LDDR Niveau 1 : TE S. GEOMETRIE PLANE

Exercice 1 $\vec{a} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ $\vec{c} = \begin{pmatrix} x \\ 3 \end{pmatrix}$

1) $\vec{a}\cdot\vec{b} = -10-4 = -14$ $\|\vec{a}\| = \sqrt{(-2)^2+4^2} = \sqrt{20}$ $\|\vec{b}\| = \sqrt{25+1} = \sqrt{26}$

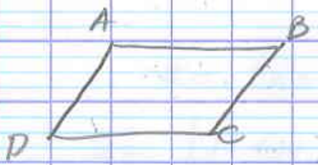
2) $\vec{u_a} = \begin{pmatrix} -2/\sqrt{20} \\ 4/\sqrt{20} \end{pmatrix}$ $\vec{u_b} = \begin{pmatrix} 5/\sqrt{26} \\ -1/\sqrt{26} \end{pmatrix}$ forment une base car $\vec{u_a} \nparallel \vec{u_b}$

mais $\vec{u_a} \not\perp \vec{u_b}$ alors n'est pas orthonormée

3) $(\vec{a}-\vec{b}) \perp \vec{c}$ $\vec{a}-\vec{b} = \begin{pmatrix} -7 \\ 5 \end{pmatrix}$

$(\vec{a}-\vec{b})\cdot\vec{c} = 0 \Rightarrow -7\cdot x + 5\cdot 3 = 0 \Rightarrow 15 = 7x \Rightarrow x = \frac{15}{7}$

Exercice 2



$A(2;-3)$ $B(1;6)$ $C(-3;7)$

1) $\vec{AB} = \vec{DC} \Rightarrow \begin{pmatrix} -1 \\ 9 \end{pmatrix} = \begin{pmatrix} -3-x \\ 7-y \end{pmatrix} \Rightarrow$

$\Rightarrow x = -2$

$y = -2$ $D(-2;-2)$

2) d_{AB}? $\vec{AB} = \begin{pmatrix} -1 \\ 9 \end{pmatrix} \Rightarrow \vec{n} = \begin{pmatrix} 9 \\ 1 \end{pmatrix}$ $9x+y+c=0$

A : $18-3+c = 0 \Rightarrow c = -15$

d_{AB} : $9x+y-15=0$

3) $\det(\vec{AB},\vec{AD}) = \begin{vmatrix} -1 & -4 \\ 9 & 1 \end{vmatrix} = -1+36 = 35$

$Aire = |35| = 35$

Exercice 3 $d : x-y+4=0$ $A(-2;-2)$ $B(8;2)$

Soit $P(x,y) \in d$: $x-y+4=0 \Rightarrow y = x+4$

$P(x;x+4)$

$dist(A,P) = dist(P,B) \Rightarrow \|\vec{AP}\| = \|\vec{BP}\| \Rightarrow$

$\Rightarrow \left\|\begin{pmatrix} x+2 \\ x+4+2 \end{pmatrix}\right\| = \left\|\begin{pmatrix} x-8 \\ x+4-2 \end{pmatrix}\right\| \Rightarrow \left\|\begin{pmatrix} x+2 \\ x+6 \end{pmatrix}\right\| = \left\|\begin{pmatrix} x-8 \\ x+2 \end{pmatrix}\right\|$

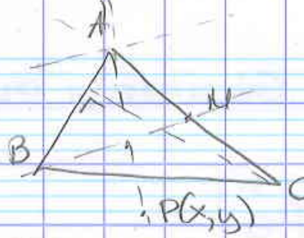
$\Rightarrow \sqrt{(x+2)^2+(x+6)^2} = \sqrt{(x-8)^2+(x+2)^2} \Rightarrow$

$\Rightarrow \cancel{x^2}+4x+4+\cancel{x^2}+12x+36 = \cancel{x^2}-16x+64+\cancel{x^2}+4x+4 \Rightarrow$

$\Rightarrow 16x+40 = -12x+68 \Rightarrow$

$\Rightarrow 28x = 28 \Rightarrow x = 1$ $y = 5$ $P(1;5)$

EXERCICE 4



$A(-5;4)$ $B(-3;-2)$ $C(7;8)$

1) Soit $P \in$ bissectrice

$\text{dist}(P; d_{AC}) = \text{dist}(P; d_{AB})$

• d_{AC}: $\vec{AC} = \begin{pmatrix} 12 \\ 4 \end{pmatrix} \parallel \begin{pmatrix} 3 \\ 1 \end{pmatrix} \Rightarrow \vec{n} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$x - 3y + c = 0$ $A: -5 - 12 + c = 0 \Rightarrow c = 17$

$d_{AC}: x - 3y + 17 = 0$

• d_{AB}: $\vec{AB} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ -3 \end{pmatrix} \Rightarrow \vec{n} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$3x + y + c = 0$ $A: -15 + 4 + c = 0 \Rightarrow c = 11$

$d_{AB}: 3x + y + 11 = 0$

$\text{dist}(P, d_{AC}) = \text{dist}(P, d_{AB}) \Rightarrow$

$$\frac{|x - 3y + 17|}{\sqrt{10}} = \frac{|3x + y + 11|}{\sqrt{10}} \Rightarrow$$

$$\begin{cases} x - 3y + 17 = 3x + y + 11 \Rightarrow 2x + 4y - 6 = 0 \Rightarrow x + 2y - 3 = 0 \ (b_1) \\ x - 3y + 17 = -3x - y - 11 \Rightarrow 4x - 2y + 28 = 0 \Rightarrow 2x - y + 14 = 0 \ (b_2) \end{cases}$$

(b_1) est la bissectrice interne.

2) $M\left(\frac{-5+7}{2} ; \frac{4+8}{2}\right) \Rightarrow M(1;6)$

$\vec{BM} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ 2 \end{pmatrix} \Rightarrow \vec{n} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $2x - y + c = 0$

$B: -6 + 2 + c = 0 \Rightarrow c = 4$

$m_B: 2x - y + 4 = 0$

3) $\vec{AB} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \vec{n}_h$ $x - 3y + c = 0$

$C: 7 - 24 + c = 0 \Rightarrow c = 17$

$h: x - 3y + 17 = 0$