

EXERCICE 1

$$\pi: \vec{n} = \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} \quad A(2; -3; 1)$$

a) $3x + y - 4z + d = 0 \quad A: 6 - 3 - 4 + d = 0 \Rightarrow d = 1$

$$\pi: 3x + y - 4z + 1 = 0$$

b) $D(k; k+1; 3) \in \pi: 3k + k+1 - 12 + 1 = 0 \Rightarrow$
 $\Rightarrow 4k - 10 = 0 \Rightarrow k = \frac{5}{2}$

c) $\vec{n}_\pi = \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} \quad \vec{n}_{\text{sol}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$\cos \alpha = \frac{|\vec{n}_\pi \cdot \vec{n}_s|}{\|\vec{n}_\pi\| \cdot \|\vec{n}_s\|} = \frac{4}{\sqrt{26}} \Rightarrow$$

$$\Rightarrow \alpha = 38.33^\circ$$

$$\vec{n}_\pi \cdot \vec{n}_s = -4$$

$$\|\vec{n}_\pi\| = \sqrt{9+1+16} = \sqrt{26}$$

$$\|\vec{n}_s\| = 1$$

EXERCICE 2

$$d: \begin{cases} x = 1 + \lambda \\ y = 3 - 2\lambda \\ z = 3 + 3\lambda \end{cases}$$

$$\pi: 4x - 3y + 6z - 24 = 0$$

1) a) $T_s: z = 0 \Rightarrow \lambda = -1 \quad T_s(0; 5; 0) = T_m$

$$T_p: y = 0 \quad \lambda = \frac{3}{2} \quad T_p\left(\frac{5}{2}; 0; \frac{15}{2}\right)$$

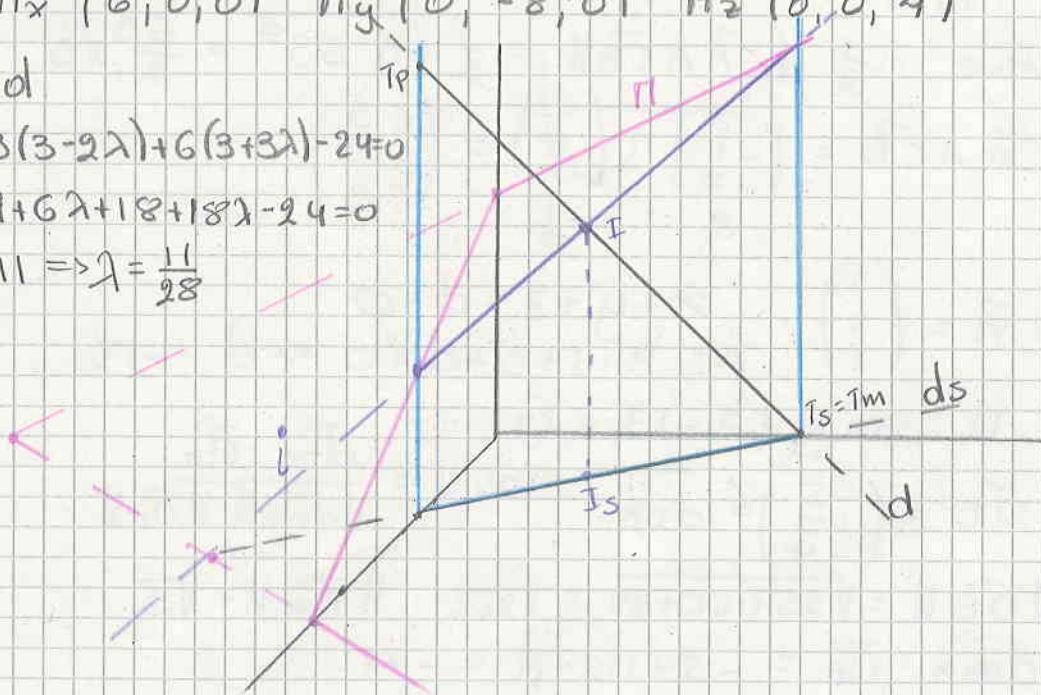
$$\pi: M_x(6; 0; 0) \quad M_y(0; -8; 0) \quad M_z(0; 0; 4)$$

