LJP : ANALYSE - DERIVEES - Solutions Classe C

EXERCICE 3

$f(x) = \frac{5}{x}$

- $f(2) = \frac{5}{2}$ $\quad$ • $f(2+\Delta x) = \frac{5}{2+\Delta x}$
- $f(2+\Delta x) - f(2) = \frac{5}{2+\Delta x} - \frac{5}{2} = \frac{10-5(2+\Delta x)}{2(2+\Delta x)} = \frac{10-10-5\Delta x}{2(2+\Delta x)} =$
  $= \frac{-5\Delta x}{2(2+\Delta x)}$
- $\frac{f(2+\Delta x)-f(2)}{\Delta x} = \frac{-5\Delta x}{2(2+\Delta x)\Delta x} = -\frac{5}{2(2+\Delta x)}$
- $\lim\limits_{\Delta x \to 0} \frac{f(2+\Delta x)-f(2)}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{-5}{2(2+\Delta x)} = -\frac{5}{4} = f'(2)$

tangente : $y = -\frac{5}{4}x + h$
$(2;\frac{5}{2})$: $\frac{5}{2} = -\frac{5}{4}\cdot 2 + h \Leftrightarrow h = \frac{10}{2} = 5$ $\Big\}$ $\underline{y = -\frac{5}{4}x + 5}$

EXERCICE 4

$f(x) = x^2 - 3x$

- $f(-2) = 4 + 6 = 10$
- $f(-2+\Delta x) = (-2+\Delta x)^2 - 3(-2+\Delta x) = 4 - 4\Delta x + \Delta x^2 + 6 - 3\Delta x$
  $= \Delta x^2 - 7\Delta x + 10$
- $f(-2+\Delta x) - f(-2) = \Delta x^2 - 7\Delta x + 10 - 10 = \Delta x(\Delta x - 7)$
- $\frac{f(-2+\Delta x)-f(-2)}{\Delta x} = \Delta x - 7$
- $\lim\limits_{\Delta x \to 0} \frac{f(-2+\Delta x)-f(-2)}{\Delta x} = \lim\limits_{\Delta x \to 0} (\Delta x - 7) = -7$

EXERCICE 5

$f(x) = \sin x$ $\quad$ $f'(x) = \cos x$

x_0	$f(x_0)$	$f'(x_0)$	
0	0	1	$y = x$
$\frac{\pi}{2}$	1	0	$y = 1$
2π	0	1	$y = x - 2\pi$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$y = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\frac{\pi}{4}$

EXERCICE 6

$f(x) = 2 + \cos x$

1. $f\left(\frac{\pi}{3}\right) = 2 + \cos\frac{\pi}{3} = 2 + \frac{1}{2} = \frac{5}{2}$

2. $f'(x) = -\sin x$

3. $f'\left(\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

4. $y = -\frac{\sqrt{3}}{2}x + h \leftarrow \left(\frac{\pi}{3} ; \frac{5}{2}\right)$

$\frac{5}{2} = -\frac{\sqrt{3}}{2}\cdot\frac{\pi}{3} + h \Leftrightarrow h = \frac{5}{2} + \frac{\sqrt{3}\pi}{6}$ $\Big\}$ $y = -\frac{\sqrt{3}}{2}x + \frac{5}{2} + \frac{\sqrt{3}\pi}{6}$

EXERCICE 7

$f(x) = x^2 + 6x - 3 \qquad f'(x) = 2x + 6$

(i) $x_0 = 1 : \quad f(1) = 4 \qquad f'(1) = 8$

$4 = 8\cdot 1 + h \Rightarrow h = -4$ $\Big\}$ $t : y = 8x - 4$

(ii) $f'(x) = 10 \Leftrightarrow 2x + 6 = 10 \Leftrightarrow 2x = 4 \Leftrightarrow x = 2$

$f(2) = 4 + 6\cdot 2 - 3 = 13$

$13 = 10\cdot 2 + h \Leftrightarrow h = 1 \Rightarrow t : y = 10x + 1$

(iii) $f'(x) = 0 \Leftrightarrow 2x + 6 = 0 \Leftrightarrow x = -3 \qquad f(-3) = 9 - 18 - 3 = -12$

Donc TH $y = -12$ la pente $= 0$

EXERCICE 8

$y = 2x \Rightarrow y' = 2 \; ; \; y = 2 \Rightarrow y' = 0 \; ; \; y = 2x^2 \Rightarrow y' = 4x$

$y = x^2 \Rightarrow y' = 2x \; ; \; y = -2 \Rightarrow y' = 0 \; ; \; y = x + 2 \Rightarrow y' = 1$

$y = 2x - 2 \Rightarrow y' = 2 \; ; \; y = x + x = 2x \Rightarrow y' = 2$

EXERCICE 9

$u(x) = x^{17} \qquad v(x) = 2x - 1$

$u'(x) = 17x^{16} \qquad v'(x) = 2$

(a) $f_1(x) = u(x) + v(x) \Rightarrow f_1'(x) = u'(x) + v'(x) = 17x^{16} + 2$

(b) $f_2(x) = u(x) - v(x) \Rightarrow f_2'(x) = u'(x) - v'(x) = 17x^{16} - 2$

(c) $f_3(x) = u(x)\cdot v(x) \Rightarrow f_3'(x) = u'(x)v(x) + u(x)\cdot v'(x) =$

$= 17x^{16}\cdot(2x-1) + x^{17}\cdot 2 = 34x^{17} - 17x^{16} + 2x^{17} = 36x^{17} - 17x^{16}$

(d) $f_4(x) = \frac{u(x)}{v(x)} \Rightarrow f_4'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)} = \frac{17x^{16}(2x-1) - x^{17}\cdot 2}{(2x-1)^2}$

$= \frac{34x^{17} - 17x^{16} - 2x^{17}}{(2x-1)^2} = \frac{32x^{17} - 17x^{16}}{(2x-1)^2}$

(e) $f_5(x) = (u \circ v)(x) \Rightarrow f_5'(x) = u'(v(x))\cdot v'(x) = 2(2x-1)^{17}$

(f) $f_6(x) = (v \circ u)(x) \Rightarrow f_6'(x) = v'(u(x))\cdot u'(x) = (2x^{17} - 1)\,17x^{16}$

EXERCICE 10

$f(x) = 7x^2 \Rightarrow f'(x) = 14x \qquad g(x) = -\frac{2}{3}x + \sqrt{5} \Rightarrow g'(x) = -\frac{2}{3}$

$h(x) = \sqrt{x} \Rightarrow h'(x) = \frac{1}{2\sqrt{x}} \qquad l(x) = x^3 \Rightarrow l'(x) = 3x^2$

$k(x) = \frac{1}{x+2} \Rightarrow k'(x) = -\frac{1}{(x+2)^2}$

$p(x) = \frac{x}{x+1} \Rightarrow p'(x) = \frac{(x+1)-x}{(x+1)^2} = \frac{1}{(x+1)^2}$

$q(x) = 10 \Rightarrow q'(x) = 0 \qquad t(x) = \frac{1}{x^2} \Rightarrow t'(x) = -\frac{2x}{x^4} = -\frac{2}{x^3}$

$r(x) = x + \sin x \Rightarrow r'(x) = 1 + \cos x$

EXERCICE 11

1. $y = 10000 \Rightarrow y' = 0$ 2. $y = x^3 - x^2 \Rightarrow y' = 3x^2 - 2x$

3. $y = x^3 - x^2 + 10 \Rightarrow y' = 3x^2 - 2x$ 4. $y = 5x^3 - 6x^2 \Rightarrow y' = 15x^2 - 12x$

5. $y = x^4 - 4x^2 + 9 \Rightarrow y' = 4x^3 - 8x$

6. $y = \sqrt[4]{x} + 5\sqrt[3]{x^2} - 9\sqrt[6]{x^2} = x^{1/4} + 5x^{2/3} - 9x^{1/3}$

$\Rightarrow y' = \frac{1}{4}x^{-3/4} + 5\cdot\frac{2}{3}x^{-1/3} - 9\frac{1}{3}x^{-2/3} =$

$= \frac{1}{4\sqrt[4]{x^3}} + \frac{10}{3\sqrt[3]{x}} - \frac{3}{\sqrt[3]{x^2}}$

7. $y = \frac{7}{\sqrt[7]{x}} + \frac{13}{\sqrt{x}} - 4\frac{x^2}{\sqrt[3]{x}} = 7x^{-1/7} + 13x^{-1/2} - 4x^2x^{-1/3}$

$= 7x^{-1/7} + 13x^{-1/2} - 4x^{5/3}$

$y' = 7\left(-\frac{1}{7}\right)x^{-8/7} + 13\left(-\frac{1}{2}\right)x^{-3/2} - 4\frac{5}{3}x^{2/3} =$

$= -\frac{1}{\sqrt[7]{x^8}} - \frac{13}{2\sqrt{x^3}} - \frac{20\sqrt[3]{x^2}}{3}$

8. $y = (x+4)(x-3) = x^2 + 4x - 3x - 12 = x^2 + x - 12.$

$y' = 2x + 1$

9. $y = (x+4)(x-3) + 30 \Rightarrow y' = 2x + 1$

10. $y = \frac{x+4}{x-3} \Rightarrow y' = \frac{x-3-(x+4)}{(x-3)^2} = \frac{x-3-x-4}{(x-3)^2} = \frac{-7}{(x-3)^2}$

11. $y = \frac{x+4}{x-3} - 20 \Rightarrow y' = -\frac{7}{(x-3)^2}$

12. $y = \frac{x}{3x^2-1} \Rightarrow y' = \frac{(3x^2-1) - x\cdot 6x}{(3x^2-1)^2} = \frac{3x^2-6x^2-1}{(3x^2-1)^2} = \frac{-3x^2-1}{(3x^2-1)^2}$

13. $y = \frac{x^2-1}{2x^2} \Rightarrow y' = \frac{2x\cdot 2x^2 - (x^2-1)4x}{4x^4} = \frac{\cancel{4x^3} - \cancel{4x^3} + 4x}{4x^{\cancel{4}3}} = \frac{1}{x^3}$

14. $y = \frac{4-5x^2}{x^2-5x+6} \Rightarrow y' = \frac{-10x\cdot(x^2-5x+6) - (4-5x^2)(2x-5)}{(x^2-5x+6)^2} =$

