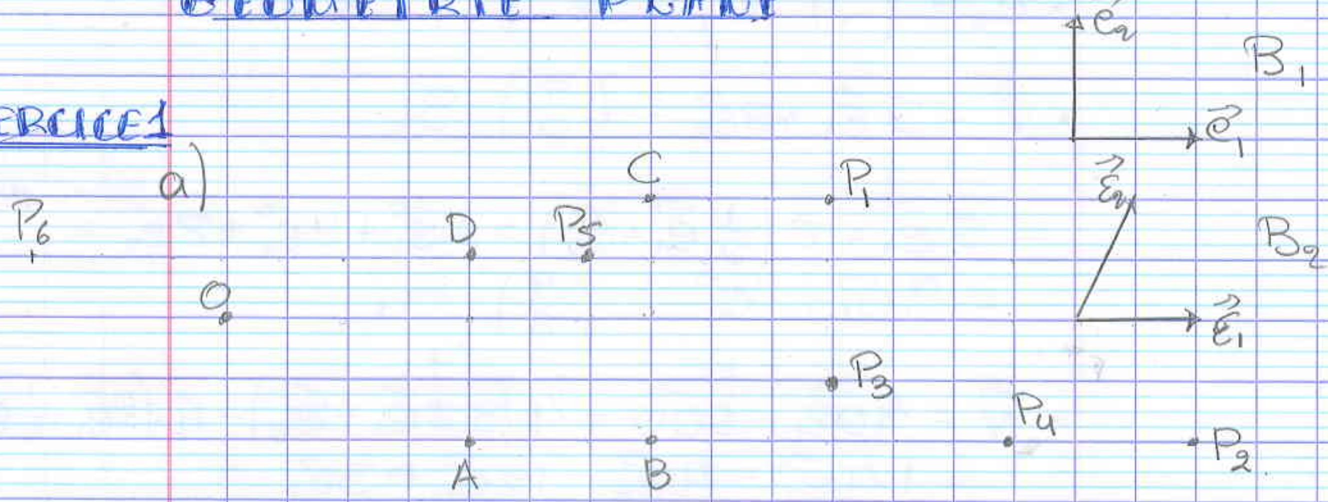
LDDR - Niveau 1 - GEOMETRIE PLANE

EXERCICE 1

a)



a) $\vec{OP_1} = \vec{OB} + \vec{AC}$ $\quad \vec{OP_2} = 2\vec{OD} + 2\vec{OA}$

$\vec{OP_3} = \vec{OC} - \vec{BD} = \vec{OC} + \vec{DB}$

$\vec{AP_4} = 3\vec{DC} - \vec{AD} = 3\vec{DC} + \vec{DA}$

$\vec{BP_5} = \vec{DC} - \vec{OA} = \vec{DC} + \vec{AO}$

$\vec{CP_6} = \frac{1}{2}\vec{OA} - 4\vec{AB} = \frac{1}{2}\vec{OA} + 4\vec{BA}$

b) $\vec{\varepsilon}_1 = \vec{e}_1 \quad \vec{\varepsilon}_2 = \frac{1}{2}\vec{e}_1 + \vec{e}_2$

$\to \vec{\varepsilon}_2 = \frac{1}{2}\vec{\varepsilon}_1 + \vec{e}_2 \Rightarrow \vec{e}_2 = -\frac{1}{2}\vec{\varepsilon}_1 + \vec{\varepsilon}_2$

- $\vec{OA} = 2\vec{e}_1 - \vec{e}_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} B_1$

  $= 2\vec{\varepsilon}_1 - \left(-\frac{1}{2}\vec{\varepsilon}_1 + \vec{\varepsilon}_2\right) = 2\vec{\varepsilon}_1 + \frac{1}{2}\vec{\varepsilon}_1 - \vec{\varepsilon}_2 =$

  $= \frac{5}{2}\vec{\varepsilon}_1 - \vec{\varepsilon}_2 = \begin{pmatrix} 5/2 \\ -1 \end{pmatrix} B_2$

- $\vec{OC} = 3.5\vec{e}_1 + 2\vec{e}_2 = \begin{pmatrix} 3.5 \\ 2 \end{pmatrix} B_1$

  $= 3.5\vec{\varepsilon}_1 + 2\left(-\frac{1}{2}\vec{\varepsilon}_1 + \vec{\varepsilon}_2\right) = 3.5\vec{\varepsilon}_1 - \vec{\varepsilon}_1 + 2\varepsilon_2 =$

  $= 2.5\vec{\varepsilon}_1 + 2\vec{\varepsilon}_2 = \begin{pmatrix} 2.5 \\ 2 \end{pmatrix} B_2$

- $\vec{AC} = 1.5\vec{e}_1 + 2\vec{e}_2 = \begin{pmatrix} 1.5 \\ 2 \end{pmatrix} B_1$

  $= 1.5\vec{\varepsilon}_1 + 2\left(-\frac{1}{2}\vec{\varepsilon}_1 + \vec{\varepsilon}_2\right) = 1.5\vec{\varepsilon}_1 - \vec{\varepsilon}_1 + 2\vec{\varepsilon}_2 = \begin{pmatrix} 0.5 \\ 2 \end{pmatrix} B_2$

- $\vec{e}_1 = \vec{\varepsilon}_1 = \begin{pmatrix}1\\0\end{pmatrix} B_2$
- $\vec{e}_2 = \begin{pmatrix}0\\1\end{pmatrix} B_1$ ou $\begin{pmatrix}-1/2\\1\end{pmatrix} B_2$
- $\vec{v} = 5\vec{\varepsilon}_1 + 8\vec{\varepsilon}_2 = \begin{pmatrix}5\\8\end{pmatrix} B_2$

$= 5\vec{e}_1 + 8\left(\frac{1}{2}\vec{e}_1 + \vec{e}_2\right) = 5\vec{e}_1 + 4\vec{e}_1 + 8\vec{e}_2 =$

$= 9\vec{e}_1 + 8\vec{e}_2 = \begin{pmatrix}9\\8\end{pmatrix} B_1$

- $\vec{w} = 4\vec{OB} - 6\vec{OD} = 4(3{,}5\vec{e}_1 - \vec{e}_2) - 6\left(2\vec{e}_1 + \frac{1}{2}e_2\right) =$

$= 14\vec{e}_1 - 4\vec{e}_2 - 12\vec{e}_1 + 3\vec{e}_2 =$

$= 2\vec{e}_1 - 7\vec{e}_2 = \begin{pmatrix}2\\-4\end{pmatrix} B_1$

$w = 2\vec{e}_1 - 7\left(-\frac{1}{2}\vec{\varepsilon}_1 + \vec{\varepsilon}_2\right) = 2\vec{\varepsilon}_1 + 3{,}5\vec{\varepsilon}_1 - 7\vec{\varepsilon}_2 =$

$= 5{,}5\vec{\varepsilon}_1 - 7\vec{\varepsilon}_2 = \begin{pmatrix}5{,}5\\-7\end{pmatrix} B_2$

Exercice 2

$\vec{a} = \vec{BE} + \vec{AD} - \vec{AE} = \cancel{\vec{OE}} - \vec{OB} + \vec{OD} - \cancel{\vec{OA}} - \cancel{\vec{OE}} + \cancel{\vec{OA}} =$

$= \vec{OD} - \vec{OB} = \vec{BD}$

$\vec{b} = \vec{AB} - \vec{DB} - \vec{AC} = \cancel{\vec{OB}} - \cancel{\vec{OA}} - \cancel{\vec{OB}} + \vec{OD} - \vec{OC} + \cancel{\vec{OA}} =$

$= \vec{CD}$

$\vec{c} = \vec{AC} - \vec{ED} + \vec{BD} - \vec{AE} =$

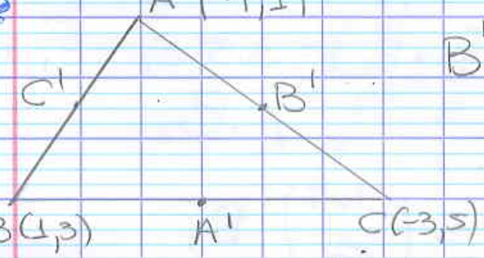
$= \vec{OC} - \cancel{\vec{OA}} - \cancel{\vec{OD}} + \cancel{\vec{OE}} + \cancel{\vec{OD}} - \vec{OB} - \cancel{\vec{OE}} + \cancel{\vec{OA}} =$

$= \vec{BC}$

$\vec{d} = \vec{BC} - \vec{DE} + \vec{CE} + \vec{DB} =$

$= \cancel{\vec{OC}} - \cancel{\vec{OB}} - \cancel{\vec{OE}} + \cancel{\vec{OD}} + \cancel{\vec{OE}} - \cancel{\vec{OC}} + \cancel{\vec{OB}} - \cancel{\vec{OD}} = \vec{0}$

Exercice 3



$B'\left(\frac{-4-3}{2} ; \frac{1+5}{2}\right) = (-3{,}5 ; 3)$

$A'\left(\frac{1-3}{2} ; \frac{3+5}{2}\right) = (-1 ; 4)$

$C'\left(\frac{-4+1}{2} ; \frac{1+3}{2}\right) = (-1{,}5 ; 2)$

$G\left(\frac{-4+1-3}{3};\frac{1+3+5}{3}\right)=(-2;3)$

Exercice 4 $A'(2;-1)$ $B'(-1;4)$ $C'(-2;2)$

$x_{A'}=\frac{x_B+x_C}{2}\Rightarrow 2=\frac{x_B+x_C}{2}\Rightarrow x_B+x_C=4$

$x_{B'}=\frac{x_A+x_C}{2}\Rightarrow -2=x_A+x_C$

$x_{C'}=\frac{x_A+x_B}{2}\Rightarrow -4=x_A+x_B$

$\begin{cases}x_A+x_B=-4\\x_C+x_B=4\end{cases}$ $(-)$ $x_A-x_C=-8$.

$\begin{cases}x_A-x_C=-8\\x_A+x_C=-2\end{cases}$ $(+)$ $2x_A=-10\Rightarrow x_A=-5$

$-5+8=x_C\Rightarrow x_C=3$

$x_B=-4+5\Rightarrow x_B=1$

$y_{A'}=\frac{y_B+y_C}{2}\Rightarrow -2=y_B+y_C$

$y_{B'}=\frac{y_A+y_C}{2}\Rightarrow 8=y_A+y_C$

$y_{C'}=\frac{y_A+y_B}{2}\Rightarrow 4=y_A+y_B$

$\begin{cases}y_B+y_C=-2\\y_A+y_C=8\end{cases}$ $(-)$ $y_A-y_B=10$ $(+)$ $2y_A=14\Rightarrow$

$y_A+y_B=4$ $\quad y_A=7$

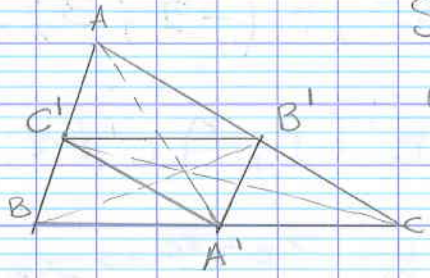
$4=7+y_B\Rightarrow y_B=-3$

$8=7+y_C\Rightarrow y_C=1$

$A(-5;7)$ $B(1;-3)$ $C(3;1)$

$G\left(\frac{-5+1+3}{3};\frac{7-3+1}{3}\right)\Rightarrow G\left(-\frac{1}{3};\frac{5}{3}\right)$

Exercice 5



Soit G le centre de gravité de ABC et G' le cdg de A'B'C'.

