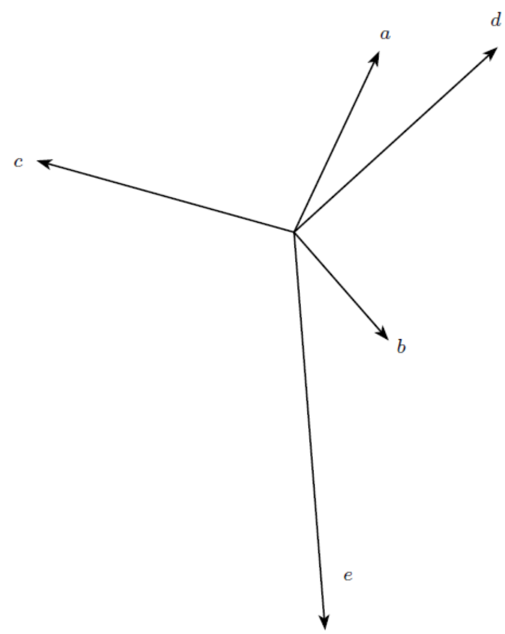
LDDR – Niveau II : Géométrie Plane

1.23 Exercises

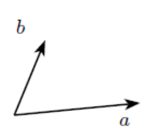
1. 1 : Given the vectors $\vec{a}$ and $\vec{b}$, precisely build the vectors

1) $\vec{c} = 2\vec{a} - \vec{b}$
2) $\vec{d} = \vec{b} - 3\vec{a}$
3) $\vec{e} = -2\vec{b} + \frac{1}{2}\vec{a}$
4) $\vec{f} = -\frac{7}{5}\vec{b}$
5) $\vec{g} = \frac{3}{5}\vec{a} + \frac{4}{3}\vec{b}$
6) $\vec{h} = \sqrt{2}\vec{a}$
7) $\vec{i} = -\sqrt{5}\vec{b}$
8) Build $\vec{m}$ so that $-\vec{a} + 2\vec{b} + \vec{m} = \vec{0}$. Then express $\vec{m}$ as a linear combination of $\vec{a}$ and $\vec{b}$.
9) $\vec{n}$ is defined by the vector equation $-4\vec{a}+3\vec{b}+2\vec{n} = \vec{0}$. Draw $\vec{n}$, then express it as a linear combination of $\vec{a}$ and $\vec{b}$.

1.2 : Use a diagram to decompose $\vec{c}$, $\vec{d}$ and $\vec{e}$ in the basis $(\vec{a};\vec{b})$. Then, indicate **an estimation** of the components of $\vec{c}=\begin{pmatrix}\cdots\\\cdots\end{pmatrix}$, $\vec{d}=\begin{pmatrix}\cdots\\\cdots\end{pmatrix}$ and $\vec{e}=\begin{pmatrix}\cdots\\\cdots\end{pmatrix}$.

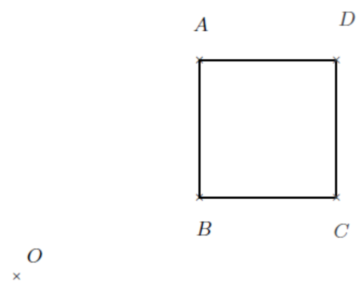


1.3 : In the basis $(\vec{a};\vec{b})$, we have $\vec{c}=-5\vec{a}+4\vec{b}$ and $\vec{d}=\begin{pmatrix}3\\y\end{pmatrix}$. Draw $\vec{c}$ and $\vec{d}$ given that they are linearly dependent. Then calculate the value of the unknown component y.



1.4 : Let $ABCD$ be a square. Place the points E, F, G and H so that :

$$\vec{AE} = \vec{AC} + \vec{BC} \qquad \vec{AF} = \vec{AO} - \vec{OC} \qquad \vec{CG} = 2\vec{CB} + \frac{1}{2}\vec{BD} \qquad \vec{OH} = \sqrt{2}\vec{CA}$$



FROM NOW ON, THE BASIS IS $(\vec{e}_1; \vec{e}_2)$

1.5 : Given the vectors $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$.

