



CALCUL ALGÈBRIQUE

Exercice 1.1

Décrire les ensembles suivants à l'aide d'intervalles. Représenter ces intervalles sur la droite réelle puis donner une expression du complémentaire des intervalles A à H

$$A = \{x \mid x \in \mathbb{R} \text{ et } -3 < x < 5\}$$

$$B = \{x \mid x \in \mathbb{R} \text{ et } x < 1\}$$

$$C = \{x \mid x \in \mathbb{R} \mid x > 3 \text{ ou } x \leq -1\}$$

$$D = \mathbb{R}$$

$$E = \{x \mid x \in \mathbb{R} \text{ et } |x| \leq 8\}$$

$$F = \{x \mid x \in \mathbb{R} \text{ et } 1 < x^2 < 5\}$$

$$G = \{x \mid x \in \mathbb{R} \text{ et } x \geq 10\}$$

$$H = \{x \mid x \in \mathbb{R} \text{ et } |x| > 4\}$$

Exercice 1.2

Sans calculatrice, évaluer

$$A = 2 - 5 \cdot 4 + 18 \div 6$$

$$B = 2 \cdot (5 \cdot 9) + (-4) - 18 \div 2$$

$$C = (\sqrt{25} - 3)^0 - 28 : 4 + 4(2 + 3 \cdot (-4))^2$$

Exercice 1.3

Evaluer les expressions quand $x = 2$ et $x = -3$.

$$A(x) = (((x + 5)x + 3)x - 8)x + 3$$

$$B(x) = [(x + 5)x + 3](x - 8)x + 3$$

$$C(x) = x(1 - x[1 - x(1 - x)])$$

$$D(x) = (40 - [30 - (20 + x)x]x)x$$

Exercice 1.4

Calculer

$$A = \frac{5}{6} + \frac{6}{7} - \frac{7}{8}$$

$$B = \frac{13}{24} \cdot \frac{36}{7}$$

$$C = \frac{85}{49} : \frac{34}{21}$$

$$D = \frac{2}{3 - \frac{5}{4}}$$

Exercice 1.5

Pour extraire les racines, on procède de la manière suivante :

$$\sqrt{80} = \sqrt{16 \cdot 5} = \sqrt{16} \cdot \sqrt{5} = 4\sqrt{5}$$

Procéder selon le même modèle pour extraire les racines suivantes :

$$A = \sqrt{12}$$

$$B = \sqrt{27}$$

$$C = \sqrt{72}$$

$$D = \sqrt{216}$$

Exercice 1.6

Calculer et simplifier au maximum

$$A = 1 - \left(\frac{7}{5} - \frac{3}{4}\right) : \left(\frac{2}{3} + \frac{1}{5}\right)$$

$$B = 3 - \left(\frac{13}{2} - \frac{5}{3}\right) : \left(\frac{7}{3} + \frac{5}{6}\right)$$

$$C = \left(2 + \frac{1}{a}\right) : \left(1 + \frac{2}{a}\right)$$

$$D = \frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}}$$

$$E = \frac{\frac{1}{ab} + \frac{1}{ac}}{\frac{1}{ab} + \frac{1}{bc}}$$

$$F = \frac{2t}{4-t^2} + \frac{1}{2-t}$$

$$G = \frac{1}{b} - \frac{1}{bc}$$

$$H = \frac{x-y}{2x} - (1-y)$$

$$I = \frac{1}{x-1} - \frac{2}{x^2-x}$$

$$J = \frac{1}{2-x} \cdot \frac{x-2}{4}$$

$$K = \frac{m^4 - 1}{(m+1)^2}$$

$$L = \frac{\frac{3}{x} - \frac{8}{x}}{\frac{x}{2} + \frac{x}{2}}$$

Exercice 1.7

Simplifier et rationaliser les expressions ci-dessous (plus petit nombre possible sous la racine, pas de racine au dénominateur et fractions réduites).

$$A = \frac{1}{\sqrt{3}}$$

$$B = \frac{\sqrt{27}}{\sqrt{15}}$$

$$C = \frac{12}{\sqrt{3}} - \sqrt{27}$$

$$D = \sqrt{63} - \sqrt{28}$$

$$E = \frac{3\sqrt{20} - 5\sqrt{15}}{\sqrt{5}}$$

$$F = \frac{\sqrt{12} - \sqrt{3}}{\sqrt{12}}$$

$$G = \frac{\sqrt{2}}{\sqrt{5} - \sqrt{2}}$$

$$H = \frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}}$$

$$I = \frac{7\sqrt{5} - 5\sqrt{7}}{\sqrt{7} + \sqrt{5}}$$

$$J = \frac{7 + 2\sqrt{10}}{\sqrt{2} + \sqrt{5}} - \frac{7 - 2\sqrt{10}}{\sqrt{2} - \sqrt{5}}$$

$$K = \frac{\sqrt{5} + \sqrt{3}}{1 - \sqrt{5}} - \frac{\sqrt{5} - \sqrt{3}}{1 + \sqrt{5}}$$