$G(x_G, y_G)$ avec $x_G = \dfrac{x_A + x_B + x_C}{3}$

$y_G = \dfrac{y_A + y_B + y_C}{3}$

$G'(x_{G'}, y_{G'})$ avec $x_{G'} = \dfrac{x_{A'} + x_{B'} + x_{C'}}{3}$ ①

$y_{G'} = \dfrac{y_{A'} + y_{B'} + y_{C'}}{3}$

$x_{A'} = \dfrac{x_B + x_C}{2}$ $\quad x_{B'} = \dfrac{x_A + x_C}{2}$ $\quad x_{C'} = \dfrac{x_A + x_B}{2}$

① $x_{G'} = \dfrac{\frac{x_B + x_C}{2} + \frac{x_A + x_C}{2} + \frac{x_A + x_B}{2}}{3} =$

$= \dfrac{x_B + x_C + x_A + x_C + x_A + x_B}{6} = \dfrac{2x_A + 2x_B + 2x_C}{6}$

$\Rightarrow x_{G'} = \dfrac{x_A + x_B + x_C}{3} = x_G$

De même $y_{G'} = y_G$ Alors $G \equiv G'$

Exercice 6

$G(5;7)$

$\vec{BC} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$

$x_G = \dfrac{x_A + x_B + x_C}{3} \Rightarrow 5 = \dfrac{1 + x_B + x_C}{3} \Rightarrow x_B + x_C = 14$ ①

$y_G = \dfrac{y_A + y_B + y_C}{3} \Rightarrow 7 = \dfrac{2 + y_B + y_C}{3} \Rightarrow y_B + y_C = 19$ ②

$\vec{BC} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} x_C - x_B \\ y_C - y_B \end{pmatrix} \Rightarrow \begin{cases} x_C - x_B = -4 & ③ \\ y_C - y_B = 6 & ④ \end{cases}$

On va résoudre le système des ①, ②, ③, ④

① $x_B + x_C = 14$

③ $-x_B + x_C = -4$

$+$ ———————

$2x_C = 10 \Rightarrow x_C = 5 \Rightarrow x_B = 9$

② $y_B + y_C = 19$

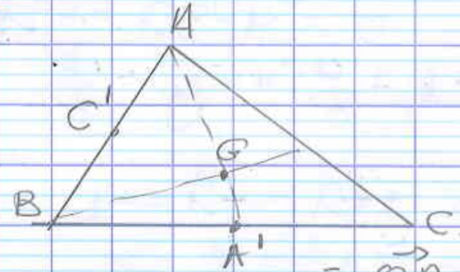
④ $-y_B + y_C = 6$

$+$ ———————

$2y_C = 25 \Rightarrow y_C = \frac{25}{2} \Rightarrow y_B = \frac{13}{2}$

$B\left(9 ; \frac{13}{2}\right) \quad C\left(5 ; \frac{25}{2}\right)$

EXERCICE



a) $OA' = \frac{OB + OC}{2}$

b) $OA' = \vec{OA} + AA' =$

$= \vec{OA} + \frac{3}{2}\vec{AG} = \vec{OA} + \frac{3}{2}(\vec{OG} - \vec{OA}) \Rightarrow$

$\vec{OA'} = \vec{OA} + \frac{3}{2}\vec{OG} - \frac{3}{2}\vec{OA} = -\frac{1}{2}\vec{OA} + \frac{3}{2}\vec{OG}$

c) $\vec{OG} = \frac{\vec{OA'} + \vec{OB'} + \vec{OC'}}{3} \Rightarrow$

$3\vec{OG} = \vec{OA'} + \vec{OB'} + \vec{OC'} \quad (1)$

$\vec{OB'} = \vec{OB} + \vec{BB'} = \vec{OB} + \frac{3}{2}BG = OB + \frac{3}{2}(OG - OB)$

$= OB + \frac{3}{2}OG - \frac{3}{2}OB \Rightarrow$

$\Rightarrow \vec{OB'} = -\frac{1}{2}OB + \frac{3}{2}OG$

(1) $3\vec{OG} = \vec{OA'} - \frac{1}{2}\vec{OB} + \frac{3}{2}\vec{OG} + \vec{OC'} \Rightarrow$

$\Rightarrow 6\vec{OG} = 2\vec{OA'} - \vec{OB} + 3\vec{OG} + 2\vec{OC'} \Rightarrow$

$\Rightarrow 3\vec{OG} = 2\vec{OA'} - \vec{OB} + 2\vec{OC'} \Rightarrow$

$\Rightarrow \vec{OB} = -3\vec{OG} + 2\vec{OA'} + 2\vec{OC'}$

EXERCICE 8

d : $A(4;1)$ $B(1;2)$

a) $\vec{AB} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ $\vec{d} = 5\vec{AB} = \begin{pmatrix} -15 \\ 5 \end{pmatrix}$

b) $d = \begin{cases} x = 4 - 3\lambda \\ y = 1 + \lambda \end{cases}$

- $C(7;2)$ $7 = 4 - 3\lambda \Rightarrow 3\lambda = -3 \Rightarrow \lambda = -1$
  $2 = 1 + \lambda \Rightarrow \lambda = 1$ $\neq \Rightarrow C \notin d$
- $D(-11;6)$ $-11 = 4 - 3\lambda \Rightarrow -15 = -3\lambda \Rightarrow \lambda = 5$
  $6 = 1 + \lambda \Rightarrow \lambda = 5$ ✓ $D \in d$
- $E(13;-2)$ $13 = 4 - 3\lambda \Rightarrow -3\lambda = 9 \Rightarrow \lambda = -3$
  $-2 = 1 + \lambda \Rightarrow \lambda = -3$ $E \in d$

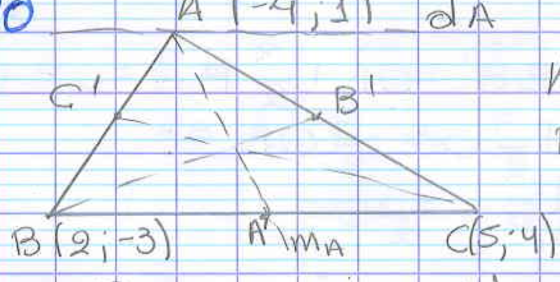
c) $P_1(0;y) \in d$ $0 = 4 - 3\lambda \Rightarrow \lambda = \frac{4}{3}$
$y = 1 + \frac{4}{3} \Rightarrow y = \frac{7}{3}$

$P_2(x;0)$ $0 = 1 + \lambda \Rightarrow \lambda = -1$
$x = 4 + 3 = 7 = x$

$P_3(10;y)$ $10 = 4 - 3\lambda \Rightarrow 3\lambda = -6 \Rightarrow \lambda = -2$
$y = 1 - 2 \Rightarrow y = -1$

$P_4(x;-12)$ $-12 = 1 + \lambda \Rightarrow \lambda = -13$
$x = 4 + 39 \Rightarrow x = 43$

EXERCICE 90



$A'\left(\frac{7}{2}; \frac{1}{2}\right)$
$B'\left(\frac{1}{2}; \frac{5}{2}\right)$
$C'(-1;-1)$

m_A : $\vec{AA'} = \begin{pmatrix} \frac{15}{2} \\ -\frac{1}{2} \end{pmatrix} \parallel \begin{pmatrix} 15 \\ -1 \end{pmatrix} \Rightarrow \vec{n}_A = \begin{pmatrix} 1 \\ 15 \end{pmatrix}$

$x + 15y + c = 0$ A : $-4 + 15 + c = 0 \Rightarrow c = -11$

m_A : $x + 15y - 11 = 0$.

m_B : $\vec{BB'} = \begin{pmatrix} -3/2 \\ 11/2 \end{pmatrix} \parallel \begin{pmatrix} -3 \\ 11 \end{pmatrix} \Rightarrow \vec{n}_B = \begin{pmatrix} 11 \\ 3 \end{pmatrix}$

$11x + 3y + c = 0$ B : $22 - 9 + c = 0 \Rightarrow c = -13$

m_B : $\underline{11x + 3y - 13 = 0}$

m_C : $\vec{CC'} = \begin{pmatrix} -6 \\ -5 \end{pmatrix} \Rightarrow \vec{n}_C = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$

$5x - 6y + c = 0$ C : $25 - 24 + c = 0 \Rightarrow c = -1$

$\underline{5x - 6y - 1 = 0}$

$\underline{d_A}$ $\vec{BC} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} \Rightarrow \vec{n}_{BC} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$

$7x - 3y + c = 0$ A : $-28 - 3 + c = 0 \Rightarrow c = 31$

$\underline{7x - 3y + 31 = 0}$

d_B : $\vec{AC} = \begin{pmatrix} 9 \\ 3 \end{pmatrix} \parallel \begin{pmatrix} 3 \\ 1 \end{pmatrix} \Rightarrow \vec{n}_{AC} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$x - 3y + c = 0$ B : $2 + 9 + c = 0 \Rightarrow c = -11$

$\underline{x - 3y - 11 = 0}$

d_C : $\vec{AB} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \parallel \begin{pmatrix} 3 \\ -2 \end{pmatrix} \Rightarrow \vec{n}_{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$2x + 3y + c = 0$ C : $10 + 12 + c = 0 \Rightarrow c = -22$

$\underline{2x + 3y - 22 = 0}$

EXERICE 11

$\underline{d_1}$ $(-1; 6)$ $m = 4$ $y = 4x + h$

$6 = -4 + h \Rightarrow h = 10$

$\underline{y = 4x + 10}$

$\underline{d_2}$ $(5; 2)$ $(1; 6)$ $m = \frac{6 - 2}{1 - 5} = \frac{4}{-4} = -1$

$y = -x + h$ $(1; 6)$ $6 = -1 + h \Rightarrow h = 7$

$\underline{y = -x + 7}$

$\underline{d_3}$ $(2;-5)$ $(2;8)$ $x=2$

$\underline{d_4}$ $(3,-5)$ $(-1;-2)$ $m=\dfrac{-2+5}{-1-3}=\dfrac{3}{-4}$

$y=-\dfrac{3}{4}x+h$

$-5=-\dfrac{3}{4}\cdot 3+h \Rightarrow h=-5+\dfrac{9}{4}=\dfrac{-11}{4}$

$y=-\dfrac{3}{4}x-\dfrac{11}{4}$

EXERCICE 12 $d_1: x+2y+3=0$ $\quad d_3: \begin{cases} x=-5+3\lambda \\ y=-3+2\lambda \end{cases}$

$d_2: 2x-3y+5=0$

$d_4 \begin{cases} x=-3+2\lambda \\ y=2-\lambda \end{cases}$

$\underline{d_1 \cap d_2}$ $\begin{cases} x+2y=-3 & *(-2) \\ 2x-3y=-5 \end{cases}$ $\quad + \begin{cases} -2x-4y=6 \\ 2x-3y=-5 \end{cases}$

$-7y=1 \Rightarrow y=-1/7$ $\quad x=-19/7$

$I_1\left(-\dfrac{19}{7};-\dfrac{1}{7}\right)$

$\underline{d_1 \cap d_3}$ $-5+3\lambda+2(-3+2\lambda)+3=0 \Rightarrow -5+3\lambda-6+4\lambda+3=0 \Rightarrow$

