L1P : GÉOMÉTRIE PLANE - SOLUTIONS

EXERCICE 1

$\vec{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ • $\vec{b} \perp \vec{a}$ $\vec{b} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

• $\vec{c} \perp \vec{a}$ $\vec{c} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ $\vec{b} \perp \vec{a} \Rightarrow \vec{b} = \begin{pmatrix} a_2 \\ -a_1 \end{pmatrix}$

$\vec{c} \perp \vec{a} \Rightarrow \vec{c} = \begin{pmatrix} -a_2 \\ a_1 \end{pmatrix}$

$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \Rightarrow \vec{n}_a = \begin{pmatrix} a_2 \\ -a_1 \end{pmatrix}$ $a_2 x - a_1 y + c = 0$

EXERCICE 2

$d_2 \perp d : -5x + 2y - 6 = 0$

$x = 4$ $-5 \cdot 4 + 2y = 6 \Rightarrow 2y = 26 \Rightarrow y = 13$

$(4;13)$ $\vec{n}_d = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \Rightarrow \vec{n}_2 = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$d_2 : 2x - 5y + c = 0$ $(4;13)$: $8 - 65 + c = 0 \Rightarrow$

$\Rightarrow c = 57$

$d_2 : 2x - 5y + 57 = 0$

EXERCICE 3

1) perpendiculaire

2) Deux axes qui forment angle droit et dont la unité est 1

3) $\vec{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ $\vec{a} \cdot \vec{b} = 20 - 3 = 17$

4) $\vec{a} = 4\vec{u}$ $\|\vec{u}\| = 1$ $\|\vec{a}\|^2 = 16\|\vec{u}\|^2 = 16$

5) $\vec{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $\vec{b} \perp \vec{a} \Rightarrow b = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\vec{c} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$\|b\| = \sqrt{9+16} = 5$ $\|\vec{c}\| = \sqrt{36+64} = 10$

6) $\vec{u}_a = \begin{pmatrix} 4/5 \\ -3/5 \end{pmatrix}$

7) $\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$

8) Le produit de $\vec{a}$ par le nombre $(\vec{a} \cdot \vec{b})$

Exercice 4 $\begin{pmatrix} k+3 \\ 4 \end{pmatrix} \perp \begin{pmatrix} k-5 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} k+3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} k-5 \\ 3 \end{pmatrix} = 0 \Rightarrow$

$\Rightarrow (k+3)(k-5)+12=0 \Rightarrow k^2+3k-5k-15+12=0$

$\Rightarrow k^2-2k-3=0 \qquad \Delta = 4+12=16$

$k_{1,2} = \frac{2 \pm 4}{2} \Rightarrow \begin{matrix} k_1 = 3 \\ k_2 = -1 \end{matrix}$

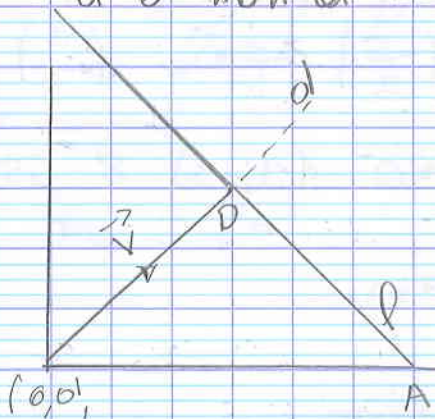
Exercice 5 $\vec{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -3 \\ 8 \end{pmatrix} \quad \|\vec{a}\| = \sqrt{4+25} = \sqrt{29} \quad \|\vec{b}\| = \sqrt{9+64} = \sqrt{73}$

a) $\vec{a} \cdot \vec{b} = -6+40 = 34$

b) $\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot b' \Rightarrow b' = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|} \Rightarrow b' = \frac{34}{\sqrt{29}}$

$\vec{a} \cdot \vec{b} = \|\vec{b}\| \cdot a' \Rightarrow a' = \frac{\vec{a} \cdot \vec{b}}{\|b\|} \Rightarrow a' = \frac{34}{\sqrt{73}}$

Exercice 6



On cherche la droite d

$\vec{d} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \vec{n}_d = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$d: x+y+c=0$

$(0,0) \quad x+y=0$

On cherche la droite l

$\vec{n}_l = \vec{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$l: x+y+c=0 \qquad A: \left(\frac{13}{2}; 0\right) \quad \frac{13}{2}+c=0 \Rightarrow c = -\frac{13}{2}$

$D = d \cap l \quad \begin{cases} x+y=0 \\ x+y-\frac{13}{2}=0 \end{cases} \quad (+) \quad 2x - \frac{13}{2} = 0 \Rightarrow$

$2x = \frac{13}{2} \Rightarrow x = \frac{13}{4}$

$x_D = y_D = \frac{13}{4} \qquad D\left(\frac{13}{4}; \frac{13}{4}\right)$

Exercice 7 $\vec{a} \cdot \vec{a} = \|\vec{a}\|^2 = 64$

$\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot b' = -8 \cdot 4 = -32$

$\vec{a} \cdot \vec{c} = 0$ car $\vec{a} \perp \vec{c}$

$\vec{a} \cdot \vec{d} = \|\vec{a}\| \cdot d' = 8 \cdot 4 = 32$

$\vec{b} \cdot \vec{b} = \|b\|^2 = \sqrt{8^2+10^2} = \sqrt{164}$

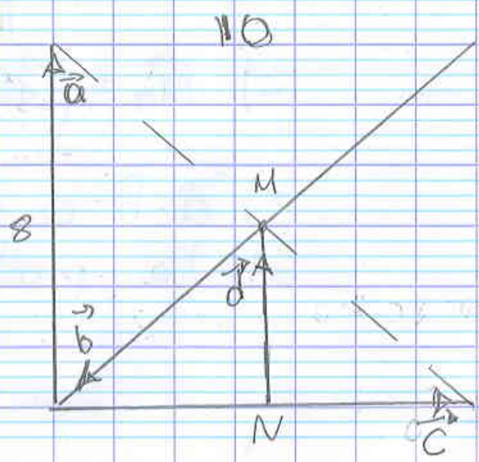
$\vec{b} \cdot \vec{c} = \|\vec{c}\| \cdot b' = +10 \cdot 5 = 50$

$\vec{b} \cdot \vec{d} = \|\vec{d}\| \cdot b' = -4 \cdot 4 = -16$

$\vec{c} \cdot \vec{c} = \|\vec{c}\|^2 = 10^2 = 100$

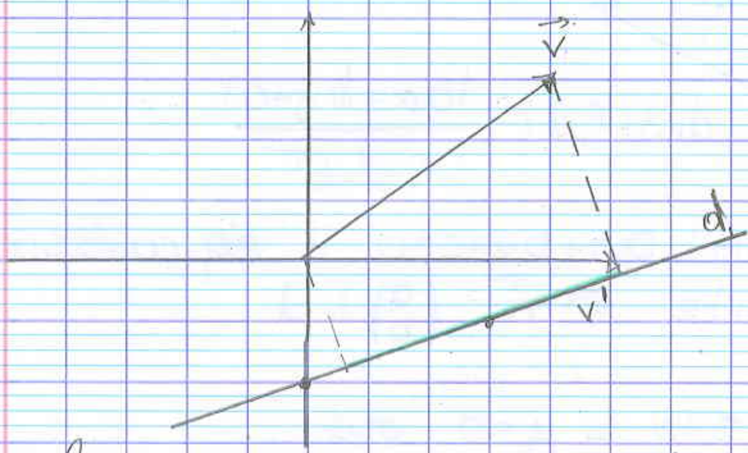
$\vec{c} \cdot \vec{d} = 0$

$\vec{d} \cdot \vec{d} = \|d\|^2 = 4^2 = 16$



Exercice 8 $\vec{v} = \begin{pmatrix}4\\3\end{pmatrix}$ $d: x-3y-6=0$ $\quad x=0 \;\; y=-2$

$x=3 \;\; y=-1$



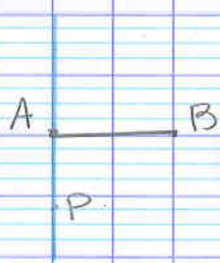
le vecteur directeur de la droite d est $\vec{d} = \begin{pmatrix}3\\1\end{pmatrix}$

$\vec{v}\cdot\vec{d} = 12+3 = 15$

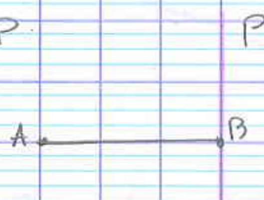
$\|\vec{d}\| = \sqrt{9+1} = \sqrt{10}$

$\vec{v}\cdot\vec{d} = \|\vec{d}\|\cdot v' \Rightarrow v' = \frac{15}{\sqrt{10}}$

Exercice 9

a) 

$\vec{AB}\cdot\vec{AP} = 0 \Rightarrow \vec{AB} \perp \vec{AP}$

b) 

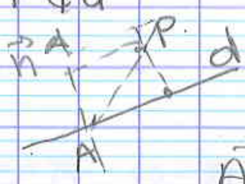
Exercice 10 $d: 4x+3y+13=0$

a) $\vec{n} = \begin{pmatrix}4\\3\end{pmatrix}$ $\quad \|\vec{n}\| = \sqrt{16+9} = 5.$

b) $A(2;-7)$ $\quad P(-1;2)$

1) $A \in d$ $\quad 4\cdot 2 - 21 + 13 = 0$ ✓

2) $P \notin d$ $\quad -4+6+13 = 15 \neq 0 \Rightarrow P \notin d$

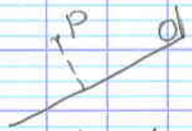
3) 