$= \frac{-\cancel{10x^3} + 50x^2 - 60x - 8x + \cancel{10x^3} + 20 - 25x^2}{(x^2-5x+6)^2} =$

$= \frac{25x^2 - 68x + 20}{(x^2-5x+6)^2} =$

15. $y = \dfrac{2x-3}{x^2+5} \Rightarrow y' = \dfrac{2(x^2+5)-(2x-3)\cdot 2x}{(x^2+5)^2} = \dfrac{2x^2+10-4x^2+6x}{(x^2+5)^2} =$

$= \dfrac{-2x^2+6x+10}{(x^2+5)^2}$

16. $y = x + \dfrac{2x-3}{x^2+5} \Rightarrow y' = 1 + \dfrac{-2x^2+6x+10}{(x^2+5)^2}$

EXERCICE 19

1. $f(x) = 5000 \Rightarrow f'(x) = 0$
2. $f(x) = 500x^{100} + 10000 \Rightarrow f'(x) = 50000x^{99}$

3) $f(x) = 8x^{-8} + 20\sqrt{x} \Rightarrow f'(x) = -64x^{-9} + \dfrac{20}{2\sqrt{x}} = -\dfrac{64}{x^9} + \dfrac{10}{\sqrt{x}}$

4. $f(x) = \dfrac{8}{x^8+20\sqrt{x}} \Rightarrow f'(x) = -8\,\dfrac{-\frac{64}{x^9}+\frac{10}{\sqrt{x}}}{(x^8+20\sqrt{x})^2}$

5. $f(x) = 9^{10} \Rightarrow f'(x) = 0$

6. $f(x) = \dfrac{\pi+\pi^2+\pi^3}{3} \Rightarrow f'(x) = 0$

7. $f(x) = \dfrac{\pi+\pi^2+\pi^3}{3}\cdot x \Rightarrow f(x) = \dfrac{\pi+\pi^2+\pi^3}{3}$

8. $f(\pi) = \dfrac{\pi+\pi^2+\pi^3}{3} \Rightarrow f'(\pi) = \frac{1}{3}(1+2\pi+3\pi^2)$

9. $f(x) = (x^3+2x)^4 \Rightarrow f'(x) = 4(x^3+2x)^3\cdot(3x^2+2)$

10. $f(x) = (x^3+2x)\sin x \Rightarrow f'(x) = (3x^2+2)\sin x + (x^3+2x)\cos x$

11. $f(x) = \sin(x^3+2x+\pi) \Rightarrow f'(x) = \cos(x^3+2x+\pi)\,(3x^2+2)$

12. $f(x) = (2-\sin(3x))^5 \Rightarrow f'(x) = 5(2-\sin(3x))^4\cdot(-\cos 3x\cdot 3) =$

$= -15\cos 3x\cdot(2-\sin(3x))^4$

13. $f(x) = \dfrac{2+\sin x}{x} \Rightarrow f'(x) = \dfrac{\cos x\cdot x-(2+\sin x)}{x^2}$

14. $f(x) = \dfrac{3x^2-x+10}{-2x+3} \Rightarrow f'(x) = \dfrac{(6x-1)(-2x+3)-(3x^2-x+10)(-2)}{(-2x+3)^2}$

$= \dfrac{-12x^2+\cancel{2x}+18x-3+6x^2-\cancel{2x}+20}{(-2x+3)^2} =$

$= \dfrac{-6x^2+18x+17}{(-2x+3)^2}$

15. $f(x) = (4x+x^2)^{1/3} - (2-5x)^3 \Rightarrow$

$\Rightarrow f'(x) = \frac{1}{3}(4x+2x^2)^{-2/3}(4+2x) - 3(2-5x)^2\cdot(-5) =$

$= \dfrac{4+2x}{3\sqrt[3]{(4x+2x^2)^2}} + 15(2-5x)^2$

16. $f(x) = \cos((x+3)^5 - x^4) \Rightarrow$

$\Rightarrow f'(x) = -\sin((x+3)^5-x^4)\cdot(5(x+3)^2-3x^2)$

17. $f(x) = ax^3 + (a-1)x + \cos(ax)$

$f'(x) = 3ax^2 + (a-1) - a\sin(ax)$

18. $f(a) = ax^3 + (a-1)x + \cos(ax)$

$f'(a) = x^3 + x - x\sin(ax)$

EXERCICE 13

$f(x) = (x+1)\cos(2x)$

$x = -\frac{3}{2}\pi \quad f\left(-\frac{3\pi}{2}\right) = \left(-\frac{3\pi}{2}+1\right)\overset{-1}{\cancel{\cos}}(-3\pi) = \frac{3\pi}{2} - 1.$

$f'(x) = \cos(2x) - 2(x+1)\sin 2x.$

$f'\left(-\frac{3\pi}{2}\right) = \cos(-3\pi) - 2\left(-\frac{3\pi}{2}+1\right)\underset{0}{\cancel{\sin}}(-3\pi) = -1.$

$\frac{3\pi}{2} - \cancel{1} = -\cancel{1} + h \Leftrightarrow h = \frac{3\pi}{2} \quad \Rightarrow t: y = -x + \frac{3\pi}{2}.$

EXERCICE 14

(a) $f(x) = \frac{2}{3}x^3 - x^2 + 7 \Rightarrow f'(x) = 2x^2 - 2x.$

(b) $f(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}}$

(c) $f(x) = (x+1)\sqrt{x} \Rightarrow f'(x) = \sqrt{x} + \frac{(x+1)}{2\sqrt{x}}$

(d) $f(x) = \frac{1-x}{2x+3} \Rightarrow f'(x) = \frac{-1(2x+3) - (1-x)2}{(2x+3)^2} = \frac{-\cancel{2x} - 3 - 2 + \cancel{2x}}{(2x+3)^2} =$

$= \frac{-5}{(2x+3)^2}$

e) $f(x) = \pi x^2 + 2\pi h x \Rightarrow f'(x) = 2\pi x + 2\pi h.$

f) $f(x) = ax^3 + bx^2 + cx + d \Rightarrow f'(x) = 3ax^2 + 2bx + c.$

(g) $f(x) = -\frac{K}{x} \Rightarrow f'(x) = \frac{K}{x^2}$

(h) $f(x) = (2x-1)^3 \Rightarrow f'(x) = 3(2x-1)^2 \cdot 2 = 6(2x-1)^2$

(i) $f(x) = \frac{x^3+2}{x^2+1} \Rightarrow f'(x) = \frac{3x^2(x^2+1) - (x^3+2)2x}{(x^2+1)^2} =$

$= \frac{3x^4 + 3x^2 - 2x^4 - 4x}{(x^2+1)^2} = \frac{x^4 + 3x^2 - 4x}{(x^2+1)^2}$

(j) $f(x) = (x^3+2)(x^2+1) = x^5 + 2x^2 + x^3 + 2$

$f'(x) = 5x^4 + 3x^2 + 4x$

(k) $f(x) = x^{1/3} \Rightarrow f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

(m) $f(x)=(x^2-7x)^5 \Rightarrow f'(x)=5(x^2-7x)^4\cdot(2x-7)$

(n) $f(x)=(\sqrt{x}-x)^4 \Rightarrow f'(x)=4(\sqrt{x}-x)^3\cdot\left(\frac{1}{2\sqrt{x}}-1\right)$

(o) $f(x)=\sqrt{x^2-7}\left(x^3-2x^2-\frac{5}{9}\right)$

$f'(x)=\frac{2x}{2\sqrt{x^2-7}}\left(x^3-2x^2-\frac{5}{9}\right)+\sqrt{x^2-7}\,(3x^2-4x)$

EXERCICE 15

▸ $f(x)=7x-3x^3+1$ $\quad f'(x)=-3x^2+7=0 \Leftrightarrow x^2=\frac{7}{3} \Leftrightarrow x=\pm\sqrt{\frac{7}{3}}$

	$-\infty$		$-\sqrt{\frac{7}{3}}$		$\sqrt{\frac{7}{3}}$	
f'		$-$	0	$+$	0	$-$
f		↘	Min	↗	Max	↘

$\text{Min}=f\left(-\sqrt{\frac{7}{3}}\right)\approx 1.035$

$\text{Max}=f\left(\sqrt{\frac{7}{3}}\right)\simeq 0.965$

▸ $g(x)=\frac{x^2}{3x-4}$ $\quad D_g=\mathbb{R}\setminus\{4/3\}$

$g'(x)=\frac{2x(3x-4)-x^2\cdot 3}{(3x-4)^2}=\frac{6x^2-8x-3x^2}{(3x-4)^2}=\frac{6x^2-8x}{(3x-4)^2}=$

$=\frac{2x\cancel{(3x-4)}}{(3x-4)^{\cancel{2}}}=\frac{2x}{3x-4}=0 \Leftrightarrow x=0.$

	$-\infty$		0		4/3		$+\infty$
g'		$+$	0	$-$	//	$+$	
g		↗	Max	↘	//	↗	

$\text{Max}=f(0)=0$

EXERCICE 16

▸ $f(x)=2x^3-x^2$ $\quad D_f=\mathbb{R}$

$f'(x)=6x^2-2x=2x(3x-1)=0 \Leftrightarrow x=0 \quad x=\frac{1}{3}.$

	$-\infty$		0		1/3		$+\infty$
f'		$+$	0	$-$	0	$+$	
f		↗	Max	↘	Min	↗	

$\text{Max}=f(0)=0$

$\text{Min}=f(1/3)=0.037.$

▸ $h(x)=\frac{2}{(1-x)^3}$ $\quad D_h=\mathbb{R}\setminus\{1\}$

$h'(x)=\frac{+2\cdot 3\cancel{(1-x)^2}}{(1-x)^{\cancel{6}\,4}}=\frac{6}{(1-x)^4}=0$ impossible.

