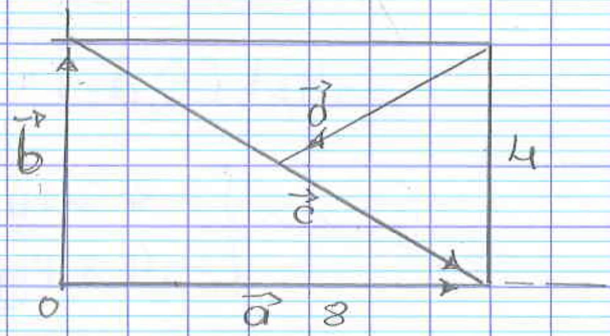
LIP : TE 6 GEOMETRIE PLANE

EXERCICE 1



$\vec{a} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

$\vec{c} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$ $\vec{d} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$

$\vec{a} \cdot \vec{b} = 0$

$\vec{a} \cdot \vec{d} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \begin{pmatrix} -4 \\ -2 \end{pmatrix} = -32$

$\vec{b} \cdot \vec{c} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \begin{pmatrix} 8 \\ -4 \end{pmatrix} = -16$

$\vec{c} \cdot \vec{d} = \begin{pmatrix} 8 \\ -4 \end{pmatrix} \begin{pmatrix} -4 \\ -2 \end{pmatrix} = -32 + 8 = -24$

EXERCICE 2

$\vec{a} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 2 \\ x \end{pmatrix}$

1) $\vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0 \Rightarrow -6 + 4x = 0 \Rightarrow x = \frac{6}{4} = \frac{3}{2}$

2) $\|\vec{a}\| = \sqrt{9+16} = 5$ $\vec{u}_a = \begin{pmatrix} -3/5 \\ 4/5 \end{pmatrix}$

3) $\vec{c} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\vec{d} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ $\vec{c} \cdot \vec{d} = -3 + 8 = 5$ $\|\vec{d}\| = \sqrt{25} = 5$

$\vec{c} \cdot \vec{d} = \|\vec{d}\| \cdot c' \Rightarrow 5 = 5 \cdot c' \Rightarrow \underline{c' = 1}$

EXERCICE 3

$d_1 : -4x + y - 7 = 0$

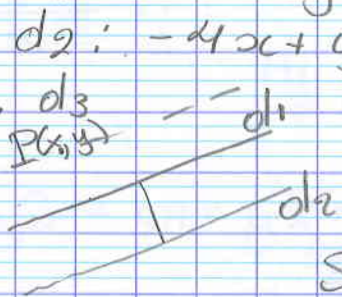
1. $\vec{n} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} \Rightarrow \vec{d} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \Rightarrow m = \frac{4}{1} = 4$

2. $d_2 \parallel d_1$ $P(-1;1)$

$d_2 : -4x + y + c = 0$ $P: 4 + 1 + c = 0 \Rightarrow c = -5$

$d_2 : -4x + y - 5 = 0$

3. d_3



On cherche la distance entre d_1 et d_2:

Soit $A \in d_1$ $x = 0$ $y = 7$ $A(0;7)$

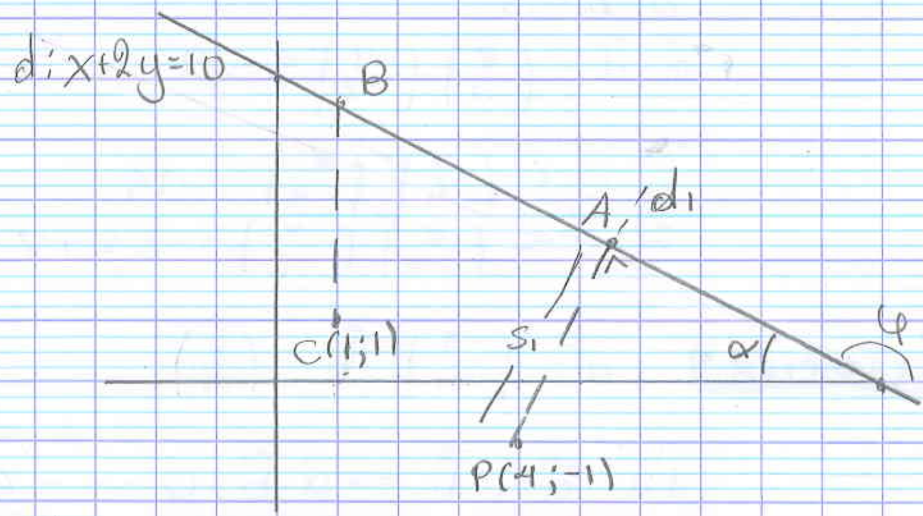
$\text{dist}(d_1, d_2) = \text{dist}(A, d_2) = \frac{|7-5|}{\sqrt{4^2+1}} = \frac{2}{\sqrt{17}}$

Soit $P(x,y)$ dont la distance à d_1 est $\frac{2}{\sqrt{17}}$

$$dist(P,d_1) = \frac{2}{\sqrt{17}} \Rightarrow \frac{|-4x+y-7|}{\sqrt{17}} = \frac{2}{\sqrt{17}} \Rightarrow$$

$$\Rightarrow \begin{cases} -4x+y-7=2 \Rightarrow -4x+y-9=0 \ (d_3) \\ -4x+y-7=-2 \Rightarrow -4x+y-5=0 \ (d_2) \end{cases}$$

EXERCICE 4



1) $S_1 = dist(P,d) = \frac{|4-2-10|}{\sqrt{1+4}} = \frac{|-8|}{\sqrt{5}} = \frac{8}{\sqrt{5}}$

On trouve la droite $d_1 \perp d$ passant par P.

$\vec{n}_d = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \Rightarrow \vec{n}_{d_1} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $2x-y+c=0$

$P: 8+1+c=0 \Rightarrow c=-9$

$d_1: 2x-y-9=0$

$A = d \cap d_1$ $\begin{cases} 2x-y=9 \\ x+2y=10 \end{cases}$ $*2$ $\begin{cases} 4x-2y=18 \\ x+2y=10 \end{cases}$ $+$

$5x = 28 \Rightarrow x = \frac{28}{5}$

$2 \cdot \frac{28}{5} - y = 9 \Rightarrow$

$y = \frac{11}{5}$ $A\left(\frac{28}{5}; \frac{11}{5}\right)$

$B: x=1$ $B \in d: 1+2y=10 \Rightarrow 2y=9 \Rightarrow y=\frac{9}{2}$

$B\left(1; \frac{9}{2}\right)$ $\vec{AB} = \begin{pmatrix} -\frac{23}{5} \\ \frac{23}{10} \end{pmatrix}$

$\|\vec{AB}\| = \sqrt{\left(\frac{23}{5}\right)^2 + \left(\frac{23}{10}\right)^2} = 5{,}14$

$\vec{BC} = \begin{pmatrix} 0 \\ -\frac{7}{2} \end{pmatrix} \Rightarrow \|\vec{BC}\| = \frac{7}{2}$

Donc la longueur est $\frac{8}{\sqrt{5}} + 5,14 + \frac{7}{2} = 12,22$

2) la pente de d est $m = -\frac{1}{2} = \tan\varphi$

$\Rightarrow \varphi = 153.43 \Rightarrow \alpha = 180 - \varphi \Rightarrow \alpha = 26.56$

3) M milieu du CP $M\left(\frac{5}{2}, 0\right)$

$\vec{CP} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \perp$ médiatrice $\Rightarrow$

$3x - 2y + c = 0$ $M: \frac{15}{2} + c = 0 \Rightarrow c = -\frac{15}{2}$

$m: 3x - 2y - \frac{15}{2} = 0 \Rightarrow 6x - 4y - 15 = 0$