$\Leftrightarrow 7\lambda=8 \Rightarrow \lambda=8/7$ $\quad I_2$ $\quad x=-5+3\cdot\dfrac{8}{7}=-\dfrac{11}{3}$

$I_2\left(-\dfrac{11}{3};-\dfrac{5}{7}\right)$ $\quad y=-3+2\cdot\dfrac{8}{7}=-\dfrac{5}{7}$

$\underline{d_1 \cap d_4}$ $-3+2\lambda+2(2-\lambda)+3=0 \Rightarrow \cancel{-3}+\cancel{2\lambda}+4-\cancel{2\lambda}+\cancel{3}=0$

$\Rightarrow 4=0$ impossible

$\underline{d_2 \cap d_3}$ $2(-5+3\lambda)-3(-3+2\lambda)+5=0 \Rightarrow$

$-10+\cancel{6\lambda}+\cancel{9}-\cancel{6\lambda}+5=0 \Rightarrow 4=0$ impossible

$\underline{d_3 \cap d_4}$ $\left.\begin{aligned} -5+3\lambda&=-3+2k \\ -3+2\lambda&=2-k \end{aligned}\right\} \Rightarrow \begin{aligned} 3\lambda-2k&=2 \\ 2\lambda+k&=5 \quad *2 \end{aligned} \Rightarrow$

$+\begin{cases} 3\lambda-2k=2 \\ 4\lambda+2k=10 \end{cases}$

$7\lambda=12 \Rightarrow \lambda=\dfrac{12}{7}$ $\quad x=-5+\dfrac{36}{7}=\dfrac{1}{7}$ $\quad y=-3+\dfrac{24}{7}=\dfrac{3}{7}$

$I(1/7;3/7)$

Exercice 13 d: A(47 ; −206) B(607 ; 4)

d' : A' (−206 ; 47) B'(4 ; 607)

d $\vec{AB} = \begin{pmatrix} 560 \\ 210 \end{pmatrix} \parallel \begin{pmatrix} 56 \\ 21 \end{pmatrix} \Rightarrow \vec{n}_d = \begin{pmatrix} 21 \\ -56 \end{pmatrix}$

$21x - 56y + c = 0$

A : $21 \cdot 47 - 56(-206) + c = 0 \Rightarrow c = -12523$

d : $21x - 56y - 12523 = 0 \Rightarrow$

$\Rightarrow 3x - 8y = 1789$

$\cap Ox$: $y = 0$ $3x = 1789 \Rightarrow x = \frac{1789}{3}$

$\cap Oy$: $x = 0$ $-8y = 1789 \Rightarrow y = -\frac{1789}{8}$

d' $\vec{A'B'} = \begin{pmatrix} 210 \\ 560 \end{pmatrix} \parallel \begin{pmatrix} 21 \\ 56 \end{pmatrix} \Rightarrow \vec{n}_{d'} = \begin{pmatrix} 56 \\ -21 \end{pmatrix}$

$56x - 21y + c = 0$

A' $56 \cdot (-206) - 21 \cdot 47 + c = 0 \Rightarrow c = 12523$

$56x - 21y + 12523 = 0$

$\Rightarrow 8x - 3y = -1789$

$d \cap d' \begin{cases} 3x - 8y = 1789 & *(-8) \\ 8x - 3y = -1789 & *(3) \end{cases} \Rightarrow$

$$\begin{array}{rl} & -24x + 64y = -14312 \\ + & 24x - 9y = -5367 \\ \hline & 55y = -19679 \Rightarrow y = -787.16 \end{array}$$

$x = 1502.76$

EXERCICE 14

$2x + 7y + 13 = 0$ $\qquad \vec{n}_1 = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$

$5x + by + c = 0$ $\qquad \vec{n}_2 = \begin{pmatrix} 5 \\ b \end{pmatrix}$

$\vec{n}_1 \parallel \vec{n}_2 \Rightarrow \begin{vmatrix} 2 & 5 \\ 7 & b \end{vmatrix} = 0 \Rightarrow 2b - 35 \Rightarrow \underset{\sim}{b = \frac{35}{2}}$

(2) $5x + \frac{35}{2}y + c = 0 \Rightarrow 10x + 35y + 2c = 0 \Rightarrow$

$\Rightarrow 2x + 7y + \frac{2c}{5} = 0$.

- la même droite $13 = \frac{2c}{5} \Rightarrow c = 32.5$
- strictement parallèles $c \neq 32.5$

$b = \frac{35}{2}$.

Exercice de revision I

d_1 $\begin{cases} x = 2+2\lambda \\ y = -3+\lambda \end{cases}$ $\vec{d} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \Rightarrow \vec{n} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\Downarrow$

$m = \frac{1}{2}$

$\lambda = 0 \quad A(2;-3) \qquad \lambda = 1 \quad B(4;-2)$

$\vec{n} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \Rightarrow d_1 : x - 2y + c = 0$

$A: \quad 2 + 6 + c = 0 \Rightarrow c = -8$

$d_1 : \quad x - 2y - 8 = 0$

d_2 $A(3;4) \quad \vec{d} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \Rightarrow m = \frac{5}{1} = 5$

éq. par $\begin{cases} x = 3 + \lambda \\ y = 4 + 5\lambda \end{cases}$ $\vec{d} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \Rightarrow \vec{n} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \Rightarrow$

$5x - y + c = 0$

$A: 15 - 4 + c = 0 \Rightarrow c = -11$

éq. cart $5x - y - 11 = 0$

d_3 $A(0;1) \quad B(3;0)$

$\vec{d_3} = \vec{AB} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \Rightarrow m = -\frac{1}{3}$

éq. par. $\begin{cases} x = 3\lambda \\ y = 1 - \lambda \end{cases}$ $\vec{d_3} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \Rightarrow \vec{n_3} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$x + 3y + c = 0 \quad A: 3 + c = 0 \Rightarrow c = -3$

éq. cart: $x + 3y - 3 = 0$

d_4 $A(2;18)$ $m=-2$

$y=-2x+h$ $A: 18=-4+h \Rightarrow h=22$

$\Rightarrow y=-2x+22 \Rightarrow$

eq. cart: $2x+y-22=0$ $\vec{n}_4=\begin{pmatrix}2\\1\end{pmatrix} \Rightarrow \vec{d}_4=\begin{pmatrix}1\\-2\end{pmatrix}$

eq. par: $\begin{cases} x=2+\lambda \\ y=18-2\lambda \end{cases}$

d_5 $A(1;-5)$ $\vec{n}=\begin{pmatrix}3\\2\end{pmatrix} \Rightarrow \vec{d}=\begin{pmatrix}2\\-3\end{pmatrix} \Rightarrow m=-\frac{3}{2}$

eq. par $\begin{cases} x=1+2\lambda \\ y=-5-3\lambda \end{cases}$

$\vec{n}_5=\begin{pmatrix}3\\2\end{pmatrix} \Rightarrow 3x+2y+c=0$

$A: 3-10+c=0 \Rightarrow c=7$

eq. cart: $3x+2y+7=0$

d_6 $y=3x+5 \Rightarrow m=3$

$x=0$ $y=5$ $A(0;5)$

$x=-1$ $y=2$ $B(-1;2)$

eq. cart $3x-y+5=0 \Rightarrow \vec{n}_5=\begin{pmatrix}3\\-1\end{pmatrix} \Rightarrow \vec{d}=\begin{pmatrix}1\\3\end{pmatrix}$

eq par $\begin{cases} x=\lambda \\ y=5+3\lambda \end{cases}$

Exercice de revision II

$d: \begin{cases} x = 2-3t \\ y = -4+t \end{cases} \quad t \in \mathbb{R}$

a) $A \overset{?}{\in} d$. $A(-7;2)$ $\quad \begin{matrix} -7 = 2-3t \Leftrightarrow t = 3 \\ 2 = -4+t \Leftrightarrow t = 6 \end{matrix} \Rightarrow A \notin d$

$B(-109;32)$ $\quad \begin{matrix} -109 = 2-3t \Rightarrow t = \frac{111}{3} \\ 32 = -4+t \Rightarrow t = 36 \end{matrix} \Rightarrow B \notin d$

$C(-139;43)$ $\quad \begin{matrix} -139 = 2-3t \Rightarrow t = 47 \\ 43 = -4+t \Rightarrow t = 47 \end{matrix} \quad C \in d$

b) $P_1(-3, y)$ $\quad -3 = 2-3t \Rightarrow t = 5/3$

$y = -4 + \frac{5}{3} \Rightarrow y = -7/3$

$P_2(x, 4)$ $\quad 4 = -4+t \Rightarrow t = 8$

$x = 2 - 3 \cdot 8 \Rightarrow x = -22$

$P_3(x,y)$ $\quad x = 2y \Rightarrow 2-3t = 2(-4+t) \Rightarrow$

$\Rightarrow 2-3t = -8+2t \Rightarrow 5t = 10 \Rightarrow t = 2$

$x = -4 \quad y = -2$

c) $\vec{d} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \Rightarrow \vec{n} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad x+3y+c=0$

$P_3(-4;-2)$ $\quad -4-6+c=0 \Rightarrow c = 10$

$d: \underbrace{x+3y+10=0}$

$m = -\frac{1}{3}$

d) $d': 3x+8y+3=0 \quad \vec{n} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} \Rightarrow \vec{d'} = \begin{pmatrix} 8 \\ -3 \end{pmatrix} \Rightarrow$

$m = -\frac{3}{8} \neq -\frac{1}{3} \Rightarrow$ sécantes

e) $I = d \cap d'$

$$\begin{cases} x+3y=-10 & \times(-3) \\ 3x+8y=-3 \end{cases} \qquad + \begin{cases} -3x-9y=30 \\ 3x+8y=-3 \end{cases}$$

$$-y = 27 \Rightarrow \underline{y=-27}$$

* $x - 81 = -10 \Rightarrow x = 71$

$I(71\,;-27)$

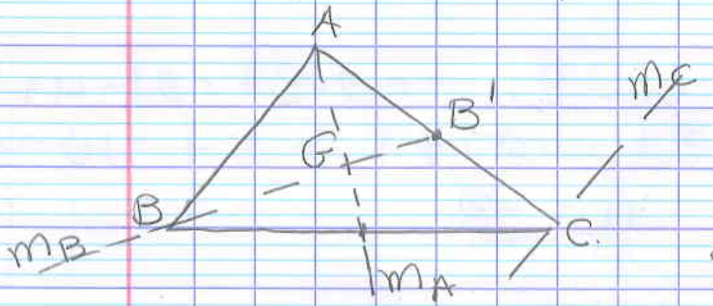
f) $d'' // d' \Rightarrow \vec{n} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} \Rightarrow 3x+8y+c=0$

$P(5\,;-1)$: $15-8+c=0 \Rightarrow c=-7$

d'' : $3x+8y-7=0$

Exercice de révision III

$A(2\,;1)$ $B(4\,;-6)$ $C(-2\,;4)$



$A'\left(\frac{4-2}{2}\,;\,\frac{-6+4}{2}\right) = (1\,;-1)$

$B'\left(\frac{2-2}{2}\,;\,\frac{1+4}{2}\right) = \left(0\,;\,\frac{5}{2}\right)$

$\underline{m_A}$ $\overrightarrow{AA'} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} \Rightarrow \vec{n}_A = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$: $2x-y+c=0$