$\vec{AP} = \begin{pmatrix}-3\\9\end{pmatrix}$ $\quad \vec{AP}\cdot\vec{n} = -12+27 = 15$

$\vec{AP}\cdot\vec{n} = \|\vec{n}\|\cdot AP' \Rightarrow AP' = \frac{15}{5} = 3$

Alors $\mathrm{dist}(P, d) = 3$

c) $A(2;-7)$ $P(x,y)$

1) 

$$dist(P,d) = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$$

EXERCICE 11

a) Soit $ax+by+c=0$ l'éq. cartésienne de la droite $\Rightarrow \vec{n} = \begin{pmatrix} a \\ b \end{pmatrix} \perp d$

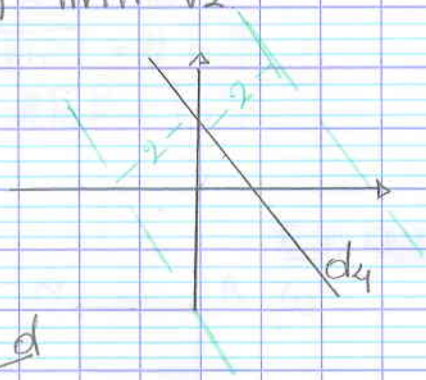
b) $x-3=0 \perp y=0$ oui

c)

d) $x+y-2=0$ $\vec{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\|\vec{n}\| = \sqrt{2}$

$\vec{n}_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ $\vec{n}_2 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

e) $d_4 : x+y=0$



f) $A \in d$ $T \notin d$

$$dist(T,d) = \frac{|\vec{n} \cdot \vec{d}|}{\|\vec{n}\|}$$

g) $$\delta(O,d) = \frac{|0.8 \cdot 0 - 0.6 \cdot 0 - 4|}{\sqrt{0.8^2 + 0.6^2}} = \frac{4}{1} = 4$$

h) tous les points $P(x,y)$ à la distance 3 pour la droite d = 2 droites parallèles à d

EXERCICE 12 $d : 5x - 12y - 25 = 0$

a) $\vec{n} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$ $A \in d$: $y=0$ $x=5$ $(5;0)$

b) $P(x,y)$ $\vec{AP} = \begin{pmatrix} x-5 \\ y \end{pmatrix}$ $\vec{AP} \cdot \vec{n} = 5(x-5) - 12y$

$$\frac{\vec{AP} \cdot \vec{n}}{\|\vec{n}\|} = \frac{5x - 12y - 25}{\sqrt{5^2 + 12^2}}$$

c) $\vec{AP}\cdot\vec{n}=0 \Rightarrow P\in d \Rightarrow AP\perp\vec{n}$

d) $\frac{\vec{AP}\cdot\vec{n}}{\|\vec{n}\|}=0 \Rightarrow P\in d$

EXERCICE 13 $h : y=\frac{1}{x}$ Soit $P(x,y)\in h$
donc $P(x;\frac{1}{x})$

$\delta(P,d)=\frac{6}{5} \Rightarrow \frac{|3x+4\cdot\frac{1}{x}-2|}{\sqrt{3^2+4^2}}=\frac{6}{5} \Leftrightarrow$

$\Leftrightarrow |3x+\frac{4}{x}-2|=6 \Rightarrow$
$\begin{cases} 3x+\frac{4}{x}-2=6 & (1) \\ 3x+\frac{4}{x}-2=-6 \end{cases}$

(1) $3x^2+4-2x=6x \Rightarrow 3x^2-8x+4=0$
$\Delta=64-48=16 \quad x_{1,2}=\frac{8\pm4}{6}= \begin{cases} 2=x_1 \\ \frac{2}{3}=x_2 \end{cases}$

(2) $3x^2+4-2x=-6x \Rightarrow 3x^2+4x+4=0$
$\Delta=16-48<0$ imp
Alors $A(2;\frac{1}{2})$ $B(\frac{2}{3};\frac{3}{2})$

EXERCICE 14 $d_1 : 4x-3y+6=0$ $d_2 : x-3=0$
Soit $P(x,y)\in$ bissectrice Alors
$\delta(P,d_1)=\delta(P,d_2) \Rightarrow$
$\frac{|4x-3y+6|}{\sqrt{16+9}}=\frac{|x-3|}{\sqrt{1}} \Rightarrow |4x-3y+6|=5|x-3|$
$\Rightarrow \begin{cases} 4x-3y+6=5x-15 \Rightarrow -x-3y+21=0 & b_1 \\ 4x-3y+6=-5x+15 \Rightarrow 9x-3y-9=0 \end{cases}$
$3x-y-3=0 \quad b_2$

Exercice 15 a) $d_1 : A(7;-2) \in d_1 \quad d_1 \perp \begin{pmatrix}3\\4\end{pmatrix} = \vec{n}_1$

$3x + 4y + c = 0 \quad A : 21 - 8 + c = 0 \Rightarrow c = -13$

$d_1 : 3x + 4y - 13 = 0 \qquad \dfrac{3x + 4y - 13}{\sqrt{9+16}} = 0$

b) $d_2 : B(0;-5) \in d_2 \quad d_2 \perp d : 3x - 6y - 1 = 0$

$\vec{n} = \begin{pmatrix}3\\-6\end{pmatrix} \parallel \begin{pmatrix}1\\-2\end{pmatrix} \Rightarrow \vec{n}_2 = \begin{pmatrix}2\\1\end{pmatrix}$

$d_2 : 2x + y + c = 0 \quad B : -5 + c = 0 \Rightarrow c = 5.$

$d_2 : 2x + y + 5 = 0 \qquad \dfrac{2x + y + 5}{\sqrt{4+1}} = 0$

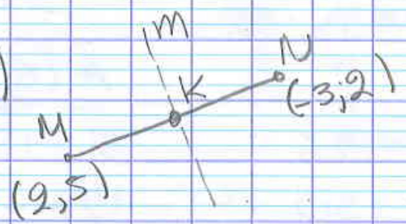
c) $d_3 : y = \frac{2}{3}x - 1 \Rightarrow 3y = 2x - 3 \Rightarrow 2x - 3y - 3 = 0$

$\dfrac{2x - 3y - 3}{\sqrt{4+9}} = 0$

d) $Ox : \ y = 0$

e) $Oy : \ x = 0$

f)



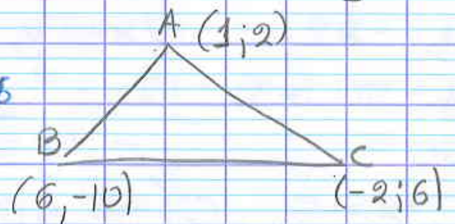
$K\left(\frac{2-3}{2} ; \frac{5+2}{2}\right) = \left(-\frac{1}{2} ; \frac{7}{2}\right)$

$\vec{n}_m = \overrightarrow{MN} = \begin{pmatrix}-5\\-3\end{pmatrix} \parallel \begin{pmatrix}5\\3\end{pmatrix}$

$5x + 3y + c = 0 \quad K : -\frac{5}{2} + \frac{21}{2} + c = 0 \Rightarrow c = -8$

$d_6 : 5x + 3y - 8 = 0$

Exercice 16



a) $\overrightarrow{AB} = \begin{pmatrix}5\\-12\end{pmatrix} \Rightarrow \vec{n}_{AB} = \begin{pmatrix}12\\5\end{pmatrix}$

$12x + 5y + c = 0$

$A : 12 + 10 + c = 0 \Rightarrow c = -22.$

$d_{AB} : 12x + 5y - 22 = 0$

- $\overrightarrow{AC} = \begin{pmatrix}-3\\4\end{pmatrix} \Rightarrow \vec{n}_{AC} = \begin{pmatrix}4\\3\end{pmatrix} \quad 4x + 3y + c = 0$

$A : 4 + 6 + c = 0 \Rightarrow c = -10$

$d_{AC} : 4x + 3y - 10 = 0$

$d_{AB}: \; 12x+5y-22=0$
$d_{AC}: \; 4x+3y-10=0$
$P(x,y) \quad \delta(P, d_{AB}) = 8 \Rightarrow \dfrac{|12x+5y-22|}{\sqrt{144+25}} = 8 \Rightarrow$

$\Rightarrow |12x+5y-22| = 104 \Rightarrow$

$\Rightarrow \begin{cases} 12x+5y-22 = 104 \Rightarrow 12x+5y = 126 \quad d_1 \\ 12x+5y-22 = -104 \Rightarrow 12x+5y = -82 \quad d_2 \end{cases}$

$\delta(P, AC) = 8 \Rightarrow \dfrac{|4x+3y-10|}{\sqrt{16+9}} = 8 \Rightarrow$

$\Rightarrow |4x+3y-10| = 40 \Rightarrow \begin{cases} 4x+3y-10 = 40 \\ 4x+3y-10 = -40 \end{cases} \Rightarrow$

$\begin{cases} 4x+3y = 50 \quad d_3 \\ 4x+3y = -30 \quad d_4 \end{cases}$

$d_1 \cap d_3: \begin{cases} 12x+5y = 126 \\ 4x+3y = 50 \; \cdot(-3) \end{cases} \quad \begin{array}{r} 12x+5y = 126 \\ -12x-9y = -150 \\ \hline -4y = -24 \Rightarrow y = 6 \end{array}$

$y = 6 \quad 4x+18 = 50 \Rightarrow 4x = 32$
$\Rightarrow x = 8$

$A(8; 6)$

$d_1 \cap d_4 \begin{cases} 12x+5y = 126 \\ 4x+3y = -30 \; \cdot(-3) \end{cases} \quad \begin{array}{r} 12x+5y = 126 \\ -12x-9y = 90 \\ \hline -4y = 216 \\ y = -54 \end{array}$