Exercice 1.8

Développer

$$A = (3a + 2b)(2a + 3b)$$

$$B = (5x + 4)(9 - 11x)$$

$$C = (a + 2b + 3c)(3a + 2b + c)$$

$$D = (x - 5y + 2)(2x + y - 2)$$

$$E = (3x + 2y)^2$$

$$F = [3(x + y)]^2$$

$$G = 3(x + y)^2$$

$$H = (a + b + c)(a + b - c)$$

$$I = (4x - \sqrt{6}y)^2$$

$$J = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$K = (\sqrt{2} - \sqrt{5})^2$$

$$L = (x - 3y)^3$$

$$M = (3x - y)(3x + y)$$

$$N = (2a + 3b)^4$$

Exercice 1.9

Compléter

$$A = 4x^2 + \dots + \dots = (\dots + 5)^2$$

$$B = \dots + 14x + \dots = (x + \dots)^2$$

$$C = x^2 - \dots + 100 = (\dots - \dots)^2$$

$$D = (3x + \dots)^2 = \dots + \dots + 25$$

$$E = 64 + \dots + 9x^4 = (\dots + \dots)^2$$

$$F = \dots - 12x + 4 = (\dots - \dots)^2$$

$$G = x^2 - 16 = (\dots + \dots)(\dots - \dots)$$

$$H = \dots - 24x + \dots = (2x - \dots)^2$$

Exercice 1.10

Simplifier

$$A = (2e^2 f^{-3})^4$$

$$B = \left(\sqrt[5]{\frac{p^2}{w^{-3}}} \right)^0$$

$$C = (100r^3 s^{-1}) : (4r^{-2} s^5 t)$$

$$D = \frac{\sqrt{x} \sqrt{y}}{\sqrt{y} \sqrt{x}}$$

$$E = \sqrt[4]{c^3} \cdot c$$

$$F = \left(125^{\frac{2}{3}} x^{-1} y^2 \right)^{-\frac{1}{2}}$$

$$G = (a^{-3} b^7)^{\frac{1}{2}} : \left(a^{\frac{1}{4}} b^{\frac{3}{2}} \right)^{-2}$$

$$H = \sqrt{\frac{\sqrt{2}}{64}}$$

$$I = \sqrt{\frac{a^{\frac{2}{3}}}{\sqrt[3]{a^{\frac{1}{2}}}}} \cdot \left(\left(a^{\frac{2}{3}} \right)^2 \right)^{\frac{1}{3}}$$

Exercice 1.11

Simplifier

$$A = a^4 \cdot a^{28}$$

$$E = \frac{h^4}{h^2}$$

$$B = x^{-25} \cdot x^{14}$$

$$F = b^{4n-7} \cdot b^{-n+7}$$

$$C = u^{x-4} \cdot u^{6-x}$$

$$G = \frac{z^5}{z^{3-6n}}$$

$$D = \frac{n^0}{n^{-5}}$$

$$H = \frac{e^x}{e^{-x}}$$

$$I = (-a)^5$$

$$N = (ab^0 c^{-3})^{-n}$$

$$J = (-x^6)^4$$

$$O = \frac{(-a)^6}{(-a)^3}$$

$$K = (a^5 b^{-8} x)^4$$

$$L = (-3x^4)^3$$

$$P = \frac{(-a)^{4n+3}}{(-a^n)^4}$$

$$M = (x^{-3} y^5)^3$$

Exercice 1.12

Simplifier

$$A = \left(\frac{a^5}{b^6} \right)^{-4} \left(\frac{a^4 b^{-4}}{c^7} \right)^3$$

$$B = \left(\frac{x^3}{y^4} \right)^6 \left(\frac{x^2 z^{-2}}{y^{-1}} \right)^{-3}$$

$$C = \left(81^{-\frac{1}{2}} \cdot (3x)^3 \right)^2 \cdot (y^{-5} xy)^2$$

$$D = \frac{(x^4 y^3)^n}{(x^2 y^4)^{2n-1}}$$

$$E = \frac{(a^{n-1} b^n)^2}{(a^2 b^3)^{n-1}}$$

$$F = \frac{a^{-2} b^{-10}}{a^7 b^{-8}} \cdot \frac{a^4 b^{-3}}{(a^{-3} b)^2}$$

$$G = \frac{(16x^4 y^2)^n}{(4x^2 y)^{2n-1}}$$

$$H = \frac{\sqrt{(2^6 \cdot x^{-12} \cdot y^{24})^{\frac{1}{3}}}}{(x^{-2} \cdot y)^{-4}}$$

Exercice 1.13

Simplifier

$$A = \frac{3a}{4b} - \frac{4b}{3a}$$

$$B = \frac{x - \frac{4}{x}}{1 - \frac{2}{x}}$$

$$C = a\sqrt{a}\sqrt{a}\sqrt{a}$$

$$D = \frac{a+2}{2a-6} - \frac{a-2}{2a+6}$$

$$E = \frac{(a^2)^{3n}}{\sqrt[3]{a^{6n-3}}}$$

$$F = \left(\frac{4a^{-2}}{9x^2} \right)^{-\frac{1}{2}}$$

$$G = \frac{(a^{-2})^3}{\sqrt{b} \cdot a^{\frac{2}{3}}} \cdot \frac{\sqrt{b^{-5}}}{\sqrt[3]{a}}$$