A : $4-1+c=0 \Rightarrow c=-3$

m_A : $\underline{2x-y-3=0}$

$\underline{m_B}$ $\overrightarrow{BB'} = \begin{pmatrix} -4 \\ \frac{17}{2} \end{pmatrix} // \begin{pmatrix} -8 \\ 17 \end{pmatrix} \Rightarrow \vec{n}_B = \begin{pmatrix} 17 \\ 8 \end{pmatrix}$

$17x+8y+c=0$ B : $68-48+c=0 \Rightarrow c=-20$

m_B : $\underline{17x+8y-20=0}$

m_c $\quad \vec{AB} = \begin{pmatrix} 2 \\ -7 \end{pmatrix} // m_c \Rightarrow \vec{n_c} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$

$7x + 2y + c = 0$

$C: -14 + 8 + c = 0 \Rightarrow c = 6$

$m_c : 7x + 2y + 6 = 0$

b) $G = m_A \cap m_B$

$\begin{cases} 2x - y = 3^* & \cdot 8 \\ 17x + 8y = 20 \end{cases} \Rightarrow \begin{cases} 16x - 8y = 24 \\ 17x + 8y = 20 \end{cases} +$

$33x = 44 \Rightarrow x_G = 4/3$

$* \; 2 \cdot \frac{4}{3} - y = 3 \Rightarrow y = 1 \qquad G(4/3; 1)$

$I_1 = m_C \cap m_B \quad \begin{cases} 7x + 2y = -6^* & \cdot(-4) \\ 17x + 8y = 20 \end{cases} \Rightarrow$

$\left.\begin{cases} -28x - 8y = 24 \\ 17x + 8y = 20 \end{cases}\right\}(+) \quad -11x = 44 \Rightarrow x = -4$

$* \; -28 + 2y = -6 \Rightarrow y = 11$

$I_1(-4; 11)$

$I_2 = m_A \cap m_C \quad \begin{cases} 2x - y = 3^* & \cdot 2 \\ 7x + 2y = -6 \end{cases} \quad \begin{cases} 4x - 2y = 6 \\ 7x + 2y = -6 \end{cases} +$

$11x = 0 \Rightarrow x = 0$

$* \; -y = 3 \Rightarrow y = -3$

$I_2(0; -3)$

c) $G = m_A \cap m_B$ centre de gravité!

d) $A\,(2,1) \quad C\,(-2;4) \quad A^*\,(x,y)$

$\vec{AC} = \vec{CA^*} \Rightarrow \begin{pmatrix} -4 \\ 3 \end{pmatrix} = \begin{pmatrix} x+2 \\ y-4 \end{pmatrix} \Rightarrow \begin{cases} x = -6 \\ y = 7 \end{cases} \quad A^*(-6; 7)$

$B\,(4;-6) \quad C\,(-2;4) \quad B^*\,(x,y)$

$\vec{BC} = \vec{CB^*} \Rightarrow \begin{pmatrix} -6 \\ 10 \end{pmatrix} = \begin{pmatrix} x+2 \\ y-4 \end{pmatrix} \Rightarrow \begin{cases} x = -8 \\ y = 14 \end{cases} \quad B^*(-8, 14)$

Exercice de révision IV

$\vec{v}_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\vec{v}_2 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ $\vec{v}_3 = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$

a) $\vec{v}_1 \not\parallel \vec{v}_2 \Rightarrow$ ils forment une base

$\vec{v}_3 = a \cdot \vec{v}_1 + b \cdot \vec{v}_2 \Rightarrow \begin{pmatrix} -2 \\ 6 \end{pmatrix} = a \begin{pmatrix} 3 \\ 4 \end{pmatrix} + b \begin{pmatrix} -2 \\ 5 \end{pmatrix} \Rightarrow$

$\begin{cases} -2 = 3a - 2b^{*} & \cdot 5 \\ 6 = 4a + 5b & \cdot 2 \end{cases} \Rightarrow + \begin{cases} -10 = 15a - 10b \\ 12 = 8a + 10b \end{cases}$

$2 = 23a \Rightarrow a = \frac{2}{23}$

$* \; -2 = 3 \cdot \frac{2}{23} - 2b \Rightarrow b = \frac{26}{23}$

Donc $\vec{v}_3 = \frac{2}{23} \vec{v}_1 + \frac{26}{23} \vec{v}_2$

b) $\vec{w}_1 \parallel \vec{v}_1 \Rightarrow \begin{vmatrix} x_1 & 3 \\ y_1 & 4 \end{vmatrix} = 0 \Rightarrow 4x_1 - 3y_1 = 0$

$\vec{w}_2 \parallel \vec{v}_2 \Rightarrow \begin{vmatrix} x_2 & -2 \\ y_2 & 5 \end{vmatrix} = 0 \Rightarrow 5x_2 + 2y_2 = 0$

$3\vec{w}_1 + 2\vec{w}_2 = \vec{v}_3 \Rightarrow \begin{pmatrix} 3x_1 \\ 3y_1 \end{pmatrix} + \begin{pmatrix} 2x_2 \\ 2y_2 \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \end{pmatrix} \Rightarrow$

$3x_1 + 2x_2 = -2$ (1)
$3y_1 + 2y_2 = 6$ (2)
$4x_1 - 3y_1 = 0$ (3)
$5x_2 + 2y_2 = 0$ (4)

(1), (3) $\begin{cases} 3x_1 + 2x_2 = -2 & \cdot(-4) \\ 4x_1 - 3y_1 = 0 & \cdot(3) \end{cases} \Rightarrow + \begin{cases} -12x_1 - 8x_2 = 8 \\ 12x_1 - 12y_1 = 0 \end{cases}$

$-8x_2 - 12y_1 = 8$

$\Rightarrow 2x_2 + 3y_1 = -2$ (5)

⑤,④ $\begin{cases} 2x_2 + 3y_1 = -2 & \cdot(-2) \\ 5x_2 + 2y_2 = 0 & \cdot 3 \end{cases} \Leftrightarrow \begin{cases} -4x_2 - 6y_1 = 4 \\ 15x_2 + 6y_2 = 0 \end{cases}$

$+$ $11x_2 = 4 \Rightarrow x_2 = \frac{4}{11}$

① $3x_1 + \frac{8}{11} = -2 \Rightarrow x_1 = -\frac{10}{11}$

③ $-\frac{40}{11} - 3y_1 = 0 \Rightarrow -\frac{40}{11} = 3y_1 \Rightarrow y_1 = -\frac{40}{33}$

④ $5 \cdot \frac{4}{11} + 2y_2 = 0 \Rightarrow y_2 = -\frac{10}{11}$

$\vec{w}_1 = \begin{pmatrix} -10/11 \\ -40/33 \end{pmatrix}$ $\vec{w}_2 = \begin{pmatrix} 4/11 \\ -10/11 \end{pmatrix}$

Exercice 15

$A(1;-4)$ $B(-8;5)$ $C(-11;2)$

$\vec{AB} = \begin{pmatrix} -9 \\ 9 \end{pmatrix}$ $\|\vec{AB}\| = \sqrt{9^2+9^2} = 9\sqrt{2}$

$\vec{AC} = \begin{pmatrix} -12 \\ 6 \end{pmatrix} \Rightarrow \|\vec{AC}\| = \sqrt{12^2+6^2} = \sqrt{180} = 6\sqrt{5}$

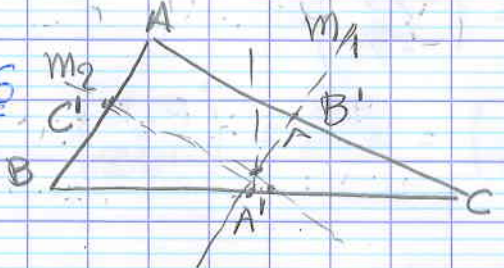
$\vec{BC} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}$ $\|\vec{BC}\| = \sqrt{3^2+3^2} = 3\sqrt{2}$

$AB^2 = 81 \cdot 2 = 162$ $AC^2 = 180$ $BC^2 = 18$

$AC^2 = AB^2 + BC^2 \Rightarrow$ rectangle

ou $\vec{AB} \cdot \vec{BC} = -9(-3) + 9 \cdot (-3) = 0 \Rightarrow \vec{AB} \perp \vec{BC}$

Exercice 16



$A'\left(-\frac{19}{2}; \frac{7}{2}\right)$

$B'(-5; -1)$

$C'\left(-\frac{7}{2}; \frac{1}{2}\right)$

$\vec{AB} = \begin{pmatrix} -9 \\ 9 \end{pmatrix} // \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \vec{n}_2$; $x - y + c = 0$

m_2 : C' : $-\frac{7}{2} - \frac{1}{2} + c = 0 \Leftrightarrow c = 4$

m_2 : $x - y + 4 = 0$

$\vec{AC} = \begin{pmatrix} -12 \\ 6 \end{pmatrix} // \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \vec{n}_1 \Rightarrow 2x - y + c = 0$

$B'(-5, -1) \quad -10 + 1 + c = 0 \Rightarrow c = 9$

$m_1 : \; 2x - y + 9 = 0$

$m_3 : \; \vec{BC} = \begin{pmatrix} -3 \\ -3 \end{pmatrix} // \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \vec{n}_3 \Rightarrow x + y + c = 0$

$A'\left(-\frac{19}{2}, \frac{7}{2}\right) : \; -\frac{19}{2} + \frac{7}{2} + c = 0 \Rightarrow c = 6$

$m_3 : \; x + y + 6 = 0$

$\begin{array}{l} m_1 \\ m_3 \end{array} \begin{cases} 2x - y + 9 = 0 \; (+) \\ x + y + 6 = 0 \end{cases} \quad 3x + 15 = 0 \Rightarrow x = -5, \; y = -1 \quad I(-5, -1)$

$m_2 : \; x - y + 4 = 0 \quad I \overset{?}{\in} m_2$

$-5 + 1 + 4 = 0 \; ✓ \Rightarrow I \in m_2$

EXERCICE 17 $\vec{v} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad \|\vec{v}\| = \sqrt{9+16} = 5$

$\vec{v}_{unitaire} = \begin{pmatrix} 3/5 \\ -4/5 \end{pmatrix}$

Donc $\|\vec{u}_1\| = 10 \begin{pmatrix} 3/5 \\ -4/5 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$

$\vec{u}_2 = 7 \cdot \begin{pmatrix} 3/5 \\ -4/5 \end{pmatrix} = \begin{pmatrix} 21/5 \\ -28/5 \end{pmatrix}$

$\vec{u}_3 = 13.5 \begin{pmatrix} 3/5 \\ -4/5 \end{pmatrix} \Rightarrow \vec{u}_3 = \begin{pmatrix} 8.1 \\ -10.8 \end{pmatrix}$

EXERCICE 18

$\vec{u}_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $\vec{u}_2 = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\vec{u}_3 = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ $\vec{u}_4 = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$\|\vec{u}_1\| = \sqrt{17}$