$y = -54 \quad 4x - 162 = -30 \Rightarrow$
$\Rightarrow 4x = 132 \Rightarrow x = 33$

$B(33; -54)$

$d_2 \cap d_3 \begin{cases} 12x+5y = -82 \\ 4x+3y = 50 \; (-3) \end{cases} \quad \begin{array}{r} 12x+5y = -82 \\ -12x-9y = -150 \\ \hline -4y = -232 \Rightarrow y = 58 \end{array}$

$y = 58 \quad 4x + 174 = 50 \Rightarrow$
$4x = -24 \Rightarrow x = -6 \quad C(-6, 58)$

$d_2 \cap d_4 \begin{cases} 12x+5y = -82 \\ 4x+3y = -30 \; \cdot(-3) \end{cases} \begin{cases} 12x+5y = -82 \\ -12x-9y = 90 \end{cases}$
$\qquad\qquad -4y = 8 \Rightarrow$
$\qquad\qquad y = -2$

$y = -2 \quad 4x - 6 = -30 \Rightarrow 4x = -24$
$x = -6$

$D(-6; -2)$

Exercice 17 $d_1 : 2x-5y-3=0 \quad \vec{n}_1 = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

a) $d_2 : 4x-10y-13=0 \quad \vec{n}_2 = \begin{pmatrix} 4 \\ -10 \end{pmatrix} \parallel \begin{pmatrix} 2 \\ 5 \end{pmatrix} \Rightarrow d_1 \parallel d_2$

b)

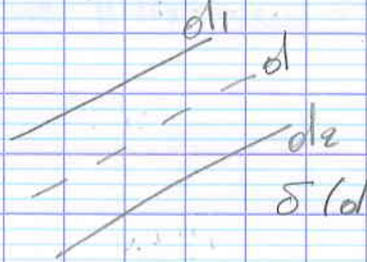
On trouve un point $A \in d_1$

$x=-1 \quad -2-5y-3=0 \Leftrightarrow 5y=-5 \Rightarrow y=-1$

$A(-1;-1) \in d_1$

$\delta(A, d_2) = \delta(d_1, d_2) = \dfrac{|-4+10-13|}{\sqrt{16+100}} = \dfrac{|-7|}{\sqrt{116}} \approx 0.65$

c)



$\delta(d_1, d) = \delta(d_2, d) = \dfrac{7}{2\sqrt{116}}$

$\delta(d, d_1) = \dfrac{7}{2\sqrt{116}} \Rightarrow \dfrac{|2x-5y-3|}{\sqrt{29}} = \dfrac{7}{2\sqrt{116}}$

$\Rightarrow |2x-5y-3| = \dfrac{7\sqrt{29}}{2\sqrt{116}} \Rightarrow |2x-5y-3| = \dfrac{7}{4} \Rightarrow$

$\begin{cases} 2x-5y-3 = \frac{7}{4} \Rightarrow 8x-20y-12=7 \Rightarrow 8x-20y-19=0 \\ 2x-5y-3 = -\frac{7}{4} \Rightarrow 8x-20y-12=-7 \Rightarrow 8x-20y-5=0 \end{cases}$

$\delta(d, d_2) = \dfrac{7}{2\sqrt{116}} \Rightarrow \dfrac{|4x-10y-13|}{\sqrt{116}} = \dfrac{7}{2\sqrt{116}} \Rightarrow$

$\begin{cases} 4x-10y-13 = \frac{7}{2} \Rightarrow 8x-20y-33=0 \\ 4x-10y-13 = -\frac{7}{2} \Rightarrow 8x-20y-19=0 \end{cases}$

La droite commune est la droite demandée

Alors $d : 8x-20y-19=0$

Exercice 18 $d_1 : 2x-y+5=0 \quad d_2 : 4x-3y+10=0$

$\Rightarrow y = 2x+5$

Soit $A \in d_1 \quad A(x; 2x+5)$

$\delta(A, d_2) = 3 \Rightarrow \dfrac{|4x-3(2x+5)+10|}{\sqrt{16+9}} = 3 \Rightarrow$

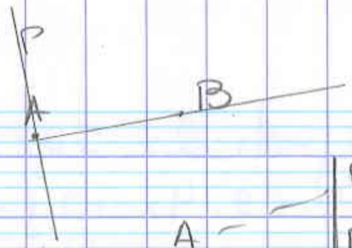
$\Rightarrow |4x-6x-15+10| = 15 \Rightarrow |-2x-5| = 15 \Rightarrow$

$\begin{cases} -2x-5=15 \Rightarrow 2x=-20 \Rightarrow x_1=-10 \\ -2x-5=-15 \Rightarrow -2x=-10 \Rightarrow x_2=5 \end{cases}$

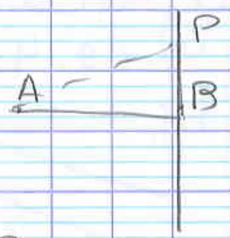
$A_1(-10;-15) \quad A_2(5;15)$

EXERCICE 19

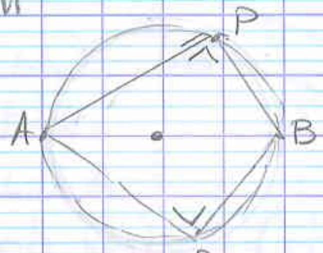
a) $\vec{AB}\cdot\vec{AP}=0$



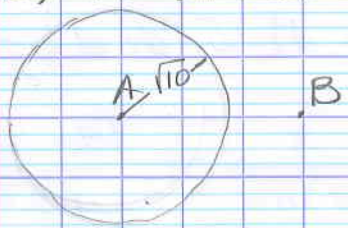
b) $\vec{AB}\cdot\vec{AP}=\|\vec{AB}\|^2$



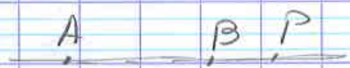
c) $\vec{AP}\cdot\vec{BP}=0$



d) $\vec{AP}\cdot\vec{AP}=10 \Rightarrow \|\vec{AP}\|^2=10 \Rightarrow \|\vec{AP}\|=\sqrt{10}$



e) $\det(\vec{AP};\vec{AB})=0 \Rightarrow \vec{AP}\,//\,\vec{AB}$



EXERCICE 20

$p: y=x^2 \quad d: y=x-1 \Rightarrow x-y-1=0$

a) $A\in p \quad A(x,x^2) \quad \delta(A,d)=2\sqrt{2} \Rightarrow$

$\Rightarrow \dfrac{|x-x^2-1|}{\sqrt{1+1}}=2\sqrt{2} \Rightarrow |x^2-x+1|=2\cdot 2$

$\Rightarrow \begin{cases} x^2-x+1=4 \Rightarrow x^2-x-3=0 \\ x^2-x+1=-4 \Rightarrow x^2-x+5=0 \end{cases}$

$\Delta=1+12 \quad x_{1,2}=\dfrac{1\pm\sqrt{13}}{2} \begin{cases} 2.3 \\ -1.3 \end{cases}$

$\Delta=1+20 \quad x_{1,2}=\dfrac{1\pm\sqrt{21}}{2} \begin{cases} 2.79 \\ -1.79 \end{cases}$

b) $\delta(B,d)=\dfrac{1}{2} \Rightarrow \dfrac{|x-x^2-1|}{\sqrt{2}}=\dfrac{1}{2} \Rightarrow$

$|x-x^2-1|=\dfrac{1}{\sqrt{2}} \Rightarrow \begin{cases} x-x^2-1=\dfrac{1}{\sqrt{2}} & (1) \\ x-x^2-1=-\dfrac{1}{\sqrt{2}} \end{cases}$

(1) $\sqrt{2}x-\sqrt{2}x^2-\sqrt{2}=1 \Rightarrow \sqrt{2}x^2-\sqrt{2}x+\sqrt{2}+1=0$

$\Delta=2-4\cdot\sqrt{2}(\sqrt{2}+1)<0$ impossible

(2) $x-x^2-1=-\dfrac{1}{\sqrt{2}} \Rightarrow \sqrt{2}x-\sqrt{2}x^2-\sqrt{2}+1=0 \Rightarrow$

$\sqrt{2}x^2-\sqrt{2}x+\sqrt{2}-1=0 \quad \Delta=2-4\cdot\sqrt{2}(\sqrt{2}-1)<0$

impossible

c) $\delta(C;d) = \frac{3\sqrt{2}}{8} \Rightarrow \frac{|x - x^2 - 1|}{\sqrt{2}} = \frac{3\sqrt{2}}{8} \Rightarrow$

$|x - x^2 - 1| = \frac{3}{4} \Rightarrow \begin{cases} 4x - 4x^2 - 4 = 3 & ① \\ 4x - 4x^2 - 4 = -3 & \end{cases}$

① $4x^2 - 4x + 7 = 0 \quad \Delta = 16 - 4\cdot4\cdot7 < 0$ impossible

② $4x^2 - 4x + 1 = 0 \quad \Delta = 0$

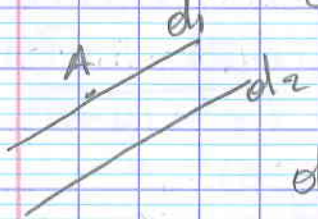
$(2x - 1)^2 = 0 \Rightarrow x = \frac{1}{2} \Rightarrow y = \frac{1}{4} \quad C\left(\frac{1}{2}; \frac{1}{4}\right)$

Exercice 21 $\quad d_1 : 4x - 3y - 12 = 0 \quad d_2 : 8x - 6y - 9 = 0$

$\vec{n}_1 = \begin{pmatrix} 4 \\ -3 \end{pmatrix} // \vec{n}_2 = \begin{pmatrix} 8 \\ -6 \end{pmatrix} // \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$d_2 : 4x - 3y - \frac{9}{2} = 0$ comme $-12 \neq -\frac{9}{2}$

$\Rightarrow d_1 // d_2$



On trouve un point $A \in d_1$

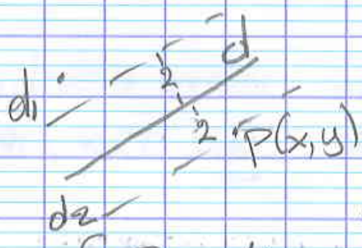
$y = 0 \quad x = 3 \quad A(3;0)$

$\text{dist}(d_1, d_2) = \text{dist}(A, d_2)$

$= \frac{|24 - 6\cdot0 - 9|}{\sqrt{64+36}} = \frac{15}{10} = 1,5$

Exercice 22 $\quad$ a) $d : 3x - 4y = 10 \quad A(8;1)$

$\cdot\ \delta(A,d) = \frac{|24 - 4 - 10|}{\sqrt{9+16}} = \frac{10}{5} = 2$



Soit $P(x,y)$ dont la distance à d est 2.

