1)} = \frac{x-3}{x-5}$

EXERCICE 7.

$kx^2 + 2(k+1)x + 1 = 0$

1. $S = \frac{-2(k+1)}{k} = -2 \Leftrightarrow k+1 = k \Rightarrow 1 = 0$ imp

2. $k \cdot 0 + 2(k+1) \cdot 0 + 1 = 0$ imp

3. racines opposées : $S = 0 \Rightarrow \frac{-2(k+1)}{k} = 0 \Leftrightarrow k = -1$

4. $x_1 = x_2 \Rightarrow \Delta = 0 \Leftrightarrow 4(k+1)^2 - 4k = 0 \Leftrightarrow$

$\Leftrightarrow (k+1)^2 - k = 0 \Leftrightarrow k^2 + 2k + 1 - k = 0 \Leftrightarrow$

$\Leftrightarrow k^2 + k + 1 = 0$ $\Delta = 1 - 4 = -3 < 0$ imp