DERIVÉES

EXERCICE 1 1. $f(x) = (2x^5-3)\cdot\sqrt{4x}$

$f'(x) = 10x^4\cdot\sqrt{4x} + (2x^5-3)\frac{4}{2\sqrt{4x}} = 10x^4\cdot\sqrt{4x} + \frac{2(2x^5-3)}{\sqrt{4x}}$

2. $g(x) = \frac{x^3}{x^2-5x+6}$

$g'(x) = \frac{3x^2(x^2-5x+6) - x^3(2x-5)}{(x^2-5x+6)^2} =$

$= \frac{x^2(3x^2+15x+18-2x^2+5x)}{(x^2-5x+6)^2} =$

$= \frac{x^2(x^2-10x+18)}{(x^2-5x+6)^2}$

3. $h(x) = 7(x^2-3)^4 \quad h'(x) = 4\cdot 7(x^2-3)^3\cdot 2x = 56x(x^2-3)^3$

4. $i(x) = \frac{1}{7}x^3 + x^{1/3} + x^{-2} - 3$

$i'(x) = \frac{3}{7}x^2 + \frac{1}{3}x^{-2/3} - 2x^{-3} =$

$= \frac{3}{7}x^2 + \frac{1}{3\sqrt[3]{x^2}} - \frac{2}{x^3}$

EXERCICE 2 a) AV: $x=3$; $x=-1$ AH: $y=2$.

$f(x) = \frac{2x^2+1}{(x-3)(x+1)}$

b) $f(x) = \frac{x^2(2x-1)}{2x^2-1}$ $\quad 2x^2-1=0 \Leftrightarrow x^2=\frac{1}{2} \Leftrightarrow x = \pm\frac{\sqrt{2}}{2}$

AV · $\lim_{x\to\frac{\sqrt{2}}{2}} f(x) = \left[\frac{\frac{\sqrt{2}}{2}-\frac{1}{2}}{0}\right] = \infty \Rightarrow$ AV: $x = \frac{\sqrt{2}}{2}$

· $\lim_{x\to-\frac{\sqrt{2}}{2}} f(x) = \left[\frac{-\frac{\sqrt{2}}{2}-\frac{1}{2}}{0}\right] = \infty \Rightarrow$ AV: $x = -\frac{\sqrt{2}}{2}$

$f(x) = \frac{2x^3-x^2}{2x^2-1}$

AO $\quad 2x^3-x^2 \;\big|\; 2x^2-1$

$-2x^3+x \;\big|\; x-\frac{1}{2}$

$-x^2+x$

$+x^2-\frac{1}{2}$

$x-\frac{1}{2}$

AO: $y = x-\frac{1}{2}$

$$-\lim_{x \to \frac{1}{\sqrt{2}}^-} f(x) = \left[\frac{\frac{\sqrt{2}-1}{2}}{0^+}\right] = -\infty.$$

x	$-\infty$		$-\frac{1}{2}$		$\frac{1}{\sqrt{2}}$		$+\infty$
x^2-1		$+$	0	$-$	0	$+$	

$$\lim_{x \to 1} f(x) = 1$$

$$\lim_{x \to +\infty} f'(x) = 1 \text{ (la pente de l'asymptote)}$$

EXERCICE 3 a) $f(x) = x^3 + x^2 + x \qquad f(3) = 39$

$f'(x) = 3x^2 + 2x + 1 \qquad f'(3) = 34$

$y = mx + h \rightarrow 39 = 34 \cdot 3 + h \Rightarrow h = -63$

$y = 34x - 63$

b) $f(x) = x^3 + 9x^2 - 21x + 17 \qquad D_f = \mathbb{R}$

$f'(x) = 3x^2 + 18x - 21$

$f''(x) = 6x + 18$

PTH : $f'(x) = 0 \Leftrightarrow 3x^2 + 18x - 21 = 0 \Leftrightarrow$

$\Leftrightarrow x^2 + 6x - 7 = 0 \Leftrightarrow x_1 = -7 \quad x_2 = 1$

$f(-7) = 262 \qquad f(1) = 6$

$(-7; 262) \qquad (1; 6)$

PI $f''(x) = 0 \Leftrightarrow 6x + 18 = 0 \Leftrightarrow x + 3 = 0 \Leftrightarrow x = -3$

$f(-3) = 134 \qquad (-3; 134)$

EXERCICE 4 a. $x = 60$

spectateurs $= 60000 - 170 \cdot 60 = 49800$

revenu $= 49800 \cdot 60 = 2988000$

coûts $= 50000 + 120000 + 6 \cdot 49800 = 468800$

bénéfice $= 2988000 - 468800 = 2519200$

b. $b(x) = (60000 - 170 \cdot x) \cdot x - 50000 - 120000 - 6 \cdot (60000 - 170x)$

$= 60000x - 170x^2 - 50000 - 120000 - 360000 + 1020x$

$= -170x^2 + 61020x - 410000$

$b'(x) = -340x + 61020 = 0 \Leftrightarrow x = \frac{61020}{340} \rightarrow$

$\rightarrow x = 179{,}5$ CHF.

EXERCICES

1. TS

x	$-\infty$		-3		0		1		$+\infty$
f'		$-$	0	$-$	0	$+$	‖	$-$	
f		↘		↘		↗	‖	↘	

TGr

x	$-\infty$		-3		$-0,7$		1		2		$+\infty$
f'		$+$	0	$-$	0	$+$	‖	$+$	0	$-$	
f		↗	Max $(-3;0)$	↘	Min $(-0,7;-1)$	↗	‖	↗	Max $(2;-1,6)$	↘	

TConv

x	$-\infty$		$-1,4$		1		$+\infty$
f''		$-$	0	$+$	‖	$-$	
f		∩	PI $(-1,4;-0,8)$	∪	‖	∩	

2. $\lim\limits_{x\to -3} f'(x) = 0$ $\lim\limits_{x\to 1^+} f(x) = -\infty$ $\lim\limits_{x\to -1,4} f(x) = -0,8$

Bonus

$f(x) = (3x-1)^3$ $D_f = \mathbb{R}$

$f'(x) = 3(3x-1)^2 \cdot 3 = 9(3x-1)^2$

$f''(x) = 18(3x-1)\cdot 3 = 54(3x-1)$

$f''(x) = 0 \Leftrightarrow 3x-1 = 0 \Leftrightarrow x = 1/3$

x	$-\infty$		$1/3$		$+\infty$
f''		$-$	0	$+$	
f		∩	PI $(\frac{1}{3};0)$	∪	