LDDR - Niveau 1 : TE 15 GEOMETRIE 3D.

Exercice 1

$d_1 : A_1(1;2;3) \quad \vec{d_1} = \begin{pmatrix} -12 \\ 6 \\ 15 \end{pmatrix} \qquad d_2 : A_2(3;3;7) \quad \vec{d_2} = \begin{pmatrix} 8 \\ -4 \\ -10 \end{pmatrix}$

a) $\vec{d_1} \parallel \begin{pmatrix} -4 \\ 2 \\ 5 \end{pmatrix} \parallel \vec{d_2} \parallel \begin{pmatrix} 4 \\ -2 \\ -5 \end{pmatrix} \Rightarrow \begin{cases} d_1 \parallel d_2 \\ \text{confondues} \end{cases}$

$A_1 \in d_1$ On regarde si $A_1 \in d_2$ $\begin{cases} \text{oui} \Rightarrow \text{confondues} \\ \text{Non} \Rightarrow d_1 \parallel d_2 \end{cases}$

$? A_1 \in d_2 : \quad d_2 = \begin{cases} x = 3 + 8\lambda \\ y = 3 - 4\lambda \\ z = 7 - 10\lambda \end{cases}$

$1 = 3 + 8\lambda \Rightarrow 8\lambda = -2 \Rightarrow \lambda = -1/4$

$2 = 3 - 4\lambda \Rightarrow 4\lambda = 1 \Rightarrow \lambda = 1/4$

Alors $A \notin d_2 \Rightarrow d_1 \parallel d_2$

b) $\left.\begin{matrix} \begin{pmatrix} 4 \\ -2 \\ -5 \end{pmatrix} \parallel \pi \Rightarrow \begin{pmatrix} 4 \\ -2 \\ -5 \end{pmatrix} \perp \vec{n_\pi} \\ \vec{A_1A_2} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \parallel \pi \Rightarrow \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \perp \vec{n_\pi} \end{matrix}\right\} \Rightarrow \vec{n_\pi} = \begin{pmatrix} 4 \\ -2 \\ -5 \end{pmatrix} \wedge \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -3 \\ -26 \\ 8 \end{pmatrix}$

$\begin{matrix} 4 & 2 \\ -2 & 1 \end{matrix}$

$\pi : 3x + 26y - 8z + d = 0$

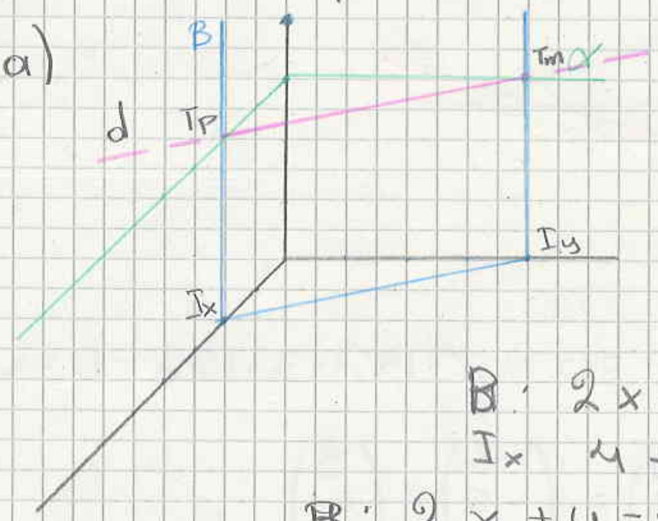
$A_1 : 3 + 52 - 24 + d = 0 \Rightarrow d = -31$

$\pi : 3x + 26y - 8z - 31 = 0$

Exercice 2

$\alpha : z = 3 \qquad \beta \parallel Oz \qquad I_x(2;0;0) \quad I_y(0;4;0)$

a)



b) $\left.\begin{matrix} \vec{I_xI_y} = \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \parallel \beta \\ Oz \parallel \beta \Rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \parallel \beta \end{matrix}\right\} \Rightarrow$

$\vec{n_\beta} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix} \parallel \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

$\begin{matrix} 1 & 0 \\ -2 & 0 \end{matrix}$

$\beta : 2x + y + d = 0$

$I_x \quad 4 + d = 0 \Rightarrow d = -4$

$\beta : 2x + y - 4 = 0$

c) $\vec{d} = \vec{I_xI_y} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$

On cherche le point Tp $y = 0$ $\quad \begin{matrix} z = 3 \\ 2x = 4 \end{matrix} \Rightarrow \begin{matrix} z = 3 \\ x = 2 \end{matrix}$

$T_p(2;0;3)$

$d \begin{cases} x = 2 + \lambda \\ y = -2\lambda \\ z = 3 \end{cases}$

EXERCICE 3.