$\|\vec{u}_2\| = \sqrt{4+9} = \sqrt{13}$

$\|\vec{u}_3\| = \sqrt{4+25} = \sqrt{29}$

$\|\vec{u}_4\| = \sqrt{9+16} = 5$

- $\cos\varphi_1 = \dfrac{\vec{u}_1 \cdot \vec{u}_2}{\|\vec{u}_1\|\,\|\vec{u}_2\|} = \dfrac{-8+3}{\sqrt{16+1}\,\sqrt{4+9}} = \dfrac{-5}{\sqrt{17}\cdot\sqrt{13}} \Rightarrow \varphi_1 = 109{,}65$
- $\cos\varphi_2 = \dfrac{\vec{u}_1 \cdot \vec{u}_3}{\|\vec{u}_1\| \cdot \|\vec{u}_3\|} = \dfrac{8-5}{\sqrt{17}\cdot\sqrt{29}} \Rightarrow \varphi_2 = 82{,}23^\circ$
- $\cos\varphi_3 = \dfrac{\vec{u}_1 \cdot \vec{u}_4}{\|\vec{u}_1\| \cdot \|\vec{u}_4\|} = \dfrac{-12+4}{\sqrt{17}\cdot 5} \Rightarrow \varphi = 112{,}83$
- $\cos\varphi_4 = \dfrac{\vec{u}_2 \cdot \vec{u}_3}{\|\vec{u}_2\| \cdot \|\vec{u}_3\|} = \dfrac{-4-15}{\sqrt{13}\cdot\sqrt{29}} \Rightarrow \varphi_4 = 168{,}1^\circ$
- $\cos\varphi_5 = \dfrac{\vec{u}_2 \cdot \vec{u}_4}{\|\vec{u}_2\|\,\|\vec{u}_4\|} = \dfrac{6+12}{\sqrt{13}\cdot 5} \Rightarrow \varphi_5 = 3{,}18^\circ$
- $\cos\varphi_6 = \dfrac{\vec{u}_3 \cdot \vec{u}_4}{\|\vec{u}_3\| \cdot \|\vec{u}_4\|} = \dfrac{-6-20}{\sqrt{29}\cdot 5} \Rightarrow \varphi_6 = 164{,}98^\circ$

EXERCICE 19

$A(2;4) \in p \perp d : 3x - 5y + 9 = 0$

$\vec{n}_d = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \Rightarrow \vec{n}_p = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

$p : 5x + 3y + c = 0$ $A : 10 + 12 + c = 0 \Rightarrow c = -22$

$p : 5x + 3y - 22 = 0$

- $B(6;-7) : 30 - 21 + c = 0 \Rightarrow c = -9$

$p : 5x + 3y - 9 = 0$

EXERCICE 20

$d_1 : 3x - 5y + 9 = 0$ $\vec{d}_1 = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$

a) $d_2 : y = -x + 7 \Rightarrow x + y - 7 = 0$

$\vec{n}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \vec{d}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\|\vec{d}_1\| = \sqrt{9+25} = \sqrt{34}$

$\|\vec{d}_2\| = \sqrt{2}$

$\cos\varphi = \dfrac{|\vec{d}_1 \cdot \vec{d}_2|}{\|\vec{d}_1\| \cdot \|\vec{d}_2\|} = \dfrac{|3-5|}{\sqrt{34}\,\sqrt{2}} \Rightarrow \varphi = 75{,}96^\circ$

$d_1 : 3x - 5y + 9 = 0 \quad \vec{d_1} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad \|\vec{d_1}\| = \sqrt{34}$

b) $A(2;4) \quad B(6;-7) \quad \vec{AB} = \begin{pmatrix} 4 \\ -11 \end{pmatrix} \perp m_{AB}$

$\vec{n}_{AB} = \begin{pmatrix} 4 \\ -11 \end{pmatrix} \Rightarrow \vec{d}_{AB} = \begin{pmatrix} 11 \\ 4 \end{pmatrix} \quad \|\vec{d}_{AB}\| = \sqrt{11^2+4^2} = \sqrt{137}$

$$\cos\varphi = \frac{\vec{d}_{AB} \cdot \vec{d_1}}{\|\vec{d}_{AB}\| \cdot \|\vec{d_1}\|} = \frac{55+12}{\sqrt{34}\,\sqrt{137}} \Rightarrow \varphi = 10.98^\circ$$

Exercice 21 $d : 2x - 3y + 10 = 0 \quad A(6;3) \quad B(5;-2)$

A

$\vec{n}_d = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \Rightarrow \vec{n}_A = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$d_A : 3x + 2y + c = 0$

$A : 18 + 6 + c = 0 \Rightarrow c = -24$

$d_A : 3x + 2y - 24 = 0$

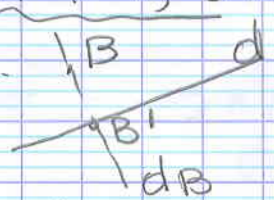
$A' = d_A \cap d \quad \begin{cases} 3x + 2y = 24 & *3 \\ 2x - 3y = -10 & *2 \end{cases}$

$+\begin{cases} 9x + 6y = 72 \\ 4x - 6y = -20 \end{cases} \quad 13x = 52 \Rightarrow x = 4$

$12 + 2y = 24 \Rightarrow y = 6$

$A'(4;6)$

B



$\vec{n}_d = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \Rightarrow \vec{n}_B = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$d_B : 3x + 2y + c = 0$

$B : 15 - 4 + c = 0 \Rightarrow c = -11$

$B(5;-2) \quad d_B : 3x + 2y - 11 = 0$

$B' = d_B \cap d \quad \begin{cases} 3x + 2y = 11 & *3 \\ 2x - 3y = -10 & *2 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} 9x + 6y = 33 \\ 4x - 6y = -20 \end{cases} + \quad 13x = 13 \Rightarrow x = 1$

$y = 4$

$B'(1;4)$

$\mathrm{dist}(A, d) = \mathrm{dist}(A, A') = \|\vec{AA'}\|$

$\vec{AA'} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\quad \|\vec{AA'}\| = \sqrt{4+9} = \sqrt{13}$

$\mathrm{dist}(B, d) = \mathrm{dist}(B, B') = \|\vec{BB'}\|$

$\vec{BB'} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \Rightarrow \|\vec{BB'}\| = \sqrt{16+36} = \sqrt{50}$

EXERCICE 22 $\quad \vec{v}_1 = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad \vec{v}_2 = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

- $\vec{v}_1 \cdot \vec{v}_2 = \|\vec{v}_1\| \cdot v_2'$ $\qquad \vec{v}_1 \cdot \vec{v}_2 = 6 + 12 = 18$

$\Rightarrow 18 = \sqrt{17} \cdot v_2' \Rightarrow \vec{v}_2{}' = \dfrac{18}{\sqrt{17}}$ $\qquad \|\vec{v}_1\| = \sqrt{1+16} = \sqrt{17}$

$\|\vec{v}_2\| = \sqrt{36+9} = \sqrt{45}$

- $\vec{v}_1 \cdot \vec{v}_2 = \|\vec{v}_2\| \cdot v_1' \Rightarrow$

$\Rightarrow 18 = \sqrt{45} \cdot v_1' \Rightarrow v_1' = \dfrac{18}{\sqrt{45}}$

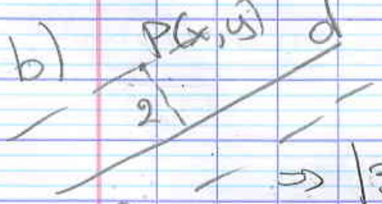
EXERCICE 23 $\quad d : 3x - 4y - 11 = 0$

$P_1(0,0) \quad \mathrm{dist}(P_1, d) = \dfrac{|3 \cdot 0 - 4 \cdot 0 - 11|}{\sqrt{9+16}} = \dfrac{11}{5}$

$P_2(-8;5) \quad \mathrm{dis}(P_2, d) = \dfrac{|-24-20-11|}{5} = \dfrac{55}{5} = 11$

$P_3(7;-2) \quad \mathrm{dis}(P_3, d) = \dfrac{|21+8-11|}{5} = \dfrac{18}{5}$

$P_4(5;1) \quad \mathrm{dis}(P_4; d) = \dfrac{|15-4-11|}{5} = 0 \Rightarrow P_4 \in d$

b) 

$\mathrm{dist}(P, d) = 2 \Rightarrow \dfrac{|3x-4y-11|}{5} = 2 \Rightarrow$

$\Rightarrow |3x - 4y - 11| = 10 \Rightarrow$

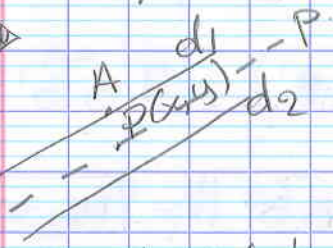
$\begin{cases} 3x - 4y - 11 = 10 & \Rightarrow d_1 : 3x - 4y = 21 \\ 3x - 4y - 11 = -10 & d_2 : 3x - 4y = 1 \end{cases}$

EXERCICE 24

$d_1 : 6x - 15y - 17 = 0 \quad \vec{n}_1 = \begin{pmatrix}6\\-15\end{pmatrix} \parallel \begin{pmatrix}2\\-5\end{pmatrix}$

$d_2 : 10x - 25y + 43 = 0 \quad \vec{n}_2 = \begin{pmatrix}10\\-25\end{pmatrix} \parallel \begin{pmatrix}2\\-5\end{pmatrix} \Rightarrow$

$d_1 \parallel d_2$



▸ On trouve un point $A \in d_1$

$y = -1 \quad 6x + 15 - 17 = 0 \Rightarrow$

$6x = 2 \Rightarrow x = 1/3$

$A\,(1/3\,;\,-1)$

$dist(d_1, d_2) = dis(A, d_2) =$

$= \dfrac{\left|\frac{10}{3} + 25 + 43\right|}{\sqrt{10^2 + 25^2}} = \dfrac{214}{3\sqrt{725}} = \dfrac{214}{15\sqrt{29}}$

▸ On cherche la droite p

On suppose $P(x,y) \in p$

$dist(P, d_1) = dist(P, d_2) = \dfrac{107}{15\sqrt{29}}$

▸ $dis(P, d_1) = \dfrac{32}{15\sqrt{29}} \Rightarrow \dfrac{|6x - 15y - 17|}{\sqrt{6^2 + 15^2}} = \dfrac{107}{15\sqrt{29}} \Leftrightarrow$

$\Rightarrow \dfrac{|6x - 15y - 17|}{\sqrt{261}} = \dfrac{107}{15\sqrt{29}} \Rightarrow$

$\Rightarrow \dfrac{|6x - 15y - 17|}{3\cancel{\sqrt{29}}} = \dfrac{107}{15\cancel{\sqrt{29}}} \Rightarrow |6x - 15y - 17| = \dfrac{107}{5}$

$\Rightarrow \begin{cases} 6x - 15y - 17 = \dfrac{107}{5} \Rightarrow 6x - 15y = \dfrac{192}{5} \\ 6x - 15y - 17 = -\dfrac{107}{5} \Rightarrow 6x - 15y = -\dfrac{22}{5} \end{cases}$

$\Rightarrow \begin{cases} 2x - 5y = \dfrac{64}{5} \\ 2x - 5y = -\dfrac{22}{15} \end{cases}$

▸ $dist(P, d_2 = \dfrac{107}{15\sqrt{29}} \Rightarrow \dfrac{|10x - 25y + 43|}{\sqrt{725}} = \dfrac{107}{15\sqrt{29}} \Rightarrow$

$\Rightarrow \dfrac{|10x - 25y + 43|}{5\cancel{\sqrt{29}}} = \dfrac{107}{15\cancel{\sqrt{29}}} \Rightarrow$