	$-\infty$		1		$+\infty$
h'		$+$	//	$+$	
h		↗	//	↗	

▶ $g(x) = \frac{4x^2-4x+10}{2x-1}$ $\quad D_g = \mathbb{R} \setminus \{1/2\}$

$g'(x) = \frac{(8x-4)(2x-1)-(4x^2-4x+10)2}{(2x-1)^2} =$

$= \frac{16x^2-8x-\cancel{8x}+4-8x^2+\cancel{8x}-20}{(2x-1)^2} = \frac{8x^2-8x-16}{(2x-1)^2} = 0$

$\Leftrightarrow 8x^2-8x-16=0 \Leftrightarrow x^2-x-2=0 \Leftrightarrow x_1=-1 \quad x_2=2$

x	$-\infty$		-1		$1/2$		2		$+\infty$
g'		+	0	−	‖	−	0	+	
g		↗	Max	↘	‖	↘	Min	↗	

Max $= f(-1) = -\frac{14}{3}$

Min $= f(2) = \frac{18}{3}$

▶ $k(x) = \sqrt{x^2+4x+4} = \sqrt{(x+2)^2}$ $\quad D_k = \mathbb{R}$

$k'(x) = \frac{\cancel{2}(x+2)}{\cancel{2}\sqrt{(x+2)^2}} = \frac{(x+2)}{|x+2|} = \begin{cases} 1 & x>-2 \\ -1 & x<-2 \end{cases}$ $\quad D_{k'} = \mathbb{R} \setminus \{-2\}$

x	$-\infty$		-2		$+\infty$
k'		+	‖	−	
k		↗	‖	↘	

EXERCICE 17

$f(x) = \frac{x^2+3x}{x-1}$ $\quad D_f = \mathbb{R} \setminus \{1\}$

· $f(x) = 0 \Leftrightarrow x^2+3x=0 \Leftrightarrow x(x+3)=0 \Leftrightarrow x=0 \quad x=-3$

· TS

x	$-\infty$		-3		0		1		$+\infty$
x^2+3x		+	0	−	0	+		+	
$x-1$		−		−		−	0	+	
f		−	0	+	0	−	‖	+	

· AV: $\lim_{x\to 1} f(x) = \text{"}\frac{4}{0}\text{"} = \infty \Rightarrow$ AV: $x=1$

· ANV:
$$\begin{array}{r|l} x^2+3x & x-1 \\ \hline -x^2+x & x+4 \\ 4x & \\ -4x-4 & \\ -4 & \end{array}$$

AO: $y = x+4$

· $f'(x) = \frac{(2x+3)(x-1)-(x^2+3x)}{(x-1)^2} = \frac{2x^2+\cancel{3x}-2x-x^2-\cancel{3x}}{(x-1)^2} =$

$= \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2} = 0 \Leftrightarrow x=0 \quad x=2$

x	$-\infty$		0		1		2		$+\infty$
f'		+	0	−	‖	−	0	+	
f		↗	Max	↘	‖	↘	Min	↗	

Max $= f(0) = 0$

Min $= f(2) = 10$

· $f(-2) = \frac{2}{3}$ $\quad f'(-2) = \frac{8}{9}$ $\quad \frac{2}{3} = \frac{8}{9}(-2)+h \Rightarrow h = \frac{22}{9} \Rightarrow t: \ y = \frac{8}{9}x + \frac{22}{9}$

EXERCICE 18 1) $f(x) = \dfrac{2x+3}{5-x}$

Domaine $D_f = \mathbb{R} \setminus \{5\}$

Zéros $\cap O_x$: $f(x) = 0 \Leftrightarrow 2x+3 = 0 \Leftrightarrow x = -\frac{3}{2}$ $\quad I_x(-\frac{3}{2};0)$

$\cap O_y$: $f(0) = \frac{3}{5}$ $\quad I_y(0;\frac{3}{5})$

TS :

x	$-\infty$	$-3/2$		5	$+\infty$
$2x+3$	$-$	0	$+$		$+$
$5-x$	$+$		$+$	0	$-$
f	$-$	0	$+$	//	$-$

AV $\lim\limits_{x \to 5^+} f(x) = -\infty$ $\quad \lim\limits_{x \to 5^-} f(x) = +\infty \Rightarrow$ $\underline{x = 5}$ AV

AH : $\lim\limits_{x \to \pm\infty} f(x) = -2 \Rightarrow$ AH : $\underline{y = -2}$

Dérivée $f'(x) = \dfrac{2(5-x) - (2x+3)(-1)}{(5-x)^2} = \dfrac{10 - \cancel{2x} + \cancel{2x} + 3}{(5-x)^2} =$

$= \dfrac{13}{(5-x)^2}$

PTH : $f'(x) = 0 \Leftrightarrow 13 = 0$ impossible

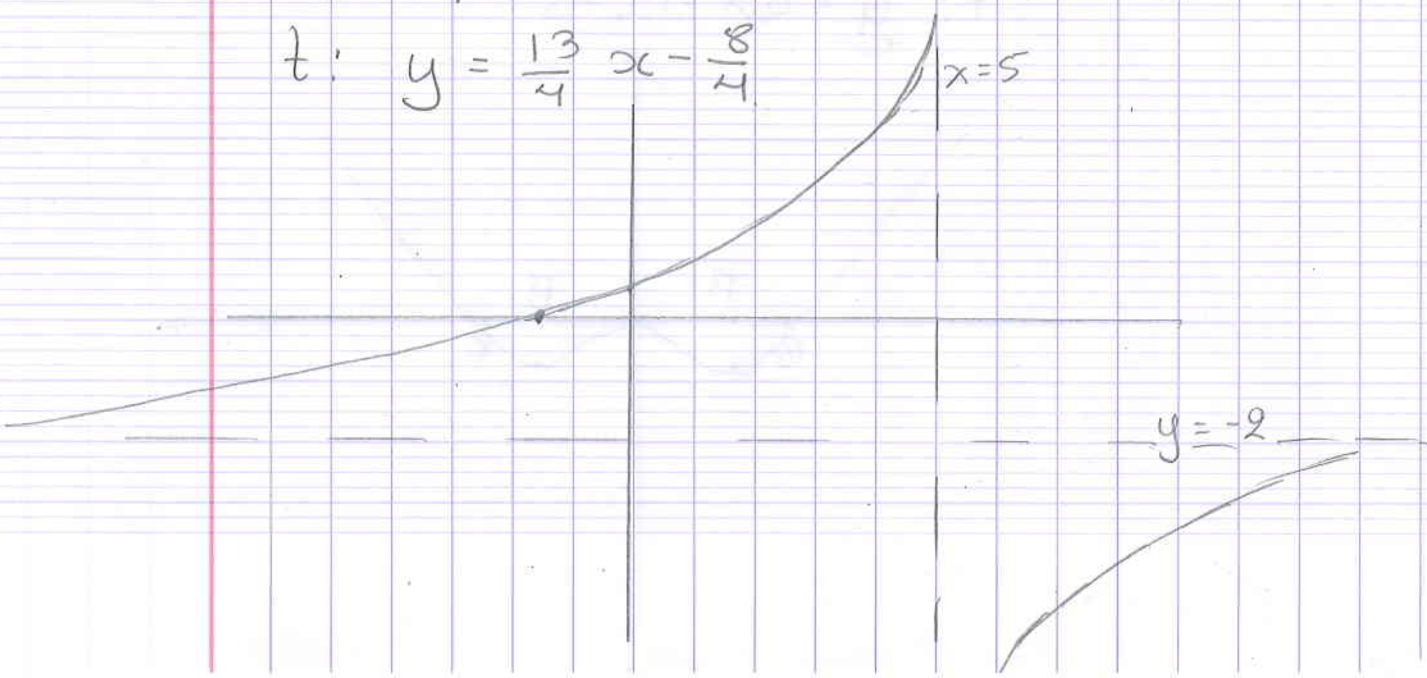
TG :

x	$-\infty$	5	$+\infty$
f'	$+$	//	$+$
f	$\nearrow$	//	$\nearrow$

tangente : $f'(3) = \frac{13}{4} = m$ $\quad f(3) = \frac{9}{2}$

$\frac{9}{2} = \frac{13}{4} \cdot 3 + h \Rightarrow h = \frac{18}{4} - \frac{26}{4} = \frac{-8}{4}$

$t : y = \frac{13}{4}x - \frac{8}{4}$



2) $g(x) = \frac{1}{12}x^4 - \frac{1}{2}x^2$

Domaine $D_g = \mathbb{R}$

$\cap Ox$: $g(x) = 0 \Leftrightarrow \frac{1}{12}x^4 - \frac{1}{2}x^2 = 0 \overset{\cdot 12}{\Leftrightarrow} x^4 - 6x^2 = 0 \Leftrightarrow$

$\Leftrightarrow x^2(x^2-6) = 0 \Leftrightarrow x = 0 \quad x = \sqrt{6} \quad x = -\sqrt{6}$

$I_{1x}(0;0) \quad I_{2x}(\sqrt{6};0) \quad I_{3x}(-\sqrt{6};0)$

$\cap Oy$: $x = 0 \quad y = 0 \quad I_y(0;0)$

TS

x	$-\infty$		$-\sqrt{6}$		0		$\sqrt{6}$		$+\infty$
$g(x)$		+	0	−	0	−	0	+	

AV $\emptyset$

AH/AO $\emptyset$

Dérivée : $g'(x) = \frac{1}{3}x^3 - x$

PTH $g'(x) = 0 \Leftrightarrow \frac{1}{3}x^3 - x = 0 \Leftrightarrow x\left(\frac{1}{3}x^2 - 1\right) = 0 \Leftrightarrow$

$x = 0 \quad x^2 = 3 \Leftrightarrow x = \sqrt{3} \quad x = -\sqrt{3}$

T Crois :

x	$-\infty$		$-\sqrt{3}$		0		$\sqrt{3}$		$+\infty$
x		−		−	0	+		+	
$\frac{1}{3}x^2 - 1$		+	0	−		−	0	+	
g'		−	0	+	0	−	0	+	
g		↘	Min	↗	Max	↘	Min	↗	