$\delta(P,d) = 2 \Rightarrow \frac{|3x - 4y - 10|}{5} = 2 \Rightarrow$

$\Rightarrow \begin{cases} 3x - 4y - 10 = 10 \\ 3x - 4y - 10 = -10 \end{cases} \Rightarrow$

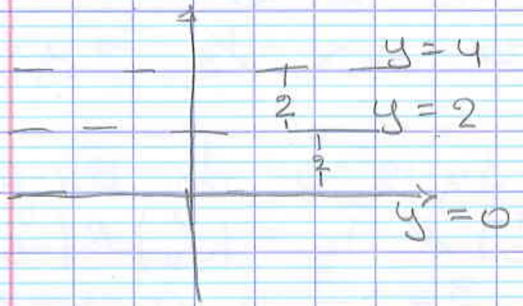
$\begin{cases} 3x - 4y - 20 = 0 & d_1 \\ 3x - 4y = 0 & d_2 \end{cases}$

b) $d: y-2=0 \quad A(0;8)$

- $\delta(A,d) = \dfrac{|8-2|}{\sqrt{1}} = 6$.

- Soit $P(x,y)$ avec $\delta(P,d)=2 \Rightarrow$

$\Rightarrow \dfrac{|y-2|}{\sqrt{2}} = 2 \Rightarrow \begin{cases} y-2=2 \Rightarrow y=4 \quad d_1 \\ y-2=-2 \Rightarrow y=0 \quad d_2 \end{cases}$



- Soit $P(x,y)$ avec $\delta(P,d)=\delta(P,A)$

$\delta(P,d) = |y-2| \quad \delta(A,P) = \|\vec{AP}\| =$
$= \sqrt{x^2+|y-8|^2}$

Alors $|y-2| = \sqrt{x^2+|y-8|^2} \Rightarrow$

$\Rightarrow (y-2)^2 = x^2+(y-8)^2 \Rightarrow$

$\Rightarrow \cancel{y^2}-4y+4 = x^2+\cancel{y^2}-16y+64 \Rightarrow$

$\Leftrightarrow 12y = x^2+60 \Rightarrow y = \dfrac{1}{12}x^2+5$

parabole

EXERCICE 23 $A(0;2)$

a) Soit d une droite non verticale qui passe par l'origine: $y=mx$, m la pente

$\Rightarrow mx-y=0$

Eq. Hessienne $\dfrac{mx-y}{\sqrt{m^2+1}} = 0$

b) $\delta(A,d)=1 \Rightarrow \dfrac{|m\cdot 0-2|}{\sqrt{m^2+1}} = 1 \Rightarrow$

$2 = \sqrt{m^2+1} \Rightarrow m^2+1=4 \Rightarrow m^2=3 \Rightarrow m=\pm\sqrt{3}$

$d_1: y=\sqrt{3}x \quad d_2: y=-\sqrt{3}x$

EXERCICE 24 $A(10;-1)$ $K(-1;1)$

a) $y = mx + h$

$A \in d$: $-1 = 10 \cdot m + h \Rightarrow h = -10m - 1$

donc $d : y = mx - 10m - 1 \Rightarrow$

$\Rightarrow mx - y - 10m - 1 = 0$

Eq. Hes : $\dfrac{mx - y - 10m - 1}{\sqrt{m^2+1}} = 0$

b) $\delta(K, d) = 5 \Rightarrow \dfrac{|-m - 1 - 10m - 1|}{\sqrt{m^2+1}} = 5 \Rightarrow$

$\Rightarrow |-11m - 2| = 5\sqrt{m^2+1} \Rightarrow$

$\Rightarrow (+11m - 2)^2 = 25(m^2+1) \Rightarrow$

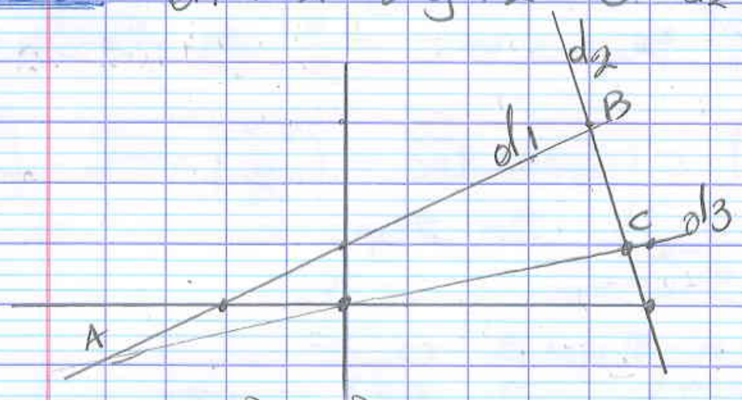
$\Rightarrow 121m^2 + 44m + 4 = 25m^2 + 25 \Rightarrow$

$\Rightarrow 96m^2 + 44m - 21 = 0$

$\Delta = 100^2$ $\quad m_{1,2} = \dfrac{-44 \pm 100}{192}$ $\quad m_1 = \dfrac{56}{192}$ $\quad m_2 = -\dfrac{144}{192}$

$m_1 = \dfrac{7}{24}$ $\quad m_2 = -\dfrac{3}{4}$

EXERCICE 25 $d_1 : x - 2y + 2 = 0$ $\quad d_2 : 3x + y - 15 = 0$ $\quad d_3 : x - 5y = 0$



$\vec{n_1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \Rightarrow \vec{d_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\vec{n_2} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \Rightarrow \vec{d_2} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\vec{n_3} = \begin{pmatrix} 1 \\ -5 \end{pmatrix} \Rightarrow \vec{d_3} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

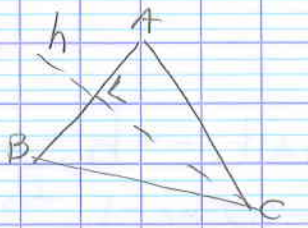
$\cos A = \dfrac{\vec{d_1} \cdot \vec{d_3}}{\|\vec{d_1}\| \, \|\vec{d_3}\|} = \dfrac{10+1}{\sqrt{5} \cdot \sqrt{26}} \Rightarrow A = 15{,}25°$

$\cos B = \dfrac{\vec{d_1} \cdot \vec{d_2}}{\|\vec{d_1}\| \cdot \|\vec{d_2}\|} = \dfrac{|2-3|}{\|\vec{d_1}\| \, \|\vec{d_2}\|} = \dfrac{1}{\sqrt{5}\sqrt{10}} \Rightarrow B = 81{,}87°$

Donc $C = 180 - 15{,}25 - 81{,}87 \Rightarrow C = 82{,}9°$

Exercice 26

$A(-1;2)$ $B(2;-3)$ $C(7;3)$



$\vec{AB} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \vec{n}_h$

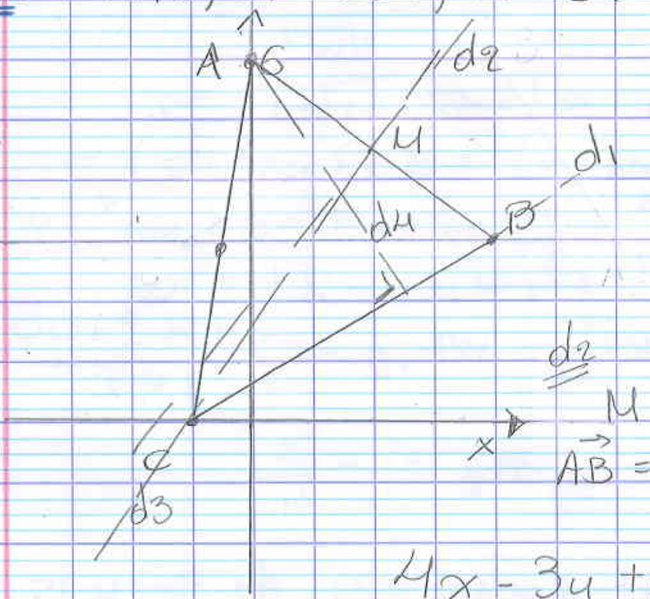
$h : 3x - 5y + c = 0$

$C : 21 - 15 + c = 0 \Rightarrow c = -6$

$h : 3x - 5y - 6 = 0$

Exercice 27

$A(0;6)$ $B(4;3)$ $C(-1;0)$



b) $\vec{BC} = \begin{pmatrix} -5 \\ -3 \end{pmatrix} \Rightarrow \vec{n}_{BC} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$

$3x - 5y + c = 0$

$C : -3 + c = 0 \Rightarrow c = 3$

$d_1 : 3x - 5y + 3 = 0$

d_2

$M\left(2 ; \frac{9}{2}\right)$

$\vec{AB} = \vec{n}_2 = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$4x - 3y + c = 0$