$\alpha : 2x+y+3z-6=0 \qquad d : A(2;2;1)\ B(\frac{1}{2};4;3)$

a) $d : \begin{cases} x = 2-3\lambda \\ y = 2+4\lambda \\ z = 1+4\lambda \end{cases} \qquad \vec{AB} = \begin{pmatrix} -3/2 \\ 2 \\ 2 \end{pmatrix} // \begin{pmatrix} -3 \\ 4 \\ 4 \end{pmatrix}$

$d \cap mur \quad x=0 \Rightarrow 2=3\lambda \Rightarrow \lambda = 2/3 \quad T_m\left(0;\frac{14}{3};\frac{11}{3}\right)$

$d \cap paroi \quad y=0 \Rightarrow 2=-4\lambda \Rightarrow \lambda = -1/2 \quad T_p\left(\frac{7}{2};0;-1\right)$

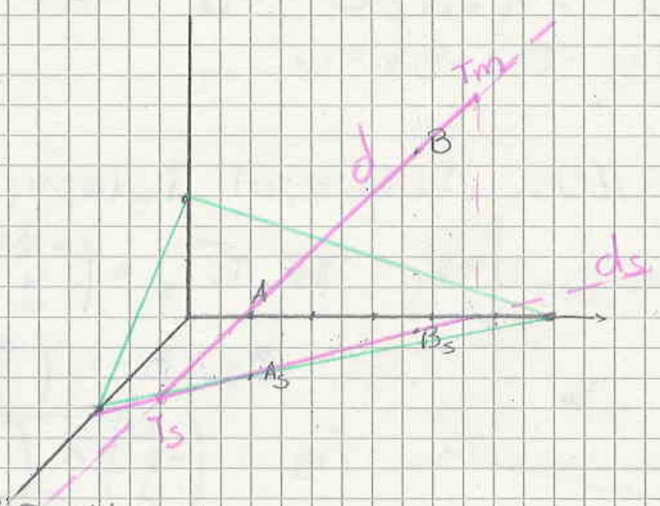
$d \cap sol \quad z=0 \Rightarrow \lambda = -1/4 \qquad T_s\left(\frac{11}{4};1;0\right)$

$d \cap \alpha : \quad 2(2-3\lambda)+2+4\lambda+3(1+4\lambda)-6=0$

$4-6\lambda+2+4\lambda+3+12\lambda-6=0 \Rightarrow 10\lambda=-3 \Rightarrow \lambda = -\frac{3}{10}$

$I\left(\frac{29}{10};\frac{4}{5};-\frac{1}{5}\right)$

b) $\alpha_x\ (3;0;0) \quad \alpha_y\ (0;6;0) \quad \alpha_z\ (0;0;2)$



c) $\beta : 4x+3z-11=0$

$4(2-3\lambda)+3(1+4\lambda)-11=0 \Rightarrow 8-\cancel{12\lambda}+3+\cancel{12\lambda}-11=0$

$0\cdot\lambda = 0 \Rightarrow d \in \beta$

$\vec{i} = \vec{n}_\beta \wedge \vec{n}_\alpha = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} \wedge \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -6 \\ 4 \end{pmatrix} // \begin{pmatrix} 3 \\ 6 \\ -4 \end{pmatrix}$

$\begin{matrix} 4 & 2 \\ 0 & 1 \end{matrix}$

sol: $z=0 \quad \alpha : 2x+y=6 \Rightarrow \frac{11}{2}+y=6 \Rightarrow y=\frac{1}{2}$

$\beta : 4x=11 \Rightarrow x=\frac{11}{4}$

$i = \begin{cases} x = \frac{11}{4} + 3\lambda \\ y = \frac{1}{2} + 6\lambda \\ z = 0 - 4\lambda \end{cases}$