$g(-\sqrt{3}) = -0.75$

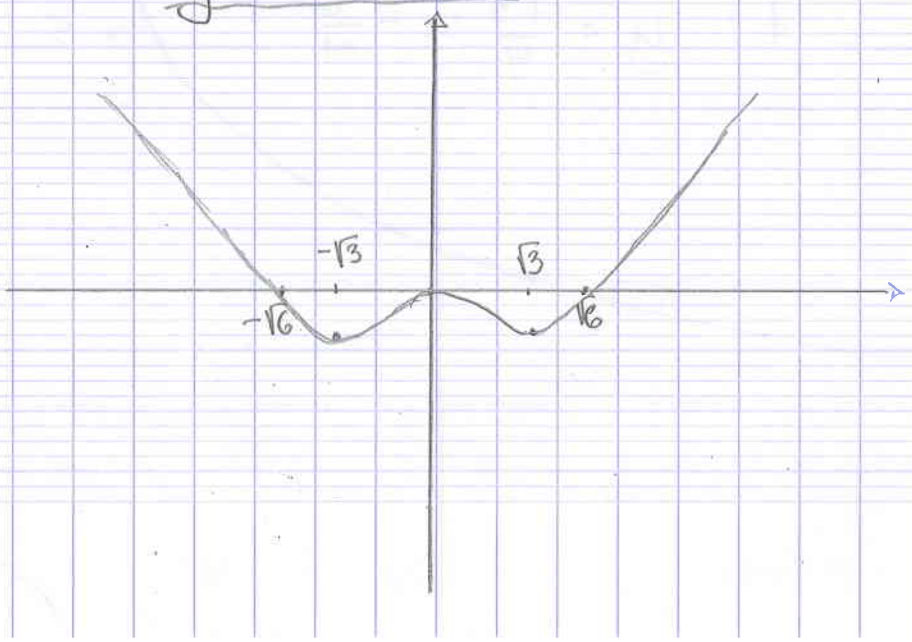
$g(0) = 0$

$g(\sqrt{3}) = -0.75$

tangente : $g(3) = 2.25 \quad g'(3) = 6$

$2.25 = 6 \cdot 3 + h \Rightarrow h = -15.75$

$t : y = 6x - 15.75$



3) $h(x) = \dfrac{3x^2+x+4}{x^2+1}$

<u>Domaine</u> $D_h = \mathbb{R}$

<u>$\cap Ox$</u> : $h(x) = 0 \Leftrightarrow 3x^2+x+4 = 0 \quad \Delta = 1 - 48 < 0 \quad \emptyset$

<u>$\cap Oy$</u> : $x = 0 \quad h(0) = 4 \quad I_y(0;4)$

<u>TS</u>

x	$-\infty$		$+\infty$
h		$+$	

AV $\emptyset$

AH : $\lim\limits_{x\to\pm\infty} h(x) = 3 \Rightarrow$ AH : $\underline{y = 3}$

<u>Dérivée</u> $h'(x) = \dfrac{(6x+1)(x^2+1) - (3x^2+x+4)\cdot 2x}{(x^2+1)^2} =$

$$= \frac{\cancel{6x^3} + x^2 + 6x + 1 - \cancel{6x^3} - 2x^2 - 8x}{(x^2+1)^2}$$

$$= \frac{-x^2 - 4x + 1}{(x^2+1)^2}$$

<u>PTH</u> : $h'(x) = 0 \Leftrightarrow -x^2 - 4x + 1 = 0 \Leftrightarrow x^2 + 4x - 1 = 0$

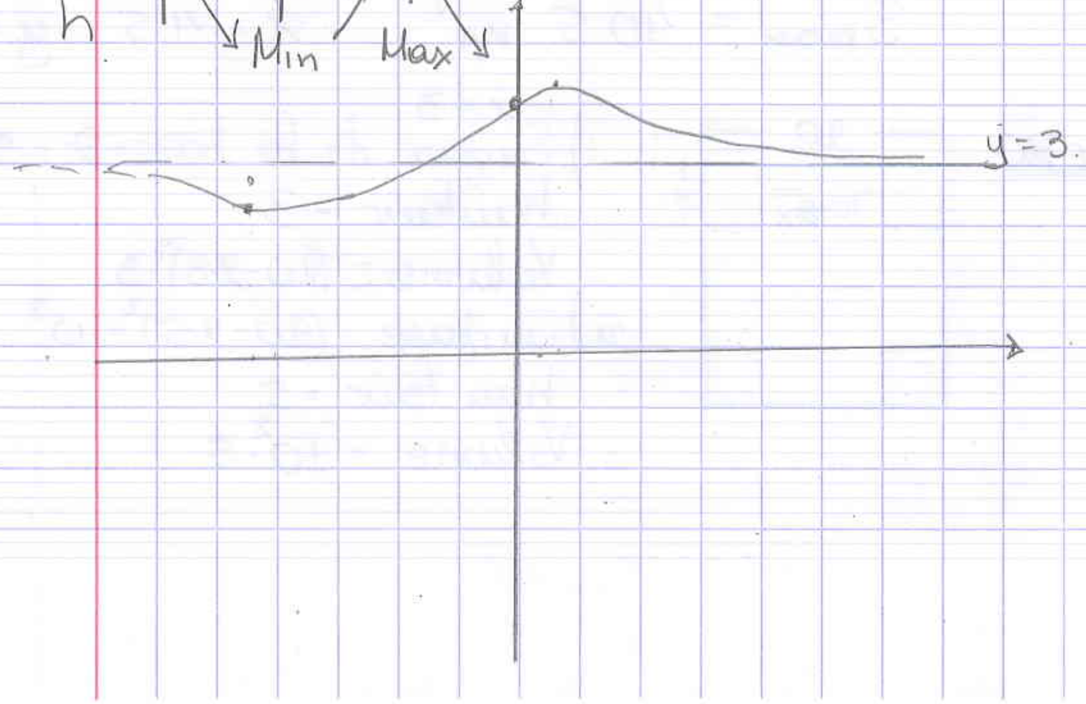
$\Delta = 16 + 4 = 20 \quad x_{1,2} = \dfrac{-4 \pm \sqrt{20}}{2} = \begin{cases} x_1 = -4.24 \\ x_2 = 0.24 \end{cases}$

$f(-4.24) = 2.83$

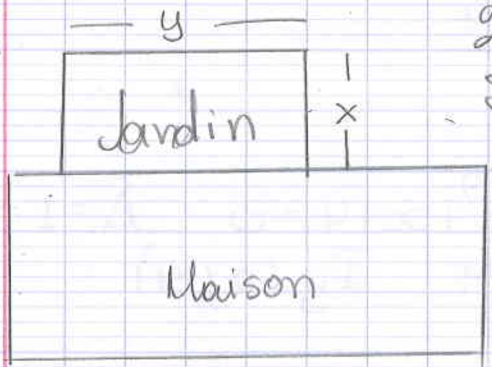
$f(0.24) = 4.17$

<u>T.G</u>

x	$-\infty$		-4.24		0.24		$+\infty$
h'		$-$	0	$+$	0	$-$	
h		$\searrow$	Min	$\nearrow$	Max	$\searrow$	



EXERCICE 19



$2x + y = 18$

$S = x \cdot y$

a) $x = 3 \qquad y = 18 - 6 = 12$

$S = 3 \cdot 12 = 36\ m^2$

b) $x = 4 \qquad y = 18 - 8 = 10$

$S = 4 \cdot 10 = 40\ m^2$

c) $x = 7 \qquad y = 18 - 14 = 4$

$S = 7 \cdot 4 = 28\ m^2$

d) $x \qquad y = 18 - 2x$

$S(x) = x \cdot (18 - 2x) = -2x^2 + 18x$

e) $S'(x) = -4x + 18$

PTH : $S'(x) = 0 \Leftrightarrow -4x + 18 = 0 \Leftrightarrow x = \frac{18}{4}$

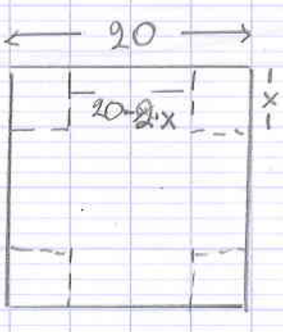
$x = \frac{9}{2} = 4.5$

x	0		4.5		18
S'		+	0	−	
S		↗	Max	↘	

f) $Max = S(4.5) = -2 \cdot 4.5^2 + 18 \cdot 4.5 = $

$S_{max} = 40.5\ m^2 \qquad x = 4.5 \qquad y = 4.5$

EXERCICE 20



$x = 3$

i) Surface de la base $= (20 - 2 \cdot 3)^2 = 14^2$

hauteur $= 3$

Volume $= (20 - 2 \cdot 3)^2 \cdot 3$

ii) Surface $= (20 - 2 \cdot 5)^2 = 10^2$

hauteur $= 5$

Volume $= 10^2 \cdot 5$

iii) $S(x) = (20-2x)^2$

$h(x) = x$

iv) $V(x) = S(x) \cdot h(x) = x \cdot (20-2x)^2$

v) $V'(x) = (20-2x)^2 + x \cdot 2(20-2x) \cdot (-2) =$

$= (20-2x)(20-2x-4x) = (20-2x)(20-6x) =$

$= 2(10-x)\,2(10-3x) = 4(10-x)(10-3x)$

$V'(x) = 0 \Leftrightarrow x = 10$ ou $x = \frac{10}{3}$

TV

x	0		$10/3$		10
V'		$+$	0	$-$	0
V		$\nearrow$	Max	$\searrow$	

$\text{Max} = V\left(\frac{10}{3}\right) = \frac{10}{3} \cdot \left(20 - \frac{20}{3}\right)^2 \triangleq 592.6 \text{ cm}^3$

pour $x = \frac{10}{3}$

EXERCICE 21 a) $y = \dfrac{x-1}{x^2+9x+20}$

$x^2+9x+20 = 0$

$\Delta = 81 - 80 = 1 \quad x_{1,2} = \begin{cases} -4 \\ -5 \end{cases}$

$D_f = \mathbb{R} \setminus \{-4; -5\}$

AV $\lim_{x \to -4} f(x) = \left[\frac{-5}{0}\right] = \infty \Rightarrow$ AV $\underline{x = -4}$

$\lim_{x \to -5} f(x) = \left[\frac{-6}{0}\right] = \infty \Rightarrow$ AV $\underline{x = -5}$

AH/AO: $\lim_{x \to \pm\infty} f(x) = 0 \Rightarrow$ AH $\underline{y = 0}$

b) $y = \dfrac{x^2+9x+20}{x-1}$ $\quad D_f = \mathbb{R} \setminus \{1\}$

AV: $\lim_{x \to 1} f(x) = \left[\frac{30}{0}\right] = \infty \rightarrow$ AV $\underline{x = 1}$