$M : 8 - \frac{27}{2} + c = 0 \Rightarrow c = \frac{11}{2}$ $\quad d_2 \quad 4x - 3y + \frac{11}{2} = 0$

$d_3 : M\left(2 ; \frac{9}{2}\right)$ $C(-1, 0)$ $\vec{MC} = \begin{pmatrix} -3 \\ -\frac{9}{2} \end{pmatrix} \Rightarrow \vec{n}_3 = \begin{pmatrix} \frac{9}{2} \\ -3 \end{pmatrix} // \begin{pmatrix} 9 \\ -6 \end{pmatrix}$

$\Rightarrow \vec{n}_3 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\quad 3x - 2y + c = 0$

$C : -3 + c = 0 \Rightarrow c = 3$

$d_3 : 3x - 2y + 3 = 0$

d_4 $\quad \vec{BC} = \begin{pmatrix} -5 \\ -3 \end{pmatrix} \Rightarrow \vec{n}_4 = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ $\quad 5x + 3y + c = 0$

$A : 18 + c = 0 \Rightarrow c = -18$ $\quad d_4 : 5x + 3y - 18 = 0$

c) $\vec{AC} = \begin{pmatrix} -1 \\ -6 \end{pmatrix}$ $\quad \vec{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $\quad \begin{vmatrix} -1 & 4 \\ -6 & -3 \end{vmatrix} = 3 + 24 = 27$

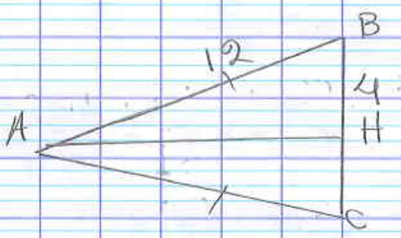
$A_{ABC} = \frac{1}{2} \cdot 27 = 13.5$

d) $A = \frac{b \cdot h}{2}$ base $= \|\vec{BC}\| = \sqrt{25+9} = \sqrt{34}$

$h = dist(A, d_1) = \frac{|-30+3|}{\sqrt{9+25}} = \frac{|-27|}{\sqrt{34}}$

$A = \frac{\sqrt{34} \cdot 27}{2\sqrt{34}} = 13{,}5$

Exercice 28



$AB = 12$ $BC = 8$

$\vec{AB} \cdot \vec{AH} = \|\vec{AH}\| \cdot \|\vec{AH}\| = \|\vec{AH}\|^2$

car la projection de $\vec{AB}$ sur le $\vec{AH}$ est le AH

$AH = \sqrt{12^2 - 4^2} = \sqrt{128}$ Alors $\vec{AH} \cdot \vec{AB} = 128$

Exercice 29

$K(2;3)$ $r = 5$

a) Cercle $(x-2)^2 + (y-3)^2 = 25$

b) Cercle $(x+2)^2 + (y-4)^2 = 4$

c) cercle $x^2 + y^2 = 1$

d) cercle $(x-1)^2 + y^2 = 169$

Exercice 30

a) Tous les points du cercle $C(0,0)$ $r = \sqrt{13}$

b) $(x-5)^2 + (y+2)^2 = 1$

c) $(x-1)^2 + (y+3)^2 = r^2$ r?

$(0,0)$ $(-1)^2 + 3^2 = 10 = r^2 \Rightarrow r = \sqrt{10}$

d) $(x-5)^2 + (y+11)^2 = 125$

$\cap Ox$: $y = 0$ $(x-5)^2 + 121 = 125 \Rightarrow (x-5)^2 = 4 \Rightarrow$

$x - 5 = 2 \Rightarrow x_1 = 7$ $x - 5 = -2 \Rightarrow x_2 = 3$

$(7;0)$ $(3;0)$

EXERCICE 31 $C \in Oy \quad C(0;y_0) \quad x^2+(y-y_0)^2=r^2$

$$\left.\begin{array}{l} A : (-6)^2+y_0^2=r^2 \Rightarrow 36+y_0^2=r^2 \; (1) \\ B : 4+(12-y_0)^2=r^2 \end{array}\right\} \Rightarrow$$

$$36+y_0^2=4+(12-y_0)^2 \Rightarrow 36+\cancel{y_0^2}=4+144-24y_0+\cancel{y_0^2}$$

$$\Rightarrow 24y_0=112 \Rightarrow y_0=\frac{3}{8}$$

Alors (1) $36+\frac{9}{64}=r^2 \Rightarrow r^2=\frac{2313}{64} \Rightarrow$

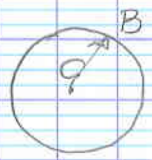
$r=\frac{\sqrt{2313}}{8}$

$$x^2+\left(y-\frac{3}{8}\right)^2=\frac{2313}{64}$$

EXERCICE 32 $C(3;1) \quad A(5;4) \quad (x-3)^2+(y-1)^2=r^2$

a) $A : (5-3)^2+(4-1)^2=r^2 \Rightarrow r^2=4+9=13 \Rightarrow r=\sqrt{13}$

b)



$\vec{CB} \,//\, \begin{pmatrix}0.6\\0.8\end{pmatrix} \,//\, \begin{pmatrix}6\\8\end{pmatrix} \,//\, \begin{pmatrix}3\\4\end{pmatrix}$

$B(x,y) \quad \vec{CB}=\begin{pmatrix}x-3\\y-1\end{pmatrix} \,//\, \begin{pmatrix}3\\4\end{pmatrix} \Rightarrow$

$$\begin{vmatrix} x-3 & 3 \\ y-1 & 4 \end{vmatrix}=0 \Rightarrow 4(x-3)-3(y-1)=0$$

$$\Rightarrow 4x-12-3y+3=0 \Rightarrow 4x-3y=9$$

$$\begin{cases} (x-3)^2+(y-1)^2=13 \\ 4x-3y=9 \Rightarrow \frac{4}{3}x-3=y \end{cases}$$

$$(x-3)^2+\left(\frac{4}{3}x-3-1\right)^2=13 \Rightarrow (x-3)^2+\left(\frac{4}{3}x-4\right)^2=13$$

$$\Rightarrow x^2-6x+9+\frac{16}{9}x^2-\frac{32}{3}x+16-13=0 \Rightarrow$$

$$\Rightarrow \frac{25}{9}x^2-\frac{50}{3}x+12=0 \Rightarrow 25x^2-150x+108=0$$

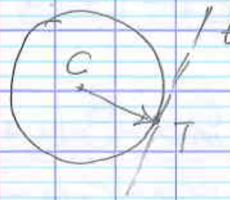
$$\Delta=11700 \quad x_{1,2}=\frac{150\pm\sqrt{11700}}{50} \begin{cases} x_1=5.16 \\ x_2=0.84 \end{cases}$$

$B_1(5{,}16\,;\,3.88) \qquad B_2(0.84\,;\,-1.88)$

Exercice 33 $C: (x-3)^2+(y+14)^2=68$ $\quad \vec{v}=\begin{pmatrix}1\\-1\end{pmatrix}$ $\quad C(3;-14)$ $\quad r=\sqrt{68}$

a) $T(1,-6): (1-3)^2+(-6+14)^2=68$ ✓ $\Rightarrow T\in C$

b)



$\vec{n}_t=\vec{CT}=\begin{pmatrix}-2\\8\end{pmatrix} \parallel \begin{pmatrix}1\\-4\end{pmatrix}$

$t: x-4y+c=0$

$T: 1+24+c=0 \Rightarrow c=-25$

$t: x-4y-25=0$

c) $t_1 \parallel t_2 \parallel \vec{v}=\begin{pmatrix}1\\-1\end{pmatrix} \Rightarrow \vec{n}=\begin{pmatrix}1\\1\end{pmatrix}$ $\quad t_1: x+y+c=0$

$dist(C,t)=\sqrt{68} \Rightarrow$

$\dfrac{|3-14+c|}{\sqrt{1+1}}=\sqrt{68} \Rightarrow |c-11|=\sqrt{136} \Rightarrow$

$$\begin{cases} c-11=\sqrt{136} \Rightarrow c_1=11+\sqrt{136} \\ c-11=-\sqrt{136} \Rightarrow c_2=11-\sqrt{136} \end{cases}$$

$t_1: x+y+11+\sqrt{136}=0$ $\quad t_2: x+y+11-\sqrt{136}=0$

Exercice 34 $K(-1;2)$ $\quad r=5$

a)

$\vec{v}=\begin{pmatrix}5\\2\end{pmatrix} \parallel t \Rightarrow \vec{n}_t=\begin{pmatrix}2\\-5\end{pmatrix}$

$2x-5y+c=0$

$dist(t,K)=5 \Rightarrow \dfrac{|-2-10+c|}{\sqrt{4+25}}=5 \Rightarrow$

$\Rightarrow |c-12|=5\sqrt{29} \Rightarrow \begin{cases} c-12=5\sqrt{29} \Rightarrow c_1=12+5\sqrt{29} \\ c-12=-5\sqrt{29} \Rightarrow c_2=12-5\sqrt{29} \end{cases}$

Exercice 35 $C: x^2+y^2-8x-2y+4=0 \Rightarrow$

$(x-4)^2-16+(y-1)^2-1+4=0 \Rightarrow (x-4)^2+(y-1)^2=13$

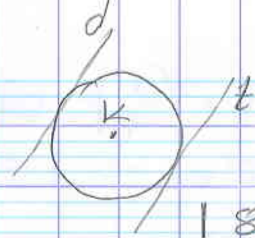
$K(4,1)$ $\quad r=\sqrt{13}$

$d: 2x-3y-18=0$

a) $K(4,1)$ $\quad r=\sqrt{13}$

b) $dist(K,d)=\dfrac{|8-3-18|}{\sqrt{4+9}}=\dfrac{13}{\sqrt{13}}=\sqrt{13}=r \Rightarrow$

d est tangente

c) 