$\Rightarrow |10x - 25y + 43| = \dfrac{107}{3} \Rightarrow$

$$\Rightarrow \begin{cases} 10x - 25y + 43 = \frac{107}{3} \\ 10x - 25y + 43 = -\frac{107}{3} \end{cases} \Rightarrow$$

$$\begin{cases} 10x - 25y = -\frac{22}{3} \Rightarrow 2x - 5y = -\frac{22}{15} \\ 10x - 25y = -\frac{236}{3} \Rightarrow 2x - 5y = -\frac{236}{15} \end{cases}$$

Alors la droite qu'on cherche est

$2x - 5y = -\frac{22}{15} \Rightarrow \underbrace{30x - 75 + 22 = 0}$

Exercice 25 a) $d_1 : 4x + 7y - 9 = 0$ $d_2 : x - 8y + 8 = 0$

Soit $P(x, y) \in$ la bissectrice $\Rightarrow$

$dist(P, d_1) = dist(P, d_2) \Rightarrow$

$$\frac{|4x + 7y - 9|}{\sqrt{4^2 + 7^2}} = \frac{|x - 8y + 8|}{\sqrt{1 + 8^2}} \Rightarrow$$

$$\Rightarrow \frac{|4x + 7y - 9|}{\cancel{\sqrt{65}}} = \frac{|x - 8y + 8|}{\cancel{\sqrt{65}}} \Rightarrow$$

$$\Rightarrow \begin{cases} 4x + 7y - 9 = x - 8y + 8 \\ 4x + 7y - 9 = -x + 8y - 8 \end{cases} \Rightarrow$$

$$\begin{cases} 3x + 15y - 17 = 0 & b_1 \\ 5x - y - 1 = 0 & b_2 \end{cases}$$

b) $d_1 : x + 2y - 3 = 0$ $d_2 : 11x - 2y - 9 = 0$

$dist(P, d_1) = dis(P, d_2) \Rightarrow$

$$\Rightarrow \frac{|x + 2y - 3|}{\sqrt{5}} = \frac{|11x - 2y - 9|}{\sqrt{125}} \Leftrightarrow$$

$$\frac{|x+2y-3|}{\cancel{\sqrt{5}}} = \frac{|11x-2y-9|}{5\cancel{\sqrt{5}}} \Leftrightarrow$$

$$\begin{cases} 5x+10y-15 = 11x-2y-9 \\ 5x+10y-15 = -11x+2y+9 \end{cases} \Rightarrow$$

$$\begin{cases} -6x+12y-6=0 \Rightarrow x-2y+1=0 \ (b_1) \\ 16x+8y-24=0 \Rightarrow 2x+y-3=0 \ (b_2) \end{cases}$$

EXERCICE 26 a) $P_0(-1;3)$ $r=3$ $(x+1)^2+(y-3)^2=9$

• $\cap Ox$: $y=0$ $(x+1)^2+9=9 \Leftrightarrow x+1=0 \Rightarrow x=-1$
$(-1,0)$

• $\cap Oy$: $x=0$ $1^2+(y-3)^2=9 \Rightarrow (y-3)^2=8 \Rightarrow$
$y-3=\pm\sqrt{8} \Rightarrow y_1=2\sqrt{2}+3$
$y_2=-2\sqrt{2}+3$

b) $P_0(3;-5)$ $r=\sqrt{18}$
$(x-3)^2+(y+5)^2=18$

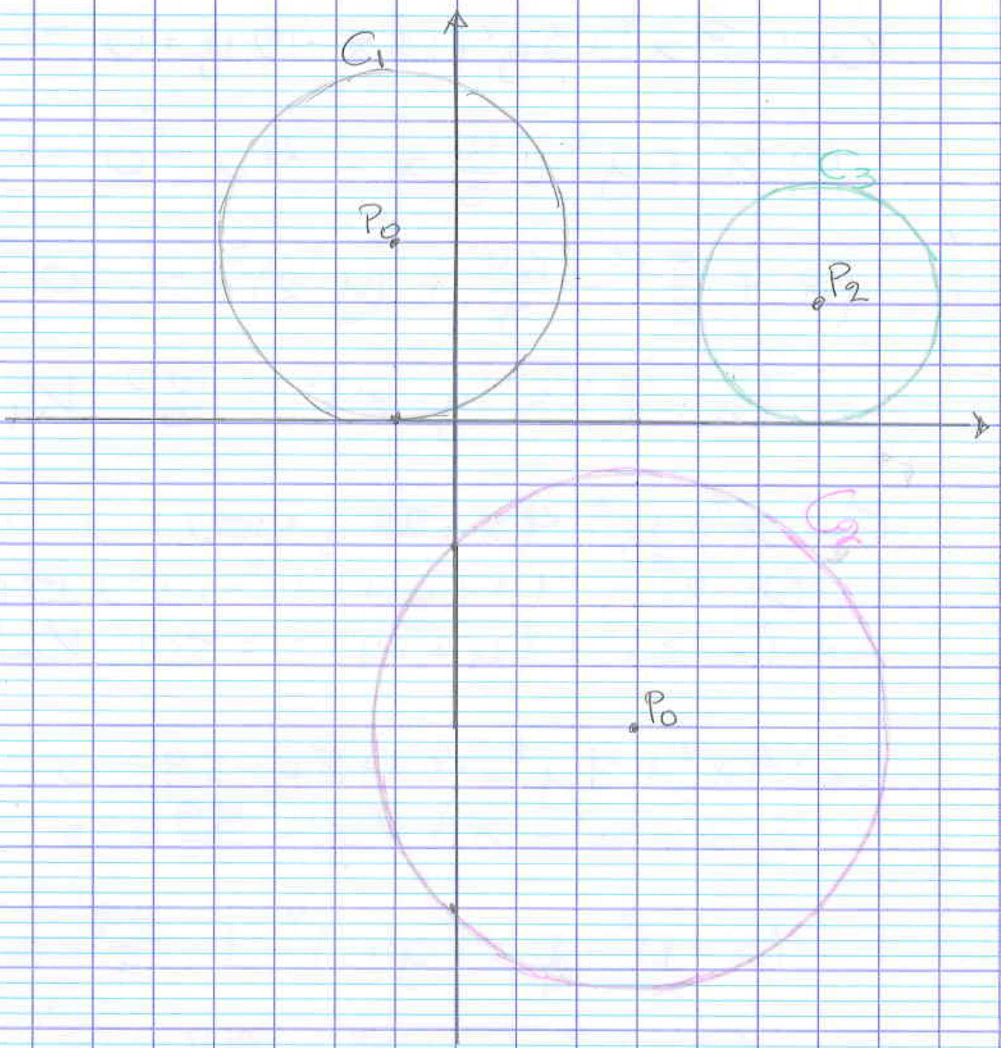
• $\cap Ox$: $y=0$ $(x-3)^2+25=18 \Rightarrow (x-3)^2=-7$ imp

• $\cap Oy$ $x=0$ $(-3)^2+(y+5)^2=18 \Rightarrow$
$\Rightarrow (y+5)^2=9 \Rightarrow y+5=\pm 3$
$y_1=-2$ $y_2=-8$ $(0;-2)$ $(0;-8)$

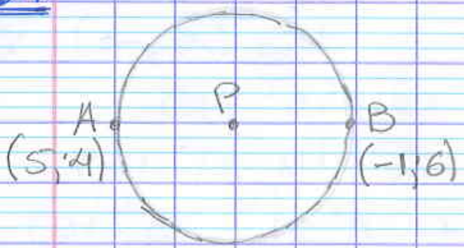
c) $P_0(6;2)$ $r=2$ $(x-6)^2+(y-2)^2=4$

• $\cap Ox$ $y=0$ $(x-6)^2+4=4 \Rightarrow x=6$ $(6;0)$

• $\cap Oy$ $x=0$ $(-6)^2+(y-2)^2=4 \Rightarrow (y-2)^2=-32$ imp



Exercice 27



P milieu de AB

$P\left(\frac{5-1}{2} ; \frac{4+6}{2}\right) \Rightarrow P(2;5)$

$r = \|\vec{PB}\|$

$\vec{PB} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \Rightarrow \|\vec{PB}\| = \sqrt{9+1} = \sqrt{10}$

$C : (x-2)^2 + (y-5)^2 = 10$

Exercice 28

$C_1 : (x+4)^2 + (y-3)^2 = 100 \quad K_1(-4;3) \quad r_1 = 10$

$C_3 : x^2 + y^2 - 14x - 10y + 70 = 0 \Rightarrow$

$(x-7)^2 - 49 + (y-5)^2 - 25 + 70 = 0 \Rightarrow$

$(x-7)^2 + (y-5)^2 = 4 \quad K_3(7,5) \quad r_3 = 2$

$C_2 : x^2 + y^2 - 4x - 6y - 3 = 0 \Rightarrow$

$(x-2)^2 - 4 + (y-3)^2 - 9 - 3 = 0 \Rightarrow$

$(x-2)^2 + (y-3)^2 = 16 \quad K_2(2;3) \quad r_2 = 4$

$C_4: 3x^2+3y^2+16x-12y=0 \Rightarrow$

$\Rightarrow x^2+y^2+\frac{16}{3}x-4y=0 \Rightarrow$

$\left(x+\frac{8}{3}\right)^2-\frac{64}{9}+(y-2)^2-4=0 \Rightarrow$

$\Rightarrow \left(x+\frac{8}{3}\right)^2+(y-2)^2=\frac{100}{9}$ $K_4\left(-\frac{8}{3};2\right)$ $r_4=\frac{10}{3}$

C_5 $x^2+y^2-4x+28y+150=0 \Rightarrow$

$(x-2)^2-4+(y+14)^2-14^2+150=0$

$\Rightarrow (x-2)^2+(y+14)^2=50$ $K_5(2;-14)$

$r_5=\sqrt{50}$

$C_6: 4x^2+4y^2-4x+8y-59=0$

$\Rightarrow x^2+y^2-x+2y-\frac{59}{4}=0$

$\left(x-\frac{1}{2}\right)^2-\frac{1}{4}+(y+1)^2-1-\frac{59}{4}=0$

$\Rightarrow \left(x-\frac{1}{2}\right)^2+(y+1)^2=\frac{64}{4}=16$

$K_6\left(\frac{1}{2};-1\right)$ $r_6=4$

EXERCICE 29

$P_0(1;2)$ $r=\|\vec{P_0P}\|=\sqrt{12^2+5^2}=13$

$\vec{P_0P}=\begin{pmatrix}12\\-5\end{pmatrix}$ $(x-1)^2+(y-2)^2=169$

- $A(1;-11)$ $(1-1)^2+(-11-2)^2=13^2=169 \Rightarrow A\in C$
- $B(11;-6)$ $(11-1)^2+(-6-2)^2=100+64=164<169$