AH/AO:

$$\begin{array}{r|l} x^2+9x+20 & x-1 \\ \cline{2-2} -\cancel{x^2}+x & x+10 \\ \hline 10x+20 & \\ -\cancel{10x}+10 & \\ \hline 30 & \end{array}$$

AO : $\underline{y = x + 10}$

c) $y = \frac{x^3-5x^2+x-9}{x^2-4x+2}$ $x^2-4x+2=0$ $\Delta = 16-8=8$

$x_{1,2} = \frac{4 \pm \sqrt{8}}{2} = \begin{cases} 3.41 \\ 0.586 \end{cases}$

$D_f = \mathbb{R} \setminus \{0.586; 3.41\}$

AV $\lim_{x \to 3.41} f(x) = \left[\frac{52}{0}\right] = \infty$ AV $x = 3.41$ ou $\underline{x = 2+\sqrt{2}}$

$\lim_{x \to 0.586} f(x) = \infty$ AV $\underline{x = 2-\sqrt{2}} \approx 0.586$

AH/AO:

$$\begin{array}{l|l} \cancel{x^3}-5x^2+x-9 & x^2-4x+2 \\ \underline{-\cancel{x^3}+4x^2-2x} & x-1 \\ \quad -\cancel{x^2}-x-9 & \\ \quad \underline{+\cancel{x^2}+4x+2} & \\ \quad\quad 3x-7 & \end{array}$$

AO : $\underline{y = x-1}$

d) $y = \frac{x-1}{(x-2)(x-7)(x+3)}$ $D_f = \mathbb{R} \setminus \{2; 7; -3\}$

AV $\lim_{x \to 2} f(x) = \left[\frac{1}{0}\right] = \infty \Rightarrow$ AV $\underline{x=2}$

$\lim_{x \to 7} f(x) = \left[\frac{6}{0}\right] = \infty \Rightarrow$ AV : $\underline{x=7}$

$\lim_{x \to -3} f(x) = \left[\frac{-4}{0}\right] = \infty \Rightarrow$ AV : $\underline{x=-3}$

AO/AH : $\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x}{x^3} = 0 \Rightarrow$ AH : $\underline{y=0}$

e) $y = \frac{x^2+16}{x-5}$ $D_f = \mathbb{R} \setminus \{5\}$

AV $\lim_{x \to 5} f(x) = \left[\frac{41}{0}\right] = \infty$ AV : $\underline{x=5}$

AH/AO :

$$\begin{array}{l|l} \cancel{x^2}+0 \cdot x+16 & x-5 \\ \underline{-\cancel{x^2}+5x} & x+5 \\ \quad \cancel{5x}+16 & \\ \quad \underline{-\cancel{5x}+25} & \\ \quad\quad 41 & \end{array}$$

AO : $\underline{y = x+5}$

f) $y = 2x + 7 - \frac{1}{x^2 - 25}$ $\qquad D_f = \mathbb{R} \setminus \{\pm 5\}$

<u>AV</u> $\quad \underline{x = 5}$ et $\underline{x = -5}$

<u>AO</u> $\quad \underline{y = 2x + 7}$

<u>EXERCICE 22</u> $\cdot f_1(x) = \frac{3}{x} + \frac{x}{3}$

$f_1'(x) = -\frac{3}{x^2} + \frac{1}{3}$

$\cdot f_2(x) = \frac{5}{x^2}(\sqrt{x} + 1) = 5\overset{u}{x^{-2}}\overset{v}{(\sqrt{x} + 1)}$

$f_2'(x) = 5(-2)x^{-3}(\sqrt{x} + 1) + 5x^{-2}\left(\frac{1}{2\sqrt{x}}\right) =$

$= \frac{-10}{x^3}(\sqrt{x} + 1) + \frac{5}{2x^2\sqrt{x}} = \frac{5}{x^2}\left(\frac{-2(\sqrt{x}+1)}{x} + \frac{1}{2\sqrt{x}}\right)$

$f_3(x) = \sin x \cdot \cos x$

$f_3'(x) = \cos x \cdot \cos x - \sin x \sin x = \cos^2 x - \sin^2 x$

$f_4(x) = \frac{\cos^3 x}{x}$

$f_4'(x) = \frac{3\cos^2 x \cdot (-\sin x) \cdot x - \cos^3 x}{x^2} =$

$= \frac{-\cos^2 x\,(+3x\sin x + \cos x)}{x^2}$

$f_5(x) = \frac{\cos(x^3)}{x}$

$f_5'(x) = \frac{-\sin(x^3) \cdot 3x^2 \cdot x - \cos(x^3)}{x^2} =$

$= \frac{-3x^3 \sin(x^3) - \cos(x^3)}{x^2}$

$f_6(x) = k x^{2n} \cos x$

$f_6'(x) = 2nkx^{2n-1}\cos x - kx^{2n}\sin x =$

$= kx^{2n-1}(2n\cos x - x\sin x)$

<u>EXERCICE 23</u> $\quad y = x^4 + 3x^3 - x^2 \qquad y' = 4x^3 + 9x^2 - 2x$

$y' = 0 \Leftrightarrow x(4x^2 + 9x - 2) = 0$

$\Delta = 81 + 32 = 113 \qquad x_{1,2} = \frac{-9 \pm \sqrt{113}}{8} = \begin{cases} x_1 = -2{,}45 \\ x_2 = 0{,}2 \end{cases}$

x	$-\infty$		-2.45		0		0.2		$+\infty$
x		$-$		$-$	0	$+$		$+$	
$4x^2+9x-2$		$+$	0	$-$		$-$	0	$+$	
f'		$-$	0	$+$	0	$-$	0	$+$	
f		$\searrow$	Min	$\nearrow$	Max	$\searrow$	Min	$\nearrow$	
			$(-2.45; -14.1)$		$(0,0)$		$(0.2; -0.014)$		

$f(-2.45) = -14.1$
$f(0) = 0$
$f(0.2) = -0.014$

tangente: $f(10) = 12900 \quad x = 10 \quad y = 12900$

$f'(10) = 4880 = m$

$y = mx + h \longrightarrow 12900 = 4880 \cdot 10 + h \Rightarrow$

$\Rightarrow h = -35900$

$t: \; y = 4880x - 35900$

EXERCICE 24 a) $y = \frac{1}{3}x^3 + \frac{1}{20}x^2 - \frac{11}{10}x \qquad D_f = \mathbb{R}$

$y' = \frac{1}{3} 3x^2 + \frac{1}{20} 2x - \frac{11}{10} = x^2 + \frac{1}{10}x - \frac{11}{10}$

$y' = 0 \Leftrightarrow 10x^2 + x - 11 = 0$

$\Delta = 1 + 4 \cdot 10 \cdot 11 = 441 \qquad x_{1,2} = \frac{-1 \pm 21}{20} = \begin{cases} 1 \\ \frac{-22}{20} = -\frac{11}{10} \end{cases}$

$S_1\left(1; f(1)\right) = \left(1; -\frac{43}{60}\right)$

$S_2\left(-\frac{11}{10}; f\left(-\frac{11}{10}\right)\right) = \left(-\frac{11}{10}; \frac{4961}{6000}\right) = (-1.1; 0.83)$

b) $y = f(x) = \frac{5x+15}{x^2-4} \qquad D_f = \mathbb{R} \setminus \{\pm 2\}$

$y' = f'(x) = \frac{5 \cdot (x^2-4) - (5x+15)(2x)}{(x^2-4)^2} =$

$= \frac{5x^2 - 20 - 10x^2 - 30x}{(x^2-4)^2} = \frac{-5x^2 - 30x - 20}{(x^2-4)^2}$

$y' = 0 \Leftrightarrow -5x^2 - 30x - 20 = 0 \Leftrightarrow x^2 + 6x + 4 = 0$

$\Delta = 36 - 16 = 20 \qquad x_{1,2} = \frac{-6 \pm \sqrt{20}}{2} \begin{cases} x_1 = -5.24 \\ x_2 = -0.76 \end{cases}$

$S_1(-5.24; +0.48) \qquad S_2 = (-0.76; +3.3)$

c) $y = 2\sin x + x \qquad D_f = \mathbb{R}$

$y' = 2\cos x + 1$

$y' = 0 \Leftrightarrow 2\cos x + 1 = 0 \Leftrightarrow \cos x = -\frac{1}{2} \Leftrightarrow$

$\begin{cases} x = \frac{2\pi}{3} + 2k\pi & ① \\ & k \in \mathbb{Z} \\ x = \frac{4\pi}{3} + 2k\pi & ② \end{cases}$

① $k=0 \quad x_1 = \frac{2\pi}{3} \approx 2.09$ (visible)

$k=1 \quad x_2 = \frac{2\pi}{3} + 2\pi = 8.38$ très grande (pas visible)

$k=-1 \quad x_3 = \frac{2\pi}{3} - 2\pi \approx -4.189$ (visible)

② $k=0 \quad x_1 = \frac{4\pi}{3} \approx 4.19$ (visible)

$k=1 \quad x_2 = \frac{4\pi}{3} + 2\pi \approx 10.5$ (pas visible)

$k=-1 \quad x_3 = \frac{4\pi}{3} - 2\pi \approx -2.09$ (visible

Alors les sommets visibles sont

$S_1\left(\frac{2\pi}{3} ; \sqrt{3} + \frac{2\pi}{3}\right) \quad S_2\left(\frac{2\pi}{3} - 2\pi ; \sqrt{3} - \frac{4\pi}{3}\right)$

$S_3\left(\frac{4\pi}{3} ; \frac{4\pi}{3} - \sqrt{3}\right) \quad S_4\left(\frac{4\pi}{3} - 2\pi ; -\sqrt{3} - \frac{2\pi}{3}\right)$

d) $y = \sin(x^3 - 2x) \qquad D = \mathbb{R}$

$y' = \cos(x^3 - 2x) \cdot (3x^2 - 2)$

$y' = 0 \Leftrightarrow \cos(x^3 - 2x) = 0$

ou $3x^2 - 2 = 0 \Leftrightarrow x^2 = \frac{2}{3} \Leftrightarrow x = \pm\sqrt{\frac{2}{3}}$

