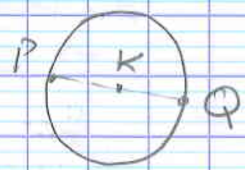
LJP : TE8 GEOMETRIE PLANE

EXERCICE 1 $r = 3\sqrt{7}$ $K(-5;2)$ $(x+5)^2+(y-2)^2=(3\sqrt{7})^2=63$

EXERCICE 2

- $x^2+y^2=144$ oui $K(0,0)$ $r=12$
- $x^2-y^2=225$ non
- $x^2+y^2-8x+18y+16=0 \Rightarrow (x-4)^2+(y+9)^2-16-81+16=0$
  $\Rightarrow (x-4)^2+(y+9)^2=81$ $K(4;-9)$ $r=9$
- $x^2+y^2-2x=111 \Rightarrow (x-1)^2-1+y^2=111 \Rightarrow$
  $(x-1)^2+y^2=112$ $K(1;0)$ $r=\sqrt{112}$
- $-2x^2-2y^2+14x-5y=0 \Rightarrow$
  $x^2+y^2-7x+\frac{5}{2}y=0 \Rightarrow$
  $\Rightarrow (x-\frac{7}{2})^2-\frac{49}{4}+(y+\frac{5}{4})^2-\frac{25}{16}=0 \Rightarrow$
  $\Rightarrow (x-\frac{7}{2})^2+(y+\frac{5}{4})^2=\frac{221}{16}$
  $K(\frac{7}{2};-\frac{5}{4})$ $r=\frac{\sqrt{221}}{4}$
- $x^2+3y^2=9$ Non
- $-x^2-16x+7y+y^2+11=0$ Non

EXERCICE 3 $P(7;-3)$ $Q(2;1)$



$K\left(\frac{7+2}{2};\frac{-3+1}{2}\right)=\left(\frac{9}{2};-1\right)$

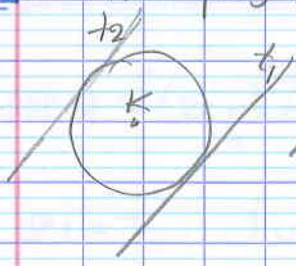
$r=\frac{\|\vec{PQ}\|}{2}=\frac{\sqrt{25+16}}{2}\Rightarrow$ $\vec{PQ}=\begin{pmatrix}-5\\4\end{pmatrix}$

$r=\frac{\sqrt{41}}{2}$

$(x-\frac{9}{2})^2+(y+1)^2=\frac{41}{4}$

EXERCICE 4

$\vec{w} = \begin{pmatrix}3\\-5\end{pmatrix}$ $\quad C: x^2+y^2-2x-24=0$



$(x-1)^2-1+y^2-24=0$

$(x-1)^2+y^2=25$

$K(1;0) \quad r=5$

$\vec{t} \parallel \vec{w} = \begin{pmatrix}3\\-5\end{pmatrix} \Rightarrow \vec{n}_t = \begin{pmatrix}5\\3\end{pmatrix}$

$t: 5x+3y+c=0$

t est tangente $dist(K,t) = r = 5 \Rightarrow$

$\Rightarrow \dfrac{|5+c|}{\sqrt{25+9}} = 5 \Rightarrow |c+5| = 5\sqrt{34} \Rightarrow$