2) $I \in \pi \cap d$

$$4(1+\lambda) - 3(3-2\lambda) + 6(3+3\lambda) - 24 = 0$$

$$\Rightarrow 4 + 4\lambda - 9 + 6\lambda + 18 + 18\lambda - 24 = 0$$

$$\Rightarrow 28\lambda = 11 \Rightarrow \lambda = \frac{11}{28}$$



$$I: x = 1 + \frac{11}{28} = \frac{39}{28} \quad y = 3 - \frac{22}{28} = \frac{62}{28} = \frac{31}{14} \quad z = 3 + \frac{33}{28} = \frac{117}{28}$$

$$I\left(\frac{39}{28}; \frac{31}{14}; \frac{117}{28}\right)$$

EXERCICE 3

$$\pi_1: 2x - y + 3z - 15 = 0$$

$$\pi_2: -4x + 2y - 6z - 7 = 0$$

$$a) \vec{n}_1 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} // \vec{n}_2 = \begin{pmatrix} -4 \\ 2 \\ -6 \end{pmatrix} // \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \Rightarrow \pi_1 // \pi_2$$

$$b) \text{ Soit } A \in \pi_1, \quad x = y = 0 \quad 3z = 15 \Rightarrow z = 5$$

$$A(0; 0; 5)$$

$$\text{dist}(\pi_1, \pi_2) = \text{dist}(A, \pi_2) = \frac{|-4 \cdot 0 + 2 \cdot 0 - 6 \cdot 5 - 7|}{\sqrt{16 + 4 + 36}} = \frac{37}{\sqrt{56}}$$

EXERCICE 4

$$A(9; 1; 3) \quad B(0; 10; 3) \quad C(1; 2; 10) \quad D(5; 0; 12)$$

$$a) \vec{AB} = \begin{pmatrix} -9 \\ 9 \\ 0 \end{pmatrix} \quad \|\vec{AB}\| = \sqrt{2 \cdot 81} = 9\sqrt{2}$$

$$\vec{AC} = \begin{pmatrix} -8 \\ 1 \\ 7 \end{pmatrix} \quad \|\vec{AC}\| = \sqrt{64 + 1 + 49} = \sqrt{114}$$

$$\vec{BC} = \begin{pmatrix} 1 \\ -9 \\ 7 \end{pmatrix} \quad \|\vec{BC}\| = \sqrt{114}$$

$$\|\vec{AC}\| = \|\vec{BC}\| \Rightarrow \text{isocèle}$$

$$\cos \hat{C} = \frac{\vec{CA} \cdot \vec{CB}}{\|\vec{CA}\| \cdot \|\vec{CB}\|}$$

$$\vec{CA} = \begin{pmatrix} 8 \\ 1 \\ -7 \end{pmatrix} \quad \vec{CB} = \begin{pmatrix} -1 \\ 8 \\ -7 \end{pmatrix}$$

$$\vec{CA} \cdot \vec{CB} = 8 - 8 + 49 = 33$$

$$= \frac{33}{114} \Rightarrow \hat{C} = 73,17^\circ \Rightarrow \hat{A} = \hat{B} = \frac{180 - 73,17}{2} = 53,41^\circ$$

$$b) \text{ Aire} = \frac{1}{2} \|\vec{CA} \wedge \vec{CB}\| = \frac{1}{2} \sqrt{3 \cdot 63^2} = \frac{63}{2} \sqrt{3}$$

$$\vec{CA} \wedge \vec{CB} = \begin{pmatrix} 8 \\ -1 \\ -7 \\ 8 \\ -1 \end{pmatrix} \wedge \begin{pmatrix} -1 \\ 8 \\ -7 \\ -1 \\ 8 \end{pmatrix} = \begin{pmatrix} 63 \\ 63 \\ 63 \end{pmatrix}$$

$$c) \vec{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad x + y + z + d = 0$$

$$A: 9 + 1 + 3 + d = 0 \Rightarrow d = -13$$

$$\pi: x + y + z - 13 = 0$$

$$d) \vec{DB} = \begin{pmatrix} -5 \\ 10 \\ -9 \end{pmatrix} = \vec{d}_{DB} \quad \cos \varphi = \frac{\vec{d}_{DB} \cdot \vec{n}_\pi}{\|\vec{d}_{DB}\| \cdot \|\vec{n}_\pi\|}$$

$$\|\vec{DB}\| = \sqrt{25 + 100 + 81} = \sqrt{206} \quad \|\vec{n}_\pi\| = \sqrt{3}$$

$$\vec{d}_{DB} \cdot \vec{n}_\pi = -5 + 10 - 9 = -4$$

$$\cos \varphi = \frac{1 - 41}{\sqrt{206} \cdot \sqrt{3}} \Rightarrow \varphi = 80,74^\circ \Rightarrow \alpha = 90 - \varphi \Rightarrow \alpha = 9,26^\circ$$

$$e) \text{dist}(D, \pi) = \frac{|5 + 12 - 13|}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

$$f) V = \frac{1}{3} \cdot \frac{4}{\sqrt{3}} \cdot \frac{63 \cdot \sqrt{3}}{2} = 42$$