$x_1 = \sqrt{\frac{2}{3}} \approx 0.82 \qquad x_2 = -0.82$

$S_1\left(\sqrt{\frac{2}{3}} ; -0.89\right) \quad S_2 = \left(\sqrt{\frac{2}{3}} ; 0.89\right)$

e) $y = \tan(x^2 + 5x - 2)$

$y' = \frac{(2x+5)}{\cos(x^2+5x-2)} = 0 \Leftrightarrow 2x + 5 = 0 \Leftrightarrow x = -\frac{5}{2}$

$S(-2.5 ; 2.4)$

f) $y = \sqrt{\cos x + \frac{x}{2}}$

$y' = \dfrac{\left(-\sin x + \frac{1}{2}\right)}{2\sqrt{\cos x + \frac{x}{2}}} = 0 \Leftrightarrow \sin x = \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x = \frac{\pi}{6} + 2k\pi & (1) \\ x = \pi - \frac{\pi}{6} + 2k\pi = \frac{5\pi}{6} + 2k\pi & (2) \end{cases}$

(1) $k = 0$ $\quad x_1 = \frac{\pi}{6} \simeq 0.52$ (visible)
$k = 1$ $\quad x_2 = \frac{\pi}{6} + 2\pi = \frac{13\pi}{6} \simeq 6,8$ (visible)

(2) $k = 0$ $\quad x_1 = \frac{5\pi}{6} \simeq 2,62$ (visible)
$k = 1$ $\quad x_2 = \frac{5\pi}{6} + 2\pi = \frac{17\pi}{6} \simeq 8,9$ (pas visible)

$S_1\left(\frac{\pi}{6}; 1.062\right)$ $S_2\left(\frac{5\pi}{6}; 0.66\right)$ $S_3\left(\frac{13\pi}{6}; 2.1\right)$

Exercice 25 a) $y = \frac{1}{3}x^3 + \frac{1}{20}x^2 - \frac{11}{10}x$ $\quad D_f = \mathbb{R}$

TS

- $y = 0 \Leftrightarrow x\left(\frac{1}{3}x^2 + \frac{1}{20}x - 11\right) = 0 \Leftrightarrow$

$x = 0$ $\quad \frac{1}{3}x^2 + \frac{1}{20}x - 11 = 0 \Leftrightarrow 200x^2 + 3x - 660 = 0$

$\Delta = 52809$ $\quad x_{1,2} = \dfrac{-3 \pm 229.8}{210}$

$x_1 = -5.82$ $\quad x_2 = 5,67$

x	$-\infty$		-5.82		0		$5,67$		$+\infty$
x		$-$	|	$-$	0	$+$	|	$+$	
$\frac{1}{3}x^2 + \frac{1}{20}x - 11$		$+$	0	$-$	|	$-$	0	$+$	
f		$-$	0	$+$	0	$-$	0	$+$	

TG

- $y' = x^2 + \frac{1}{10}x - \frac{11}{10} = 0 \Leftrightarrow 10x^2 + x - 11 = 0$

$x_1 = 1$ $\quad x_2 = -\frac{11}{10}$

x	$-\infty$		$-11/10$		1		$+\infty$
f'		$+$	0	$-$	0	$+$	
f		$\nearrow$		$\searrow$		$\nearrow$	

$(-1.1; 0.83)$ Max $\quad (1; -0,72)$ Min

T Courbure. $y'' = 2x + \frac{1}{10} = 0 \Leftrightarrow 2x = -\frac{1}{10} \Leftrightarrow x = -\frac{1}{20}$

x	$-\infty$		$-1/20$		$+\infty$
y''		$-$	0	$+$	
y		$\frown$	PI	$\smile$	

PI $\left(-\frac{1}{20} ; 0.055\right)$

b) $y = \dfrac{5x+15}{x^2-4}$ $D_f = \mathbb{R} \setminus \{\pm 2\}$

TS $y = 0 \Leftrightarrow 5x = -15 \Leftrightarrow x = -3$

x	$-\infty$		-3		-2		2		$+\infty$
$5x+15$		$-$	0	$+$		$+$		$+$	
x^2-4		$+$		$+$	0	$-$	0	$+$	
f		$-$	0	$+$	//	$-$	//	$+$	

T Cr $y' = \dfrac{-5x^2-30x-20}{(x^2-4)^2} = 0 \Leftrightarrow$ $x_1 = -5.24$
$x_2 = -0.76$

x	$-\infty$		-5.24		-2		-0.76		2		$+\infty$
N		$-$	0	$+$		$+$	0	$-$		$-$	
D		$+$		$+$	0	$+$		$+$	0	$+$	
y'		$-$	0	$+$	//	$+$	0	$-$	//	$-$	
y		$\searrow$	Min	$\nearrow$		$\nearrow$	Max	$\searrow$		$\searrow$	

Min $(-5.24; -0.48)$ Max $(-0.76; -3.3)$

T.Cour $y'' = \dfrac{10(x^3+9x^2+12x+12)}{(x^2-4)^3} = 0$ $x = -3 - \sqrt[3]{5} - \sqrt[3]{5^2} \approx$
≈ -7.63

x	$-\infty$		-7.63		$+\infty$
y''		$-$	0	$+$	
y		$\frown$	PI	$\smile$	

PI $(-7.63; -0.427)$

c) $y = 2\sin x + x \qquad D_f = \mathbb{R}$

TS $\quad y = 2\sin x + x = 0 \Leftrightarrow x = 0$

x	$-\infty$		0		$+\infty$
f		$-$	0	$+$	

TG $\quad y' = 2\cos x + 1$

$y' = 0 \Leftrightarrow 2\cos x + 1 = 0 \Leftrightarrow \cos x = -1/2$

$$\begin{cases} x = \frac{2\pi}{3} + 2k\pi \\ x = \frac{4\pi}{3} + 2k\pi \end{cases} \qquad k \in \mathbb{Z}$$

x	...	-2.09		2.09	...
y'	$-$	0	$+$	0	$-$
y	$\searrow$	Min	$\nearrow$	Max	$\searrow$

Min $\left(-2.09; -\sqrt{3} - \frac{2\pi}{3}\right)$ $\qquad$ Max $\left(2.09; \sqrt{3} + \frac{2\pi}{3}\right)$

EXERCICE 27 $\quad y = 2x^3 - \frac{x^2}{2} + x \qquad y' = 6x^2 - x + 1$

$y' = 2 \Leftrightarrow 6x^2 - x + 1 = 2 \Leftrightarrow 6x^2 - x - 1 = 0$

$\Delta = 1 + 4 \cdot 6 \cdot 1 = 25. \quad x_{1,2} = \frac{1 \pm 5}{12} \begin{cases} 1/2 \\ -\frac{1}{3} \end{cases}$

$x_1 = \frac{1}{2} \quad y_1 = 0.625 \qquad x_2 = -\frac{1}{3} \quad y_2 = 0.426$

EXERCICE 28 $\quad f(x) = \dfrac{(a+3)x^2 - 4x}{3x^2 - 6x + b}, \; a, b \in \mathbb{R}$

1. · passe par $(2;2)$: $f(2) = 2 \Leftrightarrow \dfrac{4(a+3) - 8}{12 - 12 + b} = 2 \Leftrightarrow$

$\Leftrightarrow \dfrac{4a + 4}{b} = 2 \Leftrightarrow 4a + 4 = 2b \Leftrightarrow 2a - b = -2$

· la pente de la tangente en $x = 2$ est 0

$f'(2) = 0$

$$f'(x) = \frac{(2(a+3)x - 4)(3x^2 - 6x + b) - ((a+3)x^2 - 4x)(6x - 6)}{(3x^2 - 6x + b)^2}$$

$$= \frac{2b((a+3)x - 2) - 6(a+1)x^2}{(3x^2 - 6x + b)^2}$$

$f'(2) = 0 \Leftrightarrow \dfrac{4(a(b-6)+2(b-3))}{b^2} = 0 \Leftrightarrow$

$\Leftrightarrow 4(ab-6a+2b-6) = 0 \Leftrightarrow$

$\Leftrightarrow ab-6a+2b-6=0$

On a donc le système :

$\begin{cases} 2a-b=-2 & (1) \\ ab-6a+2b-6=0 & (2) \end{cases} \Leftrightarrow$

(1) $b = 2a+2$

(2) $a(2a+2)-6a+2(2a+2)-6=0 \Leftrightarrow$

$\Leftrightarrow 2a^2+\cancel{2a}-\cancel{6a}+\cancel{4a}+4-6=0$

$\Leftrightarrow 2a^2-2=0 \Leftrightarrow a^2=1 \Leftrightarrow a=\pm 1$

- $a=1 \quad b=4 \quad / \quad a=-1 \quad b=0$

La solution qui fonctionne est $a=1$, $b=4$

Alors $f(x) = \dfrac{4x^2-4x}{3x^2-6x+4}$ et $f'(x) = -\dfrac{4(3x^2-8x+4)}{(3x^2-6x+4)^2}$

2) $x=1 \quad y=f(1)=0 \quad R(1;0)$

$f'(1) = 4 = m$

$\left.\begin{array}{l} y = mx+h \\ 0 = 4\cdot 1+h \Leftrightarrow h=-4 \end{array}\right\}$ $t: y=4x-4$

EXERCICE 29 $\quad f(x) = ax^3-5x^2+3x+b, \ a;b \in \mathbb{R}$

1. - passe pour $A(3;-1)$: $f(3)=-1 \Leftrightarrow$
   $\Leftrightarrow 27a-45+9+b=-1 \Leftrightarrow 27a+b=35$ (1)
   - $A(3;-1)$ PI : $f''(3)=0$
   $f'(x) = 3ax^2-10x+3$
   $f''(x) = 6ax-10$
   $f''(3)=0 \Leftrightarrow 18a-10=0 \Leftrightarrow a=\frac{10}{18} \Rightarrow a=\frac{5}{9}$
   En replaçant dans (1) on obtient :
   $27\cdot\frac{5}{9}+b=35 \Leftrightarrow 15+b=35 \Leftrightarrow b=20$