$K(4;1)$

$t \parallel d \Rightarrow t: 2x-3y+c=0$

$dist(K, t) = r = \sqrt{13} \Rightarrow$

$\dfrac{|8-3+c|}{\sqrt{4+9}} = \sqrt{13} \Rightarrow |c+5| = 13 \Rightarrow$

$\begin{cases} c+5=13 \Rightarrow c_1 = 8 & t: 2x-3y+8=0 \\ c+5=-13 \Rightarrow c_2=-18 & d: 2x-3y-18=0 \end{cases}$

EXERCICE 36 $C: x^2+y^2=12 \quad C_2: (x-8)^2+(y+4)^2=4$

$d: x+y-7=0$

a) $K_1(0,0) \; r_1=\sqrt{12} \quad K_2(8;-4) \; r_2=2$

$dist(K_1, d) = \dfrac{|-7|}{\sqrt{1+1}} = \dfrac{7}{\sqrt{2}} \approx 4.95 > r_1 \Rightarrow$

$\Rightarrow$ d ne coupe pas le cercle C_1

$dist(K_2, d) = \dfrac{|8-4-7|}{\sqrt{1+1}} = \dfrac{3}{\sqrt{2}} \approx 2.12 > r_2$

$\Rightarrow$ d ne coupe pas le cercle C_2

b) 

On cherche le point A

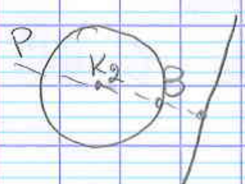
On trouve la droite $p \perp d$

$\vec{n}_d = \begin{pmatrix}1\\1\end{pmatrix} \Rightarrow \vec{n}_p = \begin{pmatrix}1\\-1\end{pmatrix}$

$p: x-y+c=0 \quad K_1: c=0 \Rightarrow p: x-y=0$

$A = p \cap d \quad + \begin{cases} x-y=0 \\ x+y=7 \end{cases} \quad 2x=7 \Rightarrow x=\frac{7}{2}$

$\frac{7}{2} = y \quad A\left(\frac{7}{2}; \frac{7}{2}\right)$

c) 

On trouve la droite $p \perp d$

$\Rightarrow p: x-y+c=0$

$K_2: 8+4+c=0 \Rightarrow c=-12$

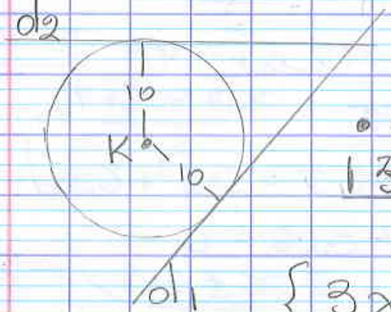
$p: x-y-12=0$

$B = p \cap d \Rightarrow + \begin{cases} x-y=12 \\ x+y=7 \end{cases} \quad 2x=19 \Rightarrow x=\frac{19}{2}$

$\frac{19}{2} - 12 = y \Rightarrow y = -\frac{5}{2}$

$B\left(\frac{19}{2}; -\frac{5}{2}\right)$

Exercice 37 $d_1 : y = \frac{3}{4}x - 2$ $\quad d_2 : y = -10$ $\quad r = 10$



$d_1 : 4y = 3x - 8 \Rightarrow 3x - 4y - 8 = 0$

• $dist(K, d_1) = 10 \Rightarrow$

$\frac{|3x_0 - 4y_0 - 8|}{\sqrt{9+16}} = 10 \Rightarrow |3x_0 - 4y_0 - 8| = 50$

$\begin{cases} 3x_0 - 4y_0 - 8 = 50 \Rightarrow 3x_0 - 4y_0 = 58 \quad (1) \\ 3x_0 - 4y_0 - 8 = -50 \Rightarrow 3x_0 - 4y_0 = -42 \quad (2) \end{cases}$

• $dist(K, d_2) = 10 \Rightarrow \frac{|y_0 + 10|}{\sqrt{1}} = 10 \Rightarrow$

$|y_0 + 10| = 10 \Rightarrow \begin{cases} y_0 + 10 = 10 \Rightarrow y_0 = 0 \\ y_0 + 10 = -10 \Rightarrow y_0 = -20 \end{cases}$

$\rightarrow y_0 = 0$ $\quad (1)$ $3x_0 = 58 \Rightarrow x_0 = \frac{58}{3}$ $\quad K_1\left(\frac{58}{3}; 0\right)$

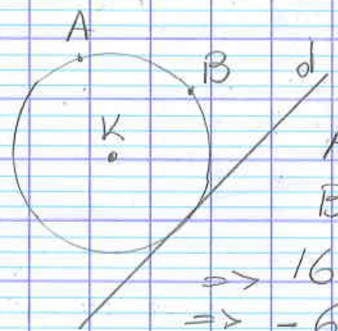
(2) $3x_0 = -42 \Rightarrow x_0 = -14$ $\quad K_2(-14; 0)$

$\rightarrow y_0 = -20$ $\quad (1)$ $3x_0 + 80 = 58 \Rightarrow 3x_0 = -22 \Rightarrow x_0 = -\frac{22}{3}$

(2) $3x_0 + 80 = -42 \Rightarrow 3x_0 = -122 \Rightarrow x_0 = -\frac{122}{3}$

$K_3\left(-\frac{22}{3}; -20\right)$ $\quad K_4\left(-\frac{122}{3}; -20\right)$

Exercice 38 $A(4;4)$ $\quad B(1;3)$ $\quad d : 2x + y - 14 = 0$



$(x - x_0)^2 + (y - y_0)^2 = r^2$

$\left.\begin{array}{l} A : (4 - x_0)^2 + (4 - y_0)^2 = r^2 \\ B : (1 - x_0)^2 + (3 - y_0)^2 = r^2 \end{array}\right\} (1)$

$\Rightarrow 16 - 8x_0 + \cancel{x_0^2} + 16 - 8y_0 + \cancel{y_0^2} = 1 - 2x_0 + \cancel{x_0^2} + 9 - 6y_0 + \cancel{y_0^2}$

$\Rightarrow -6x_0 - 2y_0 + 22 = 0 \Rightarrow$

$\Rightarrow 3x_0 + y_0 - 11 = 0 \Rightarrow y_0 = -3x_0 + 11$

$dist(K, d) = r \Rightarrow \frac{|2x_0 + y_0 - 14|}{\sqrt{4+1}} = r \Rightarrow$

$\Rightarrow |2x_0 + y_0 - 14| = r \cdot \sqrt{5} \Rightarrow$

$\xRightarrow{y_0 = -3x_0 + 11}$ $|2x_0 - 3x_0 + 11 - 14| = r \cdot \sqrt{5} \Rightarrow$

$\Rightarrow |-x_0 - 3| = r \cdot \sqrt{5} \Rightarrow (x_0 + 3)^2 = 5r^2 \Rightarrow r^2 = \frac{(x_0+3)^2}{5}$

On remplace en ①

$(4-x_0)^2+(4+3x_0-11)^2=\frac{(x_0+3)^2}{5} \Rightarrow$

$\Leftrightarrow 16-8x_0+x_0^2+9x_0^2-42x_0+49=\frac{x_0^2+6x_0+9}{5} \Leftrightarrow$

$\Leftrightarrow 10x_0^2-50x_0+65=\frac{x_0^2+6x_0+9}{5} \Rightarrow$

$\Rightarrow 50x_0^2-250x_0+325=x_0^2+6x_0+9 \Leftrightarrow$

$\Rightarrow 49x_0^2-256x_0+316=0$

$\Delta=3600 \quad x_0=\frac{256\pm60}{98}$ ⟨ $x_0=\frac{316}{98}=\frac{158}{49}$; $x_0=2$

$x_0=\frac{158}{49} \quad y_0=-3\cdot\frac{158}{49}+11=-\frac{474}{49}+11 \Rightarrow y_0=\frac{65}{49}$

$r^2=\frac{\left(\frac{158}{49}+3\right)^2}{5}=\frac{\left(\frac{305}{49}\right)^2}{5}=\frac{93025}{12005}=\frac{18605}{2401} \Rightarrow$

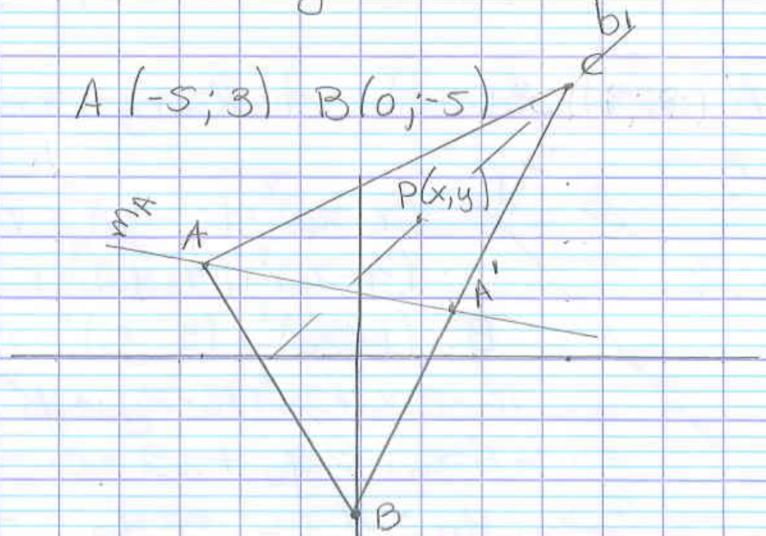
$r=\sqrt{\frac{18605}{2401}}$

$\left(x-\frac{158}{49}\right)^2+\left(y-\frac{65}{49}\right)^2=\frac{18605}{2401}$