B interieur

- $C(6;14)$ $(6-1)^2+(14-2)^2=25+12^2=169 \Rightarrow$

$C\in C$

- $D(9;14)$ $(9-1)^2+(14-2)^2=64+144=208>169$

D exterieur

EXERCICE 30 $C : (x+3)^2 + (y-5)^2 = 10$

$d_1 : 2x - y + 10 = 0 \quad d_2 : x - 3y + 8 = 0$

$d_3 : 4x + 3y - 19 = 0$

$d_1 : \ y = 2x + 10$

$d_1 \cap C : \ (x+3)^2 + (2x+10-5)^2 = 10 \Rightarrow (x+3)^2 + (2x+5)^2 = 10$

$\Leftrightarrow x^2 + 6x + 9 + 4x^2 + 20x + 25 - 10 = 0 \Leftrightarrow$

$\Leftrightarrow 5x^2 + 26x + 24 = 0 \qquad \Delta = 196$

$x_{1,2} = \dfrac{-26 \pm 14}{10}$; $+$: $x_1 = -\dfrac{12}{10} \Rightarrow x_1 = -\dfrac{6}{5}$; $-$: $x_2 = -\dfrac{40}{10} \Rightarrow x_2 = -4$

$x_1 = -\dfrac{6}{5} \quad y_1 = -\dfrac{12}{5} + 10 \Rightarrow y_1 = \dfrac{38}{5} \qquad \left(-\dfrac{6}{5} ; \dfrac{38}{5}\right)$

$x_2 = -4 \quad y_2 = -8 + 10 \Rightarrow y_2 = 2 \qquad (-4 ; 2)$

$d_2 : \ x = 3y - 8$

$d_2 \cap C : \ (3y-8+3)^2 + (y-5)^2 = 10 \Rightarrow (3y-5)^2 + (y-5)^2 = 10$

$\Rightarrow 9y^2 - 30y + 25 + y^2 - 10y + 25 = 10$

$\Rightarrow 10y^2 - 40y + 40 = 0 \Rightarrow y^2 - 4y + 4 = 0$

$\Rightarrow (y-2)^2 = 0 \Rightarrow y = 2 \quad x = -2 \quad (-2 ; 2)$

$d_3 : \ 4x + 3y = 19 \Rightarrow 3y = -4x + 19 \Rightarrow y = -\dfrac{4}{3}x + \dfrac{19}{3}$

$d_3 \cap C : \ (x+3)^2 + \left(-\dfrac{4}{3}x + \dfrac{19}{3} - 5\right)^2 = 10 \Leftrightarrow (x+3)^2 + \left(-\dfrac{4}{3}x + \dfrac{4}{3}\right)^2 = 10$

$\Rightarrow x^2 + 6x + 9 + \dfrac{16}{9}x^2 - \dfrac{32}{9}x + \dfrac{16}{9} - 10 = 0 \Rightarrow$

$\Rightarrow \dfrac{25}{9}x^2 + \dfrac{22}{9}x + \dfrac{7}{9} = 0 \Rightarrow 25x^2 + 22x + 7 = 0$

$\Delta = -216 < 0 \Rightarrow$ impossible

EXERCICE 31 $C_1 : x^2 + y^2 = 25$

$C_2 : (x-9)^2 + (y+2)^2 = 40 \Rightarrow x^2 - 18x + 81 + y^2 + 4y + 4 - 40 = 0$

$x^2 - 18x + y^2 + 4y + 45 = 0$

$C_1 \cap C_2 : \cancel{x^2} + \cancel{y^2} - 25 = \cancel{x^2} + \cancel{y^2} - 18x + 4y + 45 \Leftrightarrow$

$18x - 4y - 70$

$9x - 2y - 35 = 0 \Rightarrow y = \frac{9}{2}x - \frac{35}{2}$

axe radical

$C_1 : x^2 + \left(\frac{9}{2}x - \frac{35}{2}\right)^2 = 25 \Leftrightarrow$

$\Leftrightarrow x^2 + \frac{81}{4}x^2 - \frac{315}{2}x + \frac{1225}{4} - 25 = 0$

$\Leftrightarrow \frac{85}{4}x^2 - \frac{630}{4}x + \frac{1125}{4} = 0$

$\Leftrightarrow 85x^2 - 630x + 1125 = 0$

$\Delta = 14400 \Rightarrow x_{1,2} = \frac{630 \pm 120}{170}$

$x_1 = \frac{510}{170} = \frac{51}{17} \Rightarrow x_1 = 3 \quad y_1 = \frac{27}{2} - \frac{35}{2} = -4$

$I_1(3 ; -4) \qquad I_2\left(\frac{75}{17} ; \frac{40}{17}\right)$

$x_2 = \frac{750}{170} = \frac{75}{17} \Rightarrow y_2 = \frac{675}{34} - \frac{35}{2} = \frac{80}{34} = \frac{40}{17}$

$C_3 : (x-6)^2 + (y-7)^2 = 10 \Rightarrow$

$x^2 - 12x + 36 + y^2 - 14y + 49 - 10 = 0 \Rightarrow$

$\Rightarrow x^2 + y^2 - 12x - 14y + 75 = 0$

$C_1 \cap C_3 : \cancel{x^2} + \cancel{y^2} - 25 = \cancel{x^2} + \cancel{y^2} - 12x - 14y + 75 = 0$

$12x + 14y - 100 = 0 \Rightarrow 6x + 7y - 50 = 0$

$\Rightarrow 7y = -6x + 50 \Rightarrow y = -\frac{6}{7}x + \frac{50}{7}$

$x^2 + \left(-\frac{6}{7}x + \frac{50}{7}\right)^2 = 25 \Rightarrow$

$x^2 + \frac{36}{49}x^2 - \frac{600}{49}x + \frac{2500}{49} - 25 = 0 \Rightarrow$

$\Rightarrow \frac{85}{49}x^2 - \frac{600x}{49} + \frac{1275}{49} = 0 \Rightarrow$

$\Rightarrow 89x^2 - 600x + 1275 = 0 \quad \Delta < 0$ impossible

$\ell.$ $K_1(0,0) \quad K_3 = (6,7)$

$\vec{K_1K_3} = \begin{pmatrix} 6 \\ 7 \end{pmatrix} \quad \|\vec{K_1K_3}\| = \sqrt{36+49} = \sqrt{85} \approx 9.22$

$r_1 = 5 \quad r_3 = \sqrt{10} \quad K_1K_3 > r_1 + r_2 \quad \checkmark$

$C_2 \cap C_3 : \cancel{x^2} - 18x + \cancel{y^2} + 4y + 45 = \cancel{x^2} + \cancel{y^2} - 12x - 14y + 75 \Rightarrow$

$\Rightarrow -6x + 10y - 30 = 0 \Rightarrow$

$\Rightarrow 3x - 5y + 15 = 0 \Rightarrow 5y = 3x + 15$

$\Rightarrow y = \frac{3}{5}x + 3.$

$C_2 : (x-9)^2 + \left(\frac{3}{5}x + 3 + 2\right)^2 = 40 \Rightarrow$

$\Rightarrow (x-9)^2 + \left(\frac{3}{5}x + 5\right)^2 - 40 = 0 \Leftrightarrow$

$\Leftrightarrow x^2 - 18x + 81 + \frac{9}{25}x^2 + 6x + 25 - 40 = 0$

$\Leftrightarrow \frac{35}{25}x^2 - 12x + 66 = 0 \Rightarrow$

$\Rightarrow 35x^2 - 300x + 1650 = 0 \Rightarrow$

$\Rightarrow 7x^2 - 60x + 330 = 0. \quad \Delta = -5640 < 0$

$K_2(9;-2) \quad K_3(6,7)$

$\vec{K_2K_3} = \begin{pmatrix} -3 \\ 9 \end{pmatrix} \quad \|K_2K_3\| = \sqrt{9+81} = \sqrt{90} \approx 9.49$

$r_2 = \sqrt{40} \quad r_3 = \sqrt{10} \quad r_1 + r_3 < K_2K_3 \Rightarrow$

$\Rightarrow$ disjoints

EXERCICE 32 $\quad y = mx + 5 \quad C : x^2 + y^2 = 16 \quad K(0;0) \quad r = 4$

$\Rightarrow mx - y + 5 = 0$

$\text{dist}(K, d) = 4 \Rightarrow \frac{|m \cdot 0 - 0 + 5|}{\sqrt{m^2+1}} = 4 \Rightarrow$

$\Rightarrow 5 = 4\sqrt{m^2+1} \Rightarrow 25 = 16(m^2+1) \Rightarrow \frac{25}{16} - 1 = m^2 \Rightarrow$

$\Rightarrow \frac{9}{16} = m^2 \Rightarrow m = \pm\frac{3}{4}$

$d_1 : \quad y = \frac{3}{4}x + 5 \quad d_2 : y = -\frac{3}{4}x + 5$

EXERCICE 33 $P_0(3;-1)$ $t: d_1: 4x-3y-3=0$

r_1? $dist(P_0, d_1) = r \Leftrightarrow$

$\Leftrightarrow \frac{|12+3-3|}{\sqrt{16+9}} = r \Rightarrow r = \frac{12}{5}$

C_1 $(x-3)^2+(y+1)^2 = \left(\frac{12}{5}\right)^2$

$r_2 := dist(P_0, d_2)$ $d_2: x-y-2=0$

$\Rightarrow r_2 = \frac{|3+1-2|}{\sqrt{1+1}} \Rightarrow r_2 = \frac{2}{\sqrt{2}} \Rightarrow r_2 = \sqrt{2}$

$C_2: (x-3)^2+(y+1)^2 = 2$

EXERCICE 34 $A(-4;-3)$ $B(0;5)$ $C(6;-13)$

$P_0(x_0, y_0)$ le centre du cercle

$(x-x_0)^2+(y-y_0)^2 = r^2$

A: $(-4-x_0)^2+(-3-y_0)^2 = r^2 \Rightarrow$

$\Rightarrow 16+8x_0+x_0^2+9+6y_0+y_0^2 = r^2 \Rightarrow$

$x_0^2+y_0^2+8x_0+6y_0+25 = r^2$ ①

B: $(-x_0)^2+(5-y_0)^2 = r^2 \Rightarrow x_0^2+25-10y_0+y_0^2 = r^2$ ②

C: $(6-x_0)^2+(-13-y_0)^2 = r^2 \Rightarrow$

$36-12x_0+x_0^2+169+26y_0+y_0^2 = r^2$

$\Rightarrow x_0^2+y_0^2-12x_0+26y_0+205 = r^2$

①,② $\cancel{x_0^2}+\cancel{y_0^2}+8x_0+6y_0+\cancel{25} = \cancel{x_0^2}+\cancel{y_0^2}-10y_0+\cancel{25}$

$\Leftrightarrow 8x_0+16y_0 = 0 \Rightarrow x_0+2y_0 = 0$ ④

①,③ $\cancel{x_0^2}+\cancel{y_0^2}+8x_0+6y_0+25 = \cancel{x_0^2}+\cancel{y_0^2}-12x_0+26y_0+205$

$20x_0-20y_0-180 = 0 \Rightarrow x_0-y_0-9 = 0$ ⑤

$$\begin{array}{l} ④ \\ ⑤ \end{array}\begin{cases} x_0+2y_0 = 0 \\ x_0-y_0 = 9 \end{cases}$$

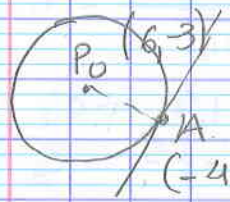
$3y_0 = -9 \Rightarrow y_0 = -3$

$x_0 = 6$ $P_0(6;-3)$

① $r^2 = (-4-6)^2 + (-3+3)^2 \Rightarrow r^2 = 100 \Rightarrow r = 10$

$C : (x-6)^2 + (y+3)^2 = 10^2$

$\underline{\underline{t_A}}$



$\vec{n}_A = \overrightarrow{P_0A} = \begin{pmatrix} -10 \\ 0 \end{pmatrix} // \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$t_A : x + c = 0$