2) $f'(x) = \frac{5}{3}x^2-10x+3$

$f'(3) = \frac{5}{\cancel{3}}\cdot \cancel{9}^{3}-30+3 = 15+3-30=-12$

$f(x) = \frac{5}{9}x^3 - 5x^2 + 3x + 20 \quad f(3) = -1$

$\left.\begin{array}{l} y = mx + h \\ -1 = -12 \cdot 3 + h \Leftrightarrow h = 35 \end{array}\right\} t : y = -12x + 35$

EXERCICE 30 $f(x) = \dfrac{ax^2+bx+c}{x+d} \quad a, b, c, d \in \mathbb{R}$

1. AV : $x = 1 \Rightarrow 1 + d = 0 \Leftrightarrow \underline{d = -1}$

AO : $y = 2x$

$$\begin{array}{r|l} ax^2 + bx + c & x-1 \\ \cline{2-2} -\cancel{ax^2} + ax & ax + a + b = y \text{ AO} \\ \cline{1-1} (a+b)x + c & \\ -\cancel{(a+b)}x + a + b & \\ \cline{1-1} a+b+c & \end{array}$$

On compare $y = ax + a + b$ et $y = 2x$

$\underline{a = 2}$ et $2 + b = 0 \Leftrightarrow \underline{b = -2}$

Alors $f(x) = \dfrac{2x^2 - 2x + c}{x-1}$

2. $P(2;0) \in G_f$: $f(2) = 0 \Leftrightarrow \dfrac{8-4+c}{2-1} = 0 \Leftrightarrow$

$\Leftrightarrow 4 + c = 0 \Leftrightarrow \underline{c = -4}$

EXERCICE 31 a) $y = x^2 \quad D_f = \mathbb{R}$

TS $y = 0 \Leftrightarrow x^2 = 0 \Leftrightarrow x = 0$

x	$-\infty$		0		$+\infty$
y		+	0	+	

TGr $y' = 2x = 0 \Leftrightarrow x = 0$

x	$-\infty$		0		$+\infty$
y'		−	0	+	
y		↘	Max (0; 0)	↗	

TCour : $y'' = 2 = 0$ imp

x	$-\infty$		$+\infty$
y''		+ ∪	

b) $y = \frac{1}{3}x - 5$ $\qquad D = \mathbb{R}$

TS $\quad y = 0 \Leftrightarrow \frac{1}{3}x - 5 = 0 \Leftrightarrow \frac{1}{3}x = 5 \Leftrightarrow x = 15$

x	$-\infty$		15		$+\infty$
y		$-$	0	$+$	

TGr $\quad y' = \frac{1}{3} = 0$ imp

x	$-\infty$		$+\infty$
y'		$+$	

TCour : $\quad y'' = 0.$

c) $y = -x^2 + 1$ $\qquad D = \mathbb{R}$

TS $\quad y = 0 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$

x	$-\infty$		-1		1		$+\infty$
y		$-$	0	$+$	0	$-$	

TGr $\quad y' = -2x = 0 \Leftrightarrow x = 0.$

$f(0) = 1$

x	$-\infty$		0		$+\infty$
y'		$+$	0	$-$	
y		↗	Max $(0;1)$	↘	

TCour : $\quad y'' = -2 = 0$ imp

x	$-\infty$		$+\infty$
y''		$-$	
y		∩	

d) $y = \frac{2}{-3x}$ $\qquad D = \mathbb{R} \setminus \{0\}$

x	$-\infty$		0		$+\infty$
y		$+$	||	$-$	

TS $\quad y = 0 \Rightarrow 2 = 0$ imp

TGr $\quad y' = -\frac{2}{3}\left(-\frac{1}{x^2}\right) = \frac{2}{3x^2} = 0$ imp

x	$-\infty$		0		$+\infty$
y'		$+$	||	$+$	
y		↗	||	↗	

Tcour $\quad y'' = \frac{2}{3} \cdot \left(-\frac{2}{x^3}\right) = -\frac{4}{3x^3} = 0$ imp.

x	$-\infty$		0		$+\infty$
y''		$+$	||	$-$	
y		∪	||	∩	

e) $y = 2x^3 + 5x^2 - x - 6$ $\qquad D = \mathbb{R}$

TS $\quad$ Pour $x = 1 \quad y = 0$

Horner :

2	5	-1	-6	1
↓	2	7	6	
2	7	6	0	

Donc $y=(x-1)(2x^2+7x+6)=0$

$\Delta = 49-4\cdot 2\cdot 6 = 1$ $\quad x_{1,2} = \frac{-7\pm 1}{4} = \left\langle \begin{array}{l} -2 \\ -\frac{6}{4} = -\frac{3}{2} \end{array}\right.$

x	$-\infty$		-2		$-3/2$		1		$+\infty$
$x-1$		$-$	|	$-$	|	$-$	0	$+$	
$2x^2+7x+6$		$+$	0	$-$	0	$+$	|	$+$	
y		$-$	0	$+$	0	$-$	0	$+$	

TG $\quad y' = 6x^2+10x-1$

$y'=0 \Leftrightarrow 6x^2+10x-1=0$

$\Delta = 100+24 = 124$

$x_{1,2} = \frac{-10\pm\sqrt{124}}{12} = \left\langle \begin{array}{l} -1.76 \\ 0.095 \end{array}\right.$

x	$-\infty$		-1.76		0.095		$+\infty$
y'		$+$	0	$-$	0	$+$	
y		$\nearrow$	Max	$\searrow$	Min	$\nearrow$	

TCour: $y'' = 12x+10 = 0 \Leftrightarrow x = -\frac{10}{12} \Rightarrow x = -\frac{5}{6}$

x	$-\infty$		$-5/6$		$+\infty$
y''		$-$	0	$+$	
y		$\frown$	PI	$\smile$	

d) $y = x^6-3x^4 \qquad D=\mathbb{R}$

TS $\quad y=0 \Leftrightarrow x^4(x^2-3)=0 \Leftrightarrow x_1=0 \quad x_2=\sqrt{3} \quad x_3=-\sqrt{3}$

x	$-\infty$		$-\sqrt{3}$		0		$\sqrt{3}$		$+\infty$
x^2		$+$	|	$+$	0	$+$	|	$+$	
x^2-3		$+$	0	$-$	|	$-$	0	$+$	
y		$+$	0	$-$	0	$-$	0	$+$	

TG $\quad y' = 6x^5-12x^3 = 6x^3(x^2-2)=0 \Leftrightarrow$

$x=0 \quad x=-\sqrt{2} \quad x=\sqrt{2}$

x	$-\infty$		$-\sqrt{2}$		0		$+\sqrt{2}$		$+\infty$
x^3		$-$		$-$	0	$+$		$+$	
x^2-2		$+$	0	$-$		$-$	0	$+$	
y'		$-$	0	$+$	0	$-$	0	$+$	
y		$\searrow$	Min	$\nearrow$	Max	$\searrow$	Min	$\nearrow$	

TCour $y'' = 30x^4 - 36x^2 = 6x^2(5x^2-6) = 0 \Leftrightarrow$

$x = 0 \quad x^2 = \frac{6}{5} \Leftrightarrow x = \pm\sqrt{\frac{6}{5}}$

x	$-\infty$		$-\sqrt{\frac{6}{5}}$		0		$\sqrt{\frac{6}{5}}$		$+\infty$
x^2		$+$		$+$	0	$+$		$+$	
$5x^2-6$		$+$	0	$-$		$-$	0	$+$	
y''		$+$	0	$-$	0	$-$	0	$+$	
y		$\cup$	PI	$\cap$		$\cap$	PI	$\cup$	

g) $y = x^3 + 4$ $\qquad D = \mathbb{R}$

TS $\quad y = 0 \Leftrightarrow x^3 = -4 \Leftrightarrow x = -\sqrt[3]{4} \approx -1.6$

x	$-\infty$		-1.6		$+\infty$
y		$-$	0	$+$	

TG $\quad y' = 3x^2 = 0 \Leftrightarrow x = 0$

	$-\infty$		0		$+\infty$
y'		$+$	0	$+$	
y		$\nearrow$	PI	$\nearrow$	

TCour $\quad y'' = 6x = 0 \Leftrightarrow x = 0$

x	$-\infty$		0		$+\infty$
y''		$-$	0	$+$	
y		$\cap$	PI	$\cup$	

h) $y = \frac{x+3}{x-4}$ $\qquad D = \mathbb{R} \setminus \{4\}$

TS $\quad y = 0 \Leftrightarrow x = -3$

x	$-\infty$		-3		4		$+\infty$
$x+3$		$-$	0	$+$		$+$	
$x-4$		$-$		$-$	0	$+$	
y		$+$	0	$-$	$\|$	$+$	

TG $y' = \dfrac{(x-4)-(x+3)}{(x-4)^2} = \dfrac{x-4-x-3}{(x-4)^2} = \dfrac{-7}{(x-4)^2}$

$y' = 0$ impossible

x	$-\infty$	4	$+\infty$
y'	$-$	//	$-$
y	$\searrow$	//	$\searrow$

TConv $y'' = \dfrac{-(-7)\cdot 2(x-4)}{(x-4)^4} = \dfrac{14\cancel{(x-4)}}{(x-4)^{\cancel{4}\,3}} = \dfrac{14}{(x-4)^3}$

$y'' = 0$ impossible

x	$-\infty$	4	$+\infty$
y''	$-$	//	$+$
y	$\cap$	//	$\cup$

EXERCICE 32 $f(x) = \frac{1}{3}x^3 - 3x^2 + 15$ $D = \mathbb{R}$

1) $f'(x) = x^2 - 6x$

$f'(x) = 0 \Leftrightarrow x(x-6) = 0 \Leftrightarrow x = 0 \quad x = 6$

x	$-\infty$		0		6		$+\infty$
f'		$+$	0	$-$	0	$+$	
f		$\nearrow$	Max $(0;15)$	$\searrow$	Min $(6;-21)$	$\nearrow$	

2) On cherche le PI

$f''(x) = 2x - 6 = 0 \Leftrightarrow x = 3$

$\left.\begin{array}{l} PI : (\overset{x}{3} ; \overset{y}{-3}) \\ f'(3) = -9 = m \end{array}\right\}$ $\begin{array}{l} -3 = -9\cdot 3 + h \Rightarrow h = 24 \\ t : y = -9x + 24 \end{array}$

EXERCICE 33 1. $x + y = 11 \Rightarrow y = 11 - x$

$P = x^3y^2 = x^3(11-x)^2$

$P'(x) = 3x^2(11-x)^2 + x^3\cdot 2(11-x)(-1) =$

$= x^2(11-x)[3(11-x) - 2x] =$

$= x^2(11-x)(33 - 3x - 2x) =$

$= x^2(11-x)(33-5x)$

$P'(x) = 0 \Rightarrow x = 0 \quad x = 11 \quad x = \frac{33}{5}$

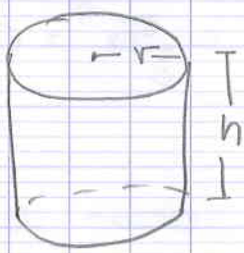
x	0		$33/5$		11		$+\infty$
x^2	0	$+$		$+$		$+$	
$11-x$		$+$		$+$	0	$-$	
$33-5x$		$+$	0	$-$		$-$	
P'	0	$+$	0	$-$	0	$+$	
P		$\nearrow$	Max	$\searrow$		$\nearrow$	

Alors pour $x = \frac{33}{5}$ on a P_{max}

et $y = 11 - \frac{33}{5} = \frac{55-33}{5} \Rightarrow$

$y = \frac{22}{5}$

2.