$x_0=2 \quad y_0=-6+11=5 \quad r^2=\frac{(2+3)^2}{5}=5$

$(x-2)^2+(y-5)^2=5$

Exercice Sup 1 $A(-5;3)$ $B(0;-5)$ $C(7;9)$



d_{AC} $\vec{AC}=\begin{pmatrix}12\\6\end{pmatrix} \parallel \begin{pmatrix}2\\1\end{pmatrix} \Rightarrow \vec{n}_{AC}=\begin{pmatrix}1\\-2\end{pmatrix}$ $x-2y+c=0$

B: $10+c=0 \Rightarrow c=-10$ $x-2y-10=0$

d_{BC} $\vec{BC}=\begin{pmatrix}7\\14\end{pmatrix}=\begin{pmatrix}1\\2\end{pmatrix} \Rightarrow \vec{n}_{BC}=\begin{pmatrix}2\\-1\end{pmatrix}$ $2x-y+c=0$

B: $5+c=0 \Rightarrow c=-5$ $2x-y-5=0$

c) $A' = \left(\frac{0+7}{2}; \frac{-5+9}{2}\right) \Rightarrow A'(3,5; 2)$

$A(-5; 3)$

$\vec{AA'} = \begin{pmatrix} 8,5 \\ -1 \end{pmatrix} \parallel \begin{pmatrix} 17 \\ -2 \end{pmatrix} \Rightarrow \vec{n}_{AA'} = \begin{pmatrix} 2 \\ 17 \end{pmatrix}$ $2x + 17y + c = 0$

m_A: A': $-10 + 51 + c = 0 \Rightarrow c = -41$ $\quad 2x + 17y - 41 = 0$

d) h_B $\vec{AC} \parallel \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \vec{n}_h \Rightarrow h: 2x + y + c = 0$

B: $-5 + c = 0 \Rightarrow c = 5$ $\quad h_B: 2x + y + 5 = 0$

e) Soit $P(x,y) \in b$

$dist(P, AC) = dist(P, BC) \Rightarrow$

$\frac{|x - 2y - 10|}{\cancel{\sqrt{1+4}}} = \frac{|2x - y - 5|}{\cancel{\sqrt{5}}} \Rightarrow$

$\begin{cases} x - 2y - 10 = 2x - y - 5 \Rightarrow x + y + 5 = 0 \quad b_1 \\ x - 2y - 10 = -2x + y + 5 \Rightarrow 3x - 3y - 15 = 0 \Rightarrow \end{cases}$

$x - y - 5 = 0 \quad b_2$

La bissectrice intérieure est la droite b_2

b_2: $x - y - 5 = 0$

EXERCICE Sup 2 a) $d: 3x - 4y + m = 0 \quad C: x^2 + y^2 - 4x - 12 = 0$

$x^2 + y^2 - 4x - 12 = 0 \Leftrightarrow (x-2)^2 - 4 + y^2 - 12 = 0 \Leftrightarrow (x-2)^2 + y^2 = 16$

$K(2; 0) \quad r = 4$

d est tangente $\Rightarrow dist(K, d) = r = 4 \Rightarrow$

$\Rightarrow \frac{|6 + m|}{\sqrt{9+16}} = 4 \Leftrightarrow |m+6| = 20 \Rightarrow \begin{cases} m + 6 = 20 \Rightarrow m_1 = 14 \\ m + 6 = -20 \Rightarrow m_2 = -26 \end{cases}$

b) $C: (x-3)^2 + (y-k)^2 = 2k^2 \quad d: x - y + 5 = 0$

$K(3; k) \quad r = \sqrt{2k^2}$

$dist(K, d) = r \Rightarrow \frac{|3 - k + 5|}{\sqrt{1+1}} = \sqrt{2k^2} \Rightarrow$

$\Rightarrow |8 - k| = \sqrt{2k^2} \cdot \sqrt{2} \Rightarrow (8-k)^2 = 2 \cdot k^2 \cdot 2 \Rightarrow$

$\Rightarrow 64 - 16k + k^2 = 4k^2 \Rightarrow 3k^2 + 16k - 64 = 0$

$\Delta = 1024 \quad k_{1,2} = \frac{-16 \pm 32}{6}$ $\begin{cases} k_1 = \frac{16}{6} = \frac{8}{3} \\ k_2 = -8 \end{cases}$

Exercice Sup 3 a) $r = 3$ $K(-2; y_0)$ $d: 4x+3y+3=0$

$dist(K,d) = r = 3 \Rightarrow \frac{|-8+3y_0+3|}{\sqrt{16+9}} = 3 \Rightarrow$

$\Rightarrow |3y_0 - 5| = 15 \Rightarrow \begin{cases} 3y_0 - 5 = 15 \Rightarrow y_0 = \frac{20}{3} \\ 3y_0 - 5 = -15 \Rightarrow y_0 = -\frac{10}{3} \end{cases}$

C_1 $(x+2)^2 + (y - \frac{20}{3})^2 = 9$

C_2 $(x+2)^2 + (y + \frac{10}{3})^2 = 9$

b) $T(-5; 7)$ $x^2+y^2+4x-6y-12=0 \Leftrightarrow$

$\Rightarrow (x+2)^2 - 4 + (y-3)^2 - 9 - 12 = 0 \Rightarrow (x+2)^2 + (y-3)^2 = 25$

$K(-2; 3)$ $r = 5$

$T \in C$? $(-5+2)^2 + (7-3)^2 = 9 + 16 = 25$ ✓

$\vec{n}_t = \vec{KT} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ $-3x + 4y + c = 0$

$T: 15 + 28 + c = 0 \Rightarrow c = -43$

$t: -3x + 4y - 43 = 0$

Exercice Sup 4 a) $d_1: 3x+4y-6=0$ $d_2: -x+5y+12=0$

$\vec{n}_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \not\parallel \vec{n}_2 = \begin{pmatrix} -1 \\ 5 \end{pmatrix} \Rightarrow$ sécantes

b) $C: x^2+y^2-8x+2y+12=0 \Rightarrow (x-4)^2 - 16 + (y+1)^2 - 1 + 12 = 0$

$\Leftrightarrow (x-4)^2 + (y+1)^2 = 5$ $K(4; -1)$ $r = \sqrt{5}$

$d_3: x - 2y + 1 = 0$

$dist(K, d_3) = \frac{|4+2-1|}{\sqrt{1+4}} = \frac{5}{\sqrt{5}} = \sqrt{5} = r$

~~donc la d~~

donc la droite est tangente

c) $C_1: x^2 + (y-2)^2 = 25$ $K_1(0; 2)$ $r_1 = 5$

$C_2: (x+4)^2 + y^2 = 49$ $K_2(-4, 0)$ $r_2 = 7$

$\vec{K_1K_2} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$ $\|\vec{K_1K_2}\| = \sqrt{16+4} = \sqrt{20} \approx 4.47 \leq r_1 + r_2$

$r_1 + r_2 = 12$

Donc les deux cercles se coupent en deux points

Exercice Sup 5 a) $C_1 : x^2+y^2-2x+4y-14=0 \Rightarrow$

$\Rightarrow (x-1)^2-1+(y+2)^2-4-14=0 \Rightarrow$

$\Rightarrow (x-1)^2+(y+2)^2=19 \quad K(1;-2) \quad r=\sqrt{19}$

$C_2 : x^2+y^2+x=0 \Leftrightarrow (x+\frac{1}{2})^2-\frac{1}{4}+y^2=0 \Rightarrow$

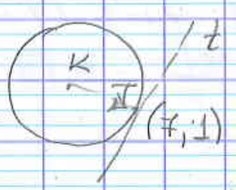
$(x+\frac{1}{2})^2+y^2=\frac{1}{4} \quad K(-\frac{1}{2};0) \quad r=\frac{1}{2}$

Exercice Sup 6 a) $C: 3x^2+3y^2-18x+12y-36=0$

$\Rightarrow x^2+y^2-6x+4y-12=0 \Rightarrow$

$\Rightarrow (x-3)^2-9+(y+2)^2-4-12=0 \Rightarrow$

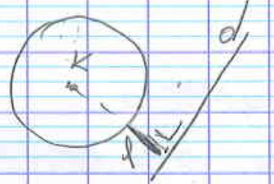
$\Rightarrow (x-3)^2+(y+2)^2=25 \quad K(3;-2) \quad r=5$

b) 

$\vec{n}_t=\vec{KT}=\begin{pmatrix}4\\3\end{pmatrix} \quad 4x+3y+c=0$

$T: 28+3+c=0 \Rightarrow c=-31$

$4x+3y-31=0$

c) 

$d_1 : y=\frac{2}{3}x+9 \Rightarrow 3y=2x+27 \Rightarrow$

$\Rightarrow 2x-3y+27=0$

$dist(K,d_1)=\frac{|6+6+27|}{\sqrt{4+9}}=\frac{27}{\sqrt{13}}$

$l=\frac{27}{\sqrt{13}}-5 \approx 2.49$

d) $C: (x-3)^2+(y+2)^2=25 \quad d_2 : y=x-2$

$(x-3)^2+(x-2+2)^2=25 \Rightarrow (x-3)^2+x^2=25 \Leftrightarrow$

$\Leftrightarrow x^2-6x+9+x^2=25 \Leftrightarrow 2x^2-6x-16=0 \Leftrightarrow$

$\Leftrightarrow x^2-3x-8=0 \qquad \Delta=9+32=41$

$x_{1,2}=\frac{+3\pm\sqrt{41}}{2}$ $\begin{cases} x_1=4.7 & y_1=2.7 \\ x_2=-1.7 & y_2=-3.7 \end{cases}$

$A(4.7;2.7) \quad B(-1.7;-3.7)$

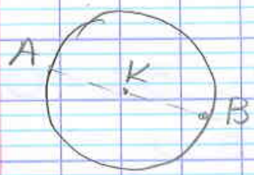
Exercice Sup 7 a) $C(2;-3)$ $r=7$ $(x-2)^2+(y+3)^2=49$

b) $C(6;-8)$ $(x-6)^2+(y+8)^2=r^2$

$O: (0-6)^2+(0+8)^2=r^2 \Rightarrow r^2=100 \Rightarrow r=10$

$(x-6)^2+(y+8)^2=100.$

c)



$A(3;2)$ $B(-1;6)$

K est le milieu de AB.

