LJP: TE: 16 GEOMETRIE PLANE

EXERCICE 1 $x^2+y^2-4x+20y=0 \Rightarrow$

$\Rightarrow (x-2)^2-4+(y+10)^2-100=0 \Rightarrow$

$(x-2)^2+(y+10)^2=104 \quad K(2;-10) \quad r=\sqrt{104}$

$d: y=\frac{1}{2}x \Rightarrow x=2y$

$d \cap C: (2y-2)^2+(y+10)^2=104 \Leftrightarrow$

$\Leftrightarrow 4y^2-8y+\cancel{4}+y^2+20y+1\cancel{0}0-1\cancel{0}4=0 \Leftrightarrow$

$\Leftrightarrow 5y^2+12y=0 \Leftrightarrow y(5y+12)=0 \Leftrightarrow$

$\Leftrightarrow y=0 \quad y=-\frac{12}{5}$

$y=0 \quad x=0 \quad A(0,0) \qquad y=-\frac{12}{5} \quad x=-\frac{24}{5} \quad B\left(-\frac{24}{5};-\frac{12}{5}\right)$

EXERCICE 2 a) $C_1\ x^2+y^2=14 \qquad C_2: x^2+y^2-8y+12x=14$

$\cancel{x^2}+\cancel{y^2}=\cancel{x^2}+\cancel{y^2}-8y+12x \Leftrightarrow$

$\Leftrightarrow \quad 12x=8y \Rightarrow 3x-2y=0$ axe radical

b) $C_1: 3x-x^2+7y-y^2=0 \Leftrightarrow$

$x^2+y^2-3x-7y=0$

$C_2: x^2+y^2-\frac{3}{5}x+2y=4$

$\cancel{x^2}+\cancel{y^2}-3x-7y=\cancel{x^2}+\cancel{y^2}-\frac{3}{5}x+2y-4 \Rightarrow$

$\Leftrightarrow -15x-35y=-3x+10y-20 \Leftrightarrow$

$\Leftrightarrow -12x-45y+20=0 \Leftrightarrow$

$\Leftrightarrow 12x+45y-20=0$ axe radical

Bonus axe radical $3x-2y=0 \Rightarrow \vec{d}=\begin{pmatrix}2\\3\end{pmatrix}$

$C_1: x^2+y^2=14 \quad K_1(0,0) \quad r_1=\sqrt{14}$

$C_2: x^2+y^2-8y+12x-14=0 \Leftrightarrow$

$(x+6)^2-36+(y-4)^2-16-14=0$

$(x+6)^2+(y-4)^2=66 \quad K_2(-6;4) \quad r_2=\sqrt{66}$

$\overrightarrow{K_1K_2}=\begin{pmatrix}-6\\4\end{pmatrix} \parallel \begin{pmatrix}-3\\2\end{pmatrix}$

$\vec{d}\cdot\overrightarrow{K_1K_2}=2\cdot(-3)+3\cdot2=0 \Rightarrow \overrightarrow{K_1K_2}\perp\vec{d}$

EXERCICE 4

$x^2+y^2-2x-24=0 \Leftrightarrow (x-1)^2-1+y^2-24=0 \Leftrightarrow$

$(x-1)^2+y^2=25 \quad K(1;0) \quad r=5$

$\vec{w}\begin{pmatrix}3\\-5\end{pmatrix} \quad t \parallel \vec{w} \Rightarrow \vec{n}_t=\begin{pmatrix}5\\3\end{pmatrix}$

$t:\ 5x+3y+c=0$

$\text{dist}(K,t)=r=5 \Rightarrow \dfrac{|5+c|}{\sqrt{25+9}}=5 \Rightarrow$

$\Rightarrow |c+5|=5\sqrt{34} \Rightarrow \begin{cases} c_1+5=5\sqrt{34} \Rightarrow c_1=5\sqrt{34}-5 \\ c_2+5=-5\sqrt{34} \Rightarrow c_2=-5\sqrt{34}-5 \end{cases}$

$t_1:\ 5x+3y = 5\sqrt{34}-5$

$t_2:\ 5x+3y=-5\sqrt{34}-5$

EXERCICE 5

$A(1;4) \quad B(3;-2) \quad K \in 3x+4y+16=0$

$K(x_0,y_0) \quad 3x_0+4y_0+16=0 \Leftrightarrow 4y_0=-3x_0-16$

$\Rightarrow y_0=-\frac{3}{4}x_0-4$

$\|\vec{AK}\|=\|\vec{BK}\|$

$\vec{AK}=\begin{pmatrix}x_0-1\\-\frac{3}{4}x_0-4-4\end{pmatrix}=\begin{pmatrix}x_0-1\\-\frac{3}{4}x_0-8\end{pmatrix}$

$\vec{BK}=\begin{pmatrix}x_0-3\\-\frac{3}{4}x_0-4+2\end{pmatrix}=\begin{pmatrix}x_0-3\\-\frac{3}{4}x_0-2\end{pmatrix}$

$\sqrt{(x_0-1)^2+\left(\frac{3}{4}x_0+8\right)^2}=\sqrt{(x_0-3)^2+\left(\frac{3}{4}x_0+2\right)^2} \Leftrightarrow$

$\cancel{x_0^2}-2x_0+1+\cancel{\frac{9}{16}x_0^2}+12x_0+64=\cancel{x_0^2}-6x_0+9+\cancel{\frac{9}{16}x_0^2}+3x_0+4$

$\Leftrightarrow 10x_0+65=-3x_0+13 \Leftrightarrow 13x_0=-52 \Rightarrow x_0=-4$

$y_0=3-4=-1 \quad K(-4;-1)$

$r=\|\vec{AK}\|=\sqrt{(-4-1)^2+(-3+8)^2}=\sqrt{25+25}=5\sqrt{2}$

$(x+4)^2+(y+1)^2=50$

EXERCICE 6 $K(1;0)$ $r = 3\sqrt{5}$

$d_1 : y = \frac{1}{2}x + 7$ $d_2 : y = -\frac{1}{2}x - 7$

$2y = x + 14 \Rightarrow$ $2y = -x - 14 \Rightarrow$

$d_1 : x - 2y + 14 = 0$ $d_2 : x + 2y + 14 = 0$

1. $dis(d_1, K) = \dfrac{|1 + 14|}{\sqrt{1 + 4}} = \dfrac{15}{\sqrt{5}} = \dfrac{15 \cdot \sqrt{5}}{5} = 3\sqrt{5} = r$

$dist(d_2, K) = \dfrac{|1 + 14|}{\sqrt{5}} = 3\sqrt{5} = r$

$\Rightarrow$ donc les deux droites sont tangentes au cercle

2. $\vec{d_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\vec{d_2} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\vec{d_1} \cdot \vec{d_2} = 4 - 1 = 3$ $\|\vec{d_1}\| = \sqrt{5} = \|\vec{d_2}\|$

$\cos \varphi = \dfrac{\vec{d_1} \cdot \vec{d_2}}{\|d_1\| \|d_2\|} = \dfrac{3}{\sqrt{5}^2} = \dfrac{3}{5} \Rightarrow \varphi = 53.13°$