$c+5 = 5\sqrt{34} \Rightarrow c_1 = -5+5\sqrt{34}$

$c+5 = -5\sqrt{34} \Rightarrow c_2 = -5-5\sqrt{34}$

$t_1: 5x+3y-5+5\sqrt{34}=0$

$t_2: 5x+3y-5-5\sqrt{34}=0$

EXERCICE 5

a) $C_1: x^2+y^2=14 \quad C_2: x^2+y^2-8y+11x=14$

$x^2+y^2 = x^2+y^2-8y+11x \Rightarrow$

$\Rightarrow 11x-8y=0$ axe radical

b) C_1 $3x-x^2+7y-y^2=0 \Leftrightarrow x^2+y^2-3x-7y=0$

$C_2: 5x^2+5y^2-3x+10y=20 \Leftrightarrow$

$\Rightarrow x^2+y^2-\frac{3}{5}x+2y-4=0$

$\cancel{x^2}+\cancel{y^2}-3x-7y = \cancel{x^2}+\cancel{y^2}-\frac{3}{5}x+2y-4$

$\Rightarrow -\frac{11}{5}x - 9y = -4 \Rightarrow -11x-45y=-20$

$11x+45y=20$ axe radical

EXERCICE 6

$d: x+y=2$

$C_1: (x-5)^2+(y+11)^2=100 \quad C_2: x^2+y^2=2$

$C_3: (x-7)^2+(y+8)^2=1$

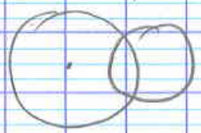
$C_1, C_2 \quad K_1(5;-11) \quad r_1=10 \quad K_2(0;0) \quad r_2=\sqrt{2}$

$\vec{K_1K_2} = \begin{pmatrix}-5\\11\end{pmatrix} \quad \|\vec{K_1K_2}\| = \sqrt{25+121} = \sqrt{146} \approx 12.08$

$r_1+r_2 \approx 11,41 < K_1K_2 \Rightarrow$ disjoints

C_1, C_3 $\quad K_1(5;-11)$ $\quad r_1=10$ $\quad K_3(7;-8)$ $\quad r_3=1$

$\vec{K_1K_3}=\begin{pmatrix}2\\3\end{pmatrix}$ $\quad \|\vec{K_1K_3}\|=\sqrt{4+9}=\sqrt{13}\approx 3.6 < r_1+r_3=11$



sécants

C_3, C_2 $\quad K_3(7;-8)$ $\quad r_3=1$ $\quad K_2(0,0)$ $\quad r_2=\sqrt{2}$

$\vec{K_1K_3}=\begin{pmatrix}7\\-8\end{pmatrix}$ $\quad \|\vec{K_1K_3}\|=\sqrt{49+64}=\sqrt{113}\approx 10.63 > r_1+r_3$

d, C_2 $\quad K_2(0,0)$ $\quad r_2=\sqrt{2}$ $\quad d: x+y-2=0$

$\text{dist}(K_2, d)=\dfrac{|0+0-2|}{\sqrt{1+1}}=\dfrac{2}{\sqrt{2}}=\sqrt{2}=r$

la droite est tangente

Bonus $\quad A(1,4)$ $\quad B(3;-2)$ $\quad K\in d: 3x+4y+16=0$

Soit $K(x_0, y_0)\in d$: $\quad 3x_0+4y_0+16=0 \Rightarrow$

$\Rightarrow 4y_0=-3x_0-16 \Rightarrow y_0=-\frac{3}{4}x_0-4$

$K\left(x_0; -\frac{3}{4}x_0-4\right)$

$\text{dist}(K,A)=\text{dist}(B,K)$

$\vec{AK}=\begin{pmatrix}x_0-1\\-\frac{3}{4}x_0-4-4\end{pmatrix}=\begin{pmatrix}x_0-1\\-\frac{3}{4}x_0\end{pmatrix}$ $\quad \vec{BK}=\begin{pmatrix}x_0-3\\-\frac{3}{4}x_0-4+2\end{pmatrix}=\begin{pmatrix}x_0-3\\-\frac{3}{4}x_0-2\end{pmatrix}$

$\|\vec{AK}\|=\|\vec{BK}\| \Rightarrow \sqrt{(x_0-1)^2+\frac{9}{16}x_0^2}=\sqrt{(x_0-3)^2+\left(-\frac{3}{4}x_0-2\right)^2} \Rightarrow$

$\Rightarrow \cancel{x_0^2}-2x_0+1+\cancel{\frac{9}{16}x_0^2}=\cancel{x_0^2}-6x_0+9+\cancel{\frac{9}{16}x_0^2}+3x_0+4$

$\Leftrightarrow 4x_0+1=3x_0+13 \Rightarrow x_0=12$

Alors $\quad y_0=-\frac{3}{\cancel{4}}\cdot \cancel{12}^{3}-4 \Rightarrow y_0=-13$ $\quad K(12;-13)$

$\|\vec{AK}\|=\left\|\begin{pmatrix}11\\-17\end{pmatrix}\right\|=\sqrt{11^2+17^2}=\sqrt{410}=r$

$(x-12)^2+(y+13)^2=410$