$K\left(\frac{3-1}{2};\frac{2+6}{2}\right) \Rightarrow K(1;4)$

$r=\frac{\|\vec{AB}\|}{2}$ $\vec{AB}=\begin{pmatrix}-4\\4\end{pmatrix}$ $\|\vec{AB}\|=\sqrt{16+16}=\sqrt{32}$

$r=\frac{\sqrt{32}}{2}=\frac{4\sqrt{2}}{2}=2\sqrt{2}$

$(x-1)^2+(y-4)^2=8$

d) $C(1;-1)$ $t: 5x-12y+9=0$

On cherche le rayon du cercle

$dist(C,t)=r \Rightarrow r=\frac{|5+12+9|}{\sqrt{25+144}}=\frac{26}{13}=2$

$(x-1)^2+(y+1)^2=4$

e) $A(3;1)$ $B(-1;3)$ $C\in d: 3x=y+2 \Rightarrow y=3x-2$

Alors $C(x_0;3x_0-2)$

$(x-x_0)^2+(y-3x_0+2)^2=r^2$

$A: (3-x_0)^2+(3-3x_0)^2=r^2$ ①

$B: (-1-x_0)^2+(5-3x_0)^2=r^2$ $\Rightarrow$

$\Leftrightarrow (3-x_0)^2+(3-3x_0)^2=(1+x_0)^2+(5-3x_0)^2 \Leftrightarrow$

$\Leftrightarrow 9-6x_0+x_0^2+9-18x_0+9x_0^2=1+2x_0+x_0^2+25-30x_0+9x_0^2$

$\Rightarrow \cancel{10x_0^2}-24x_0+18=\cancel{10x_0^2}-28x_0+26 \Leftrightarrow$

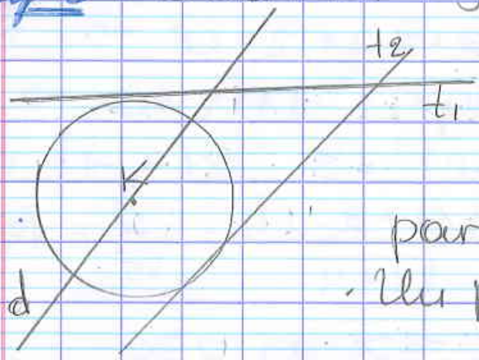
$\Rightarrow 4x_0=8 \Rightarrow x_0=4$ $y_0=3\cdot4-2=10$

① $(3-4)^2+(3-30)^2=1+27^2=730=r^2$

$(x-4)^2+(y-10)^2=730$

EXERCICE Sup 8 $C \in d : 4x - 5y = 3$ $\quad t_1 : 2x - 3y - 10 = 0$

$t_2 : 3x - 2y + 5 = 0$



On trouve l'équation paramétrique de d

· Un point $y = 1$ $\quad 4x = 8 \Rightarrow x = 2$

$(2 ; 1)$

· vecteur directeur

$\vec{n} = \begin{pmatrix} 4 \\ -5 \end{pmatrix} \Rightarrow \vec{d} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

$d : \begin{cases} x = 2 + 5\lambda \\ y = 1 + 4\lambda \end{cases}$ Donc $C(2 + 5\lambda ; 1 + 4\lambda)$

$\text{dist}(C, t_1) = \text{dist}(C, t_2) \Rightarrow$

$\dfrac{|2(2+5\lambda) - 3(1+4\lambda) - 10|}{\sqrt{4+9}} = \dfrac{|3(2+5\lambda) - 2(1+4\lambda) + 5|}{\sqrt{9+4}} \Rightarrow$

$\Rightarrow |4 + 10\lambda - 3 - 12\lambda - 10| = |6 + 15\lambda - 2 - 8\lambda + 5| \Rightarrow$

$\Rightarrow |-2\lambda - 9| = |7\lambda + 9| \Rightarrow$

$\begin{cases} -2\lambda - 9 = 7\lambda + 9 \Rightarrow 9\lambda = -18 \Rightarrow \lambda_1 = -2 \\ -2\lambda - 9 = -7\lambda - 9 \Rightarrow 5\lambda = 0 \Rightarrow \lambda_2 = 0 \end{cases}$

$\lambda_1 = -2 \quad C_1(-8 ; -7) \quad \lambda_2 = 0 \quad C_2(2 ; 1)$

$r_1 = \text{dist}(C_1, t_1) = \dfrac{|-16 + 21 - 10|}{\sqrt{4+9}} = \dfrac{5}{\sqrt{13}}$

$C_1 : (x+8)^2 + (y+7)^2 = \dfrac{25}{13}$

$r_2 = \text{dist}(C_2, t_1) = \dfrac{|4 - 3 - 10|}{\sqrt{9+4}} = \dfrac{|-9|}{\sqrt{13}} = \dfrac{9}{\sqrt{13}}$

$C_2 : (x-2)^2 + (y-1)^2 = \dfrac{81}{13}$

Exercice Sup 9 a) $d: x-2y-1=0$

$C: x^2+y^2-8x+2y+12=0 \Rightarrow$

$(x-4)^2-16+(y+1)^2-1+12=0 \Rightarrow$

$\Rightarrow (x-4)^2+(y+1)^2=5 \quad K(4;-1) \quad r=\sqrt{5}$

$dist(K,d) = \dfrac{|4+2-1|}{\sqrt{1+4}} = \dfrac{5}{\sqrt{5}} = \sqrt{5} = r \Rightarrow$ tangent

b) $d_2: y=x+10 \Rightarrow x-y+10=0$

$C_2: x^2+y^2=1 \quad K(0;0) \quad r=1$

$dist(C_2,d_2) = \dfrac{|10|}{\sqrt{1+1}} = \dfrac{10}{\sqrt{2}} \approx 7.07 > r \Rightarrow$

$\Rightarrow$ extérieurs

Exercice Sup 10 $C: x^2+y^2=25 \quad d: 2x-y-5=0 \Rightarrow y=2x-5$

$C \cap d: x^2+(2x-5)^2=25 \Rightarrow$

$\Rightarrow x^2+4x^2-20x+\cancel{25}-\cancel{25}=0$

$\Rightarrow 5x^2-20x=0 \Rightarrow 5x(x-4)=0 \Rightarrow x=0$, $x=4$

$x_1=0 \quad y_1=-5 \quad A(0;-5)$

$x_2=4 \quad y_2=3 \quad B(4;3)$

Exercice Sup 11 a) $T(-1;2) \quad C: x^2+y^2=5 \quad K(0,0) \quad r=\sqrt{5}$

$T \in C$? $(-1)^2+2^2=5$ ✓

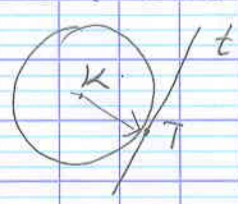
$\vec{n}_t = \vec{KT} = \begin{pmatrix}-1\\2\end{pmatrix} \quad -x+2y+c=0$

$T: 1+4+c=0 \Rightarrow c=-5$

$t: -x+2y-5=0$

b) $T(-5;7) \in C: (x+2)^2+(y-3)^2=25 \quad K(-2;3)$

$(-5+2)^2+(7-3)^2 = 9+16=25$ ✓



$\vec{n}_t = \vec{KT} = \begin{pmatrix}-3\\4\end{pmatrix}$

$-3x+4y+c=0$

$T: 15+28+c=0 \Rightarrow c=-41$

$t: -3x+4y-41=0$

Exercice Sup 12 a) $x^2+y^2+10x-2y+6=0 \Rightarrow$

$(x+5)^2-25+(y-1)^2-1+6=0 \Rightarrow$

$\Leftrightarrow (x+5)^2+(y-1)^2=20 \quad K(-5;1) \quad r=\sqrt{20}$

$d: 2x+y=7 \quad t \parallel d \Rightarrow t: 2x+y+c=0$

$dist(K;t)=\sqrt{20} \Rightarrow \dfrac{|-10+1+c|}{\sqrt{4+1}}=\sqrt{20} \Rightarrow$

$|c-9|=\sqrt{100} \Rightarrow |c-9|=10 \Rightarrow \begin{cases} c-9=10 \\ c-9=-10 \end{cases} \Rightarrow$

$c_1=19 \quad c_2=-1$

$t_1: 2x+y+19=0 \quad t_2: 2x+y-1=0$

b) $x^2+y^2-2x+4y=0 \Rightarrow (x-1)^2-1+(y+2)^2-4=0 \Rightarrow$

$\Leftrightarrow (x-1)^2+(y+2)^2=5 \quad K(1;-2) \quad r=\sqrt{5}$

$d: x-2y-345=0 \quad t \perp d$

$\vec{n}_d=\begin{pmatrix}1\\-2\end{pmatrix} \Rightarrow \vec{n}_t=\begin{pmatrix}2\\1\end{pmatrix}$

$t: 2x+y+c=0$

$dist(K,t)=\sqrt{5} \Rightarrow \dfrac{|2-2+c|}{\sqrt{4+1}}=\sqrt{5} \Rightarrow |c|=5 \Rightarrow$

$c=\pm 5$

$t_1: 2x+y+5=0 \quad t_2: 2x+y-5=0$