$A : -4 + c = 0 \Rightarrow c = 4$

$t_A : x + 4 = 0$

$\underline{\underline{t_B}}$ $\vec{n}_B = \overrightarrow{P_0B} = \begin{pmatrix} -6 \\ 8 \end{pmatrix} // \begin{pmatrix} -3 \\ 4 \end{pmatrix} // \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

$t_B : 3x - 4y + c = 0$

$B(0;5)$ $-20 + c = 0 \Rightarrow c = 20$

$t_B : 3x - 4y + 20 = 0$

$t_C : \vec{n}_C = \overrightarrow{P_0C} = \begin{pmatrix} 0 \\ -10 \end{pmatrix} // \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow t_C : y + c = 0$

$C(6,-13) : -13 + c = 0 \Rightarrow c = 13$

$t_C : y + 13 = 0$

<u>EXERCICE 35</u> a) $x^2 + y^2 - 4x + 10y + 12 = 0$

$\Leftrightarrow (x-2)^2 - 4 + (y+5)^2 - 25 + 12 = 0 \Rightarrow$

$\Leftrightarrow (x-2)^2 + (y+5)^2 = 17$ $P_0(2;-5)$ $r = \sqrt{17}$

• $(6,y) : (6-2)^2 + (y+5)^2 = 17 \Rightarrow (y+5)^2 = 1 \Rightarrow$

$y + 5 = 1 \Rightarrow y_1 = -4$ $A(6;-4)$

$y + 5 = -1 \Rightarrow y_2 = -6$ $B(6;-6)$

t_A: $\vec{n}_A = \overrightarrow{P_0A} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $4x + y + c = 0$

$A(6,-4)$ $24 - 4 + c = 0 \Rightarrow c = -20$

$t_A : 4x + y - 20 = 0$

t_B: $\vec{n}_B = \overrightarrow{P_0B} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ $4x - y + c = 0$

$B(6,-6)$ $24 + 6 + c = 0 \Rightarrow c = -30$

$t_B : 4x - y - 30 = 0$

$P_0(2;-5)$ $r=\sqrt{17}$ $t \parallel d: x-4y+10=0$

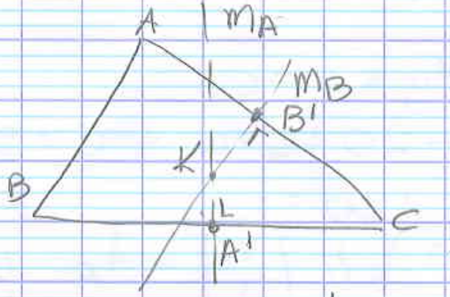
$t: x-4y+c=0$

$\text{dist}(P_0, t)=\sqrt{17} \Rightarrow \dfrac{|2+20+c|}{\sqrt{1+16}}=\sqrt{17} \Rightarrow$

$|c+22|=17 \Rightarrow \begin{cases} c+22=17 \Rightarrow c_1=-5 \\ c+22=-17 \Rightarrow c_2=-39 \end{cases}$

$t_1: x-4y-5=0$ $t_2: x-4y-39=0$

b) $A(-1,4)$ $B(7,6)$ $C(-2;3)$



$\underline{m_A}$ $A'\left(\frac{5}{2};\frac{9}{2}\right)$

$\vec{n}_A=\vec{BC}=\begin{pmatrix}-9\\-3\end{pmatrix} \parallel \begin{pmatrix}3\\1\end{pmatrix}$

$m_A: 3x+y+c=0$

$A': \frac{15}{2}+\frac{9}{2}+c=0 \Rightarrow c=-12$

$\underline{m_A}: 3x+y-12=0$

$\underline{m_B}$; $B'\left(\frac{-3}{2};\frac{7}{2}\right)$ $\vec{n}_B=\vec{AC}=\begin{pmatrix}-1\\-1\end{pmatrix} \parallel \begin{pmatrix}1\\1\end{pmatrix}$

$x+y+c=0$ $-\frac{3}{2}+\frac{7}{2}+c=0 \Rightarrow c=-2$

$\underline{m_B}$ $x+y-2=0$

$K=m_A \cap m_B \begin{cases} 3x+y-12=0 \\ x+y-2=0 \end{cases}$ $y=-3$

$2x-10=0 \Rightarrow x=5$

$K(5;-3)$ $r=\|\vec{KA}\|=\sqrt{25+49}=\sqrt{74}$

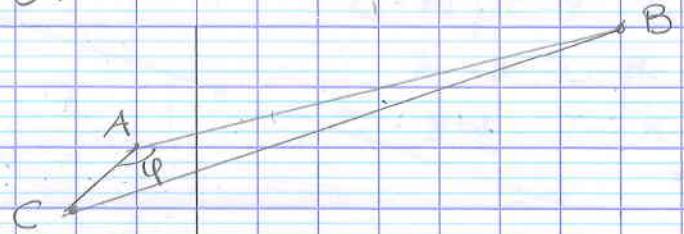
$\vec{KA}=\begin{pmatrix}-5\\7\end{pmatrix}$

$(x-5)^2+(y+3)^2=74$

$\overrightarrow{AB} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$ $\|\overrightarrow{AB}\| = \sqrt{64+4} = \sqrt{68}$

$\overrightarrow{AC} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ $\|\overrightarrow{AC}\| = \sqrt{1+1} = \sqrt{2}$

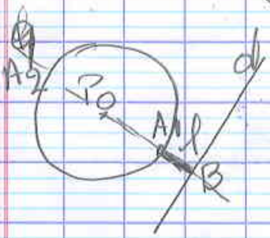
$\overrightarrow{BC} = \begin{pmatrix} -9 \\ -3 \end{pmatrix}$ $\|\overrightarrow{BC}\| = \sqrt{81+9} = \sqrt{90}$



$\cos\varphi = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{\|\overrightarrow{AB}\| \, \|\overrightarrow{AC}\|} = \dfrac{-8-2}{\sqrt{68}\ \sqrt{2}} \Rightarrow \varphi = 149°$

c) $d: x - 3y + 7 = 0$ $\quad C: (x-4)^2 + (y+2)^2 = 10$

$P_0(4, -2)$ $\quad r = \sqrt{10}$



$\text{dist}(P_0, d) = \dfrac{|4+6+7|}{\sqrt{1+9}} = \dfrac{17}{\sqrt{10}} = 5.37 > r$

$l = 5.37 - \sqrt{10} = 2.21$

On cherche la droite $q \perp d$

$d: x - 3y + 7 = 0 \Rightarrow \vec{n}_d = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \Rightarrow \vec{n}_q = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$q: 3x + y + c = 0$

$P_0: 12 - 2 + c = 0 \Rightarrow c = -10$

$q: 3x + y - 10 = 0$

$B = q \cap d: \begin{cases} 3x + y = 10 \\ x - 3y = -7 \end{cases} \cdot (+3) \quad \begin{cases} 9x + 3y = 30 \\ x - 3y = -7 \end{cases}$

$10x = 23 \Rightarrow x = 2.3$ $\quad y = 10 - 3 \cdot 2.3 \Rightarrow$

$y = 3.1$ $\quad \underline{B(2.3; 3.1)}$

$A: C \cap q: \quad q: y = -3x + 10$

$(x-4)^2 + (-3x+12)^2 = 10 \Leftrightarrow$

$x^2 - 8x + 16 + 9x^2 - 72x + 144 - 10 = 0 \Rightarrow$

$10x^2 - 80x + 150 = 0 \Rightarrow x^2 - 8x + 15 = 0$

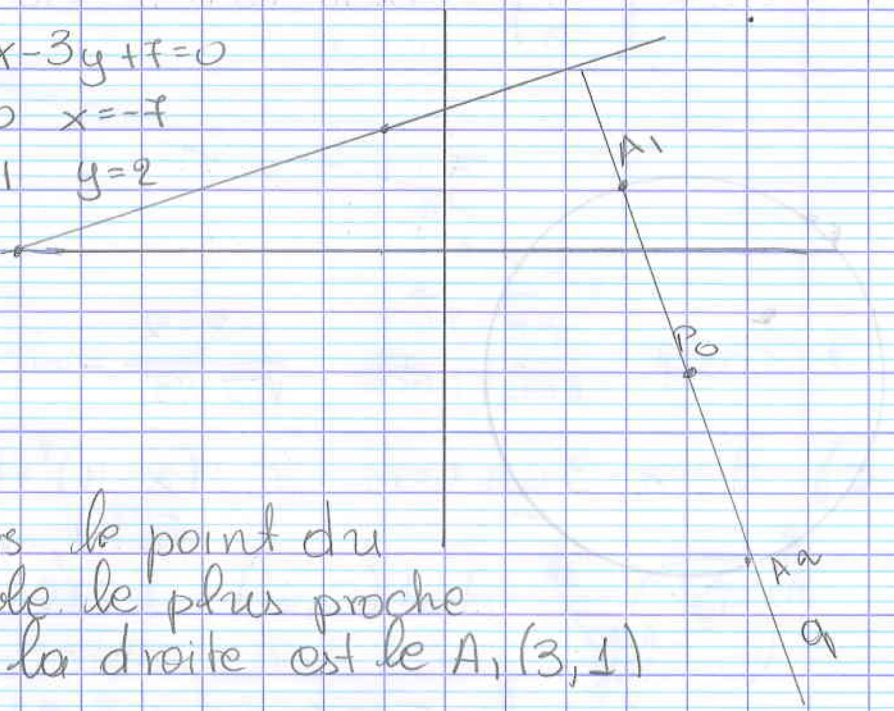
$x_1 = 3 \quad x_2 = 5$

$A_1 \quad y_1 = 1 \quad y_2 = -5 \; A_2$

$d: x - 3y + 7 = 0$

$y = 0 \quad x = -7$

$x = -1 \quad y = 2$



Alors le point du cercle le plus proche de la droite est le $A_1(3,1)$

d) $M(5;-1) \quad (x-5)^2 + (y+1)^2 = r^2$

$t: 3x + 4y - 36 = 0$

$r = dist(M,t) = \frac{|15 - 4 - 36|}{\sqrt{9+16}} = \frac{25}{5} = 5 = r$

$(x-5)^2 + (y+1)^2 = 5^2$

$d: 2x + 3y - 8 = 0 \Rightarrow 3y = -2x + 8 \Rightarrow$

$y = -\frac{2}{3}x + \frac{8}{3}$

$C: (x-5)^2 + \left(-\frac{2}{3}x + \frac{8}{3} + 1\right)^2 = 25 \Rightarrow$

$\Rightarrow (x-5)^2 + \left(\frac{4}{9}x^2 - \frac{44}{9}x + \frac{121}{9}\right) - 25 = 0$

$\Rightarrow x^2 - 10x + \cancel{25} + \frac{4}{9}x^2 - \frac{44}{9}x + \frac{121}{9} - \cancel{25} = 0$

$\Rightarrow \frac{13}{9}x^2 - \frac{134}{9}x + \frac{121}{9} = 0 \Rightarrow$

$13x^2 - 134x + 121 = 0 \quad \Delta = 11664$

$x_{1,2} = \frac{134 \pm 108}{26} < \begin{matrix} x_1 = 9.3 & y_1 = -3.53 \\ x_2 = 1 & y_2 = 2 \end{matrix}$