Aire totale $= 2\pi r \cdot h + 2\pi r^2$ (1)

$V = \pi r^2 \cdot h = 1000 \Rightarrow h = \frac{1000}{\pi r^2}$

(1) $A(r) = 2\cancel{\pi}\cancel{r} \cdot \frac{1000}{\cancel{\pi} r^{\cancel{2}}} + 2\pi r^2 \Rightarrow$

$\Rightarrow A(r) = \frac{2000}{r} + 2\pi r^2$

$A'(r) = -\frac{2000}{r^2} + 4\pi r = 0 \Leftrightarrow$

$\Leftrightarrow -2000 + 4\pi r^3 = 0 \Leftrightarrow 4\pi r^3 = 2000 \Leftrightarrow$

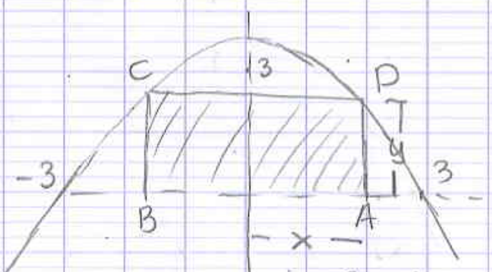
$\Leftrightarrow r^3 = 159.15 \Rightarrow r = 5.42$

r	0		5.42		$+\infty$
A'		$-$	0	$+$	
A		$\searrow$	Min	$\nearrow$	

$r = 5.42$ cm

$h = \frac{1000}{\pi \cdot 5.42^2} \approx 10.83$ cm

EXERCICE 34 $\quad y = -\frac{4}{9}x^2 + 4$



ii) $A(x;0) \quad B(-x;0) \quad C(-x; -\frac{4}{9}x^2+4), D(x; -\frac{4}{9}x^2+4)$

iii) longueur $= 2x$. largeur: $y = -\frac{4}{9}x^2 + 4$

$p(x) = 4x + 2 \cdot (-\frac{4}{9}x^2 + 4) = 4x - \frac{8}{9}x^2 + 8$

$p(x) = -\frac{8}{9}x^2+4x+8$ $\quad p'(x) = -\frac{16}{9}x+4$

$p'(x)=0 \Leftrightarrow +\frac{16x}{9} = 4 \Leftrightarrow x = \frac{4\cdot 9}{16} \Rightarrow x = \frac{9}{4}$

x	0		$9/4$		3
p'		$+$	0	$-$	
p		$\nearrow$	Max	$\searrow$	

Max pour $x = \frac{9}{4}$

IV) $A(x) = 2x\cdot y = 2x\left(-\frac{4}{9}x^2+4\right) \Rightarrow$

$A(x) = -\frac{8}{9}x^3+6x$

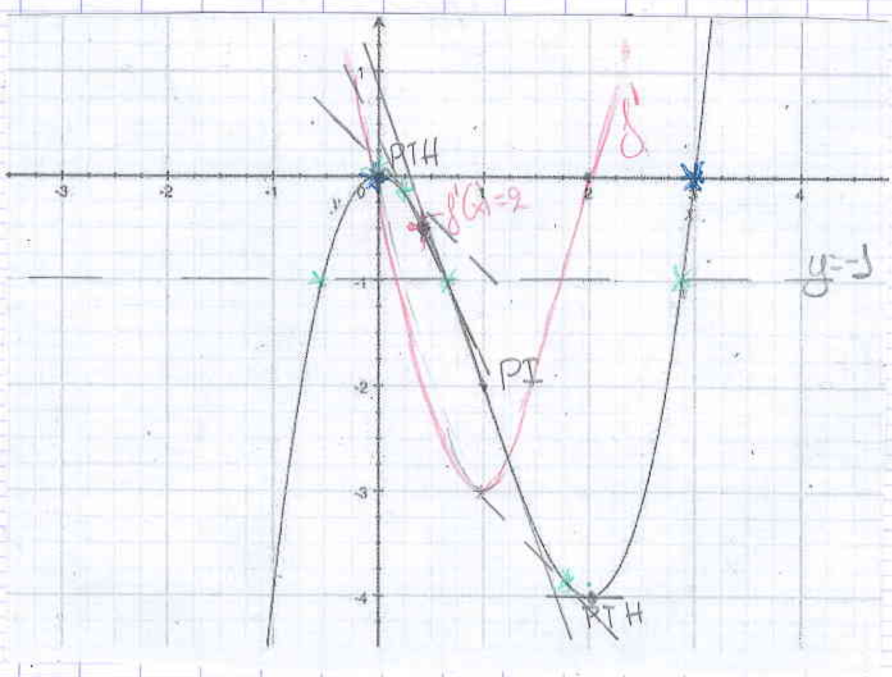
$A'(x) = -\frac{8}{3}x^2+6 = 0 \Leftrightarrow \frac{8}{3}x^2 = 6 \Leftrightarrow x^2 = \frac{18}{8} \Leftrightarrow$

$x^2 = \frac{9}{4} \Rightarrow x = \pm\frac{3}{2}$

	0		$3/2$		$+\infty$
$A'(x)$		$+$	0	$-$	
$A(x)$		$\nearrow$	Max	$\searrow$	

Max pour $x = \frac{3}{2}$

EXERCICE 35



a) PTH: $(0;0)$ $\quad (2;-4)$

$P'(x) = 0 \Rightarrow x = 0 \quad x = 11 \quad x = \frac{33}{5}$

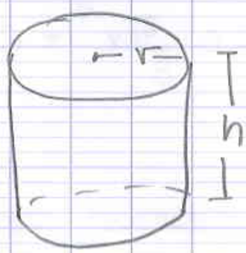
x	0		$33/5$		11		$+\infty$
x^2	0	$+$		$+$		$+$	
$11-x$		$+$		$+$	0	$-$	
$33-5x$		$+$	0	$-$		$-$	
P'	0	$+$	0	$-$	0	$+$	
P		$\nearrow$	Max	$\searrow$		$\nearrow$	

Alors pour $x = \frac{33}{5}$ on a P_{max}

et $y = 11 - \frac{33}{5} = \frac{55-33}{5} \Rightarrow$

$y = \frac{22}{5}$

2.



Aire totale $= 2\pi r \cdot h + 2\pi r^2$ (1)

$V = \pi r^2 \cdot h = 1000 \Rightarrow h = \frac{1000}{\pi r^2}$

(1) $A(r) = 2\cancel{\pi}\cancel{r} \cdot \frac{1000}{\cancel{\pi} r^{\cancel{2}}} + 2\pi r^2 \Rightarrow$

$\Rightarrow A(r) = \frac{2000}{r} + 2\pi r^2$

$A'(r) = -\frac{2000}{r^2} + 4\pi r = 0 \Leftrightarrow$

$\Leftrightarrow -2000 + 4\pi r^3 = 0 \Leftrightarrow 4\pi r^3 = 2000 \Leftrightarrow$

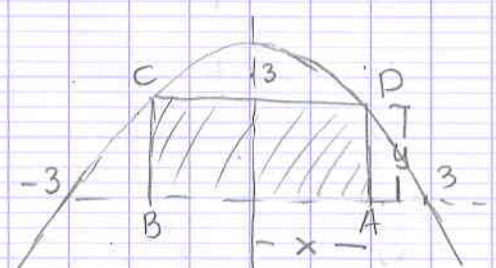
$\Leftrightarrow r^3 = 159.15 \Rightarrow r = 5.42$

r	0		5.42		$+\infty$
A'		$-$	0	$+$	
A		$\searrow$	Min	$\nearrow$	

$r = 5.42$ cm

$h = \frac{1000}{\pi \cdot 5.42^2} \approx 10.83$ cm

EXERCICE 34 $\quad y = -\frac{4}{9}x^2 + 4$



ii) $A(x;0)$ $\quad B(-x;0)$ $\quad C\left(-x; -\frac{4}{9}x^2+4\right)$, $D\left(x; -\frac{4}{9}x^2+4\right)$

iii) longueur $= 2x$. $\quad$ largeur : $y = -\frac{4}{9}x^2 + 4$

$p(x) = 4x + 2 \cdot \left(-\frac{4}{9}x^2 + 4\right) = 4x - \frac{8}{9}x^2 + 8$

b) TG

x	$-\infty$		0		2		$+\infty$
f'		$+$	0	$-$	0	$+$	
f		$\nearrow$	Max	$\searrow$	Min	$\nearrow$	

TCour

x	$-\infty$		1		$+\infty$
f''		$-$	0	$+$	
f		$\frown$	PI $(1;-2)$	$\smile$	

c) $f'(1) \simeq -3$ = la pente de la tangente en $x=1$.

d) $f(x) = -1$: 3 solutions ; les points d'intersection du graphe avec la droite $y=-1$

e) $f'(x) = -1$: 2 solutions : les points où la pente de la tangente est -1