1) Complete $\vec{a} \mathbin{/\!/} \begin{pmatrix} -6 \\ \dots \end{pmatrix}$ $\quad \vec{b} \mathbin{/\!/} \begin{pmatrix} 7 \\ \dots \end{pmatrix}$ $\quad \vec{c} \mathbin{/\!/} \begin{pmatrix} \dots \\ -11 \end{pmatrix}$

2) Calculate the components of the vectors : $2\vec{a} - 3\vec{b}$ $\quad \frac{1}{3}\vec{a} + \frac{3}{2}\vec{c}$ $\quad -4\left(\vec{a} - \vec{b}\right) + 3\left(-\vec{b} + \vec{c}\right)$

3) Show that $(\vec{a}; \vec{b})$ is a basis. Then determine the components of $\vec{c}$ in the basis $(\vec{a}; \vec{b})$.
Hint : look for α, β so that $\vec{c} = \alpha\vec{a} + \beta\vec{b}$ and solve a 2x2 system.

4) Write $\vec{b}$ as a linear combination of $\vec{a}$ and $\vec{c}$.

1.6 :

1) Show that the vectors $\vec{a} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ are linearly independent.

2) Decompose $\vec{c} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ in the basis $(\vec{a}; \vec{b})$, by calculation and by drawing.

3) Determine m so that $\begin{pmatrix} 7 \\ m \end{pmatrix} \mathbin{/\!/} \vec{a}$

4) Determine n so that $\begin{pmatrix} n \\ -12 \end{pmatrix}$ and $(\vec{a} + \vec{b})$ are linearly dependent.

1.7 : Determine the vectors $\vec{a}$ and $\vec{b}$ that simultaneously satisfy the following three conditions : $\vec{a} \mathbin{/\!/} \vec{e_1}$, $\vec{b} \mathbin{/\!/} (2\vec{e_1} + \vec{e_2})$ and $3\vec{a} + \vec{b} = 7\vec{e_1} - \vec{e_2}$.

1.8 : We consider the points $A(3;4)$ and $B(-2;1)$, and we define some more points by :

$$\vec{OC} = \vec{AB},\ \ \vec{OD} = -\vec{AB},\ \ \vec{BE} = \vec{OA},\ \ \vec{BF} = -\vec{OA},\ \ \vec{AG} = \vec{OB},\ \ \vec{AH} = -\vec{OB}$$

Draw these points and use Chasles' relation to calculate their coordinates.

1.9 :

1) Complete thanks to Chasles' relation :

$$\begin{array}{llcl}
A(7;5) & B(-4;1) & \longrightarrow & \vec{AB} = \begin{pmatrix} \cdots \\ \cdots \end{pmatrix} \\
A(\ldots;\ldots) & B\left(2;\frac{1}{3}\right) & \longrightarrow & \vec{AB} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \\
A(\ldots;-2) & B(0;\ldots) & \longrightarrow & \vec{AB} = \begin{pmatrix} \cdots \\ 5 \end{pmatrix} \mathbin{/\!/} \begin{pmatrix} 7 \\ -4 \end{pmatrix} \\
A\left(\frac{1}{2};3\right) & B(\ldots;-1) & \longrightarrow & \vec{AB} = \begin{pmatrix} \cdots \\ \cdots \end{pmatrix} \mathbin{/\!/} \vec{e_2}
\end{array}$$

2) Determine by computations the coordinates of the midpoint of the segment CD with $C(2;-3)$ and $D(-1;-2)$

3) Determine the coordinates of the point E, the reflection point of $C(2;-3)$ in $D(-1;-2)$. Sketch the situation.

4) Determine the coordinates of $P'(x';y')$, the reflection point of $P(x;y)$ in $M(m_1;m_2)$.

5) $(6.5;-3)$ is the midpoint of DF, with $D(-1;-2)$. Find, by computations, the point F.

1.10 : We consider $A(4;-6)$, $\vec{AB} = 3\vec{e_1} + 2\vec{e_2}$, $\vec{BC} = -5\vec{e_1} + 4\vec{e_2}$, $\vec{OD} = -\vec{CB}$ and $\vec{OE} = \vec{OD} - \vec{BC}$. Determine the coordinates of the points B, C, D and E by computations.

1.11 : Let's consider $A(-2;5)$, $B(1;-3)$ and $C\left(\frac{3}{2};\frac{3}{2}\right)$.

1) Calculate the coordinates of the vertex D of the parallelogram with vertices $ABCD$.

2) Calculate the coordinates of the midpoint M of the parallelogram.

1.12 : Complete that table, each column being a separate question (M_{AB} is the midpoint of the segment AB):

A	$(4;9)$	$(0.5;-2)$	$(...;...)$	$(2;7)$	$(...;...)$	$(...;...)$
B	$(-2;5)$	$(...;...)$	$(0;7)$	$(...;...)$	$(-5.5;17)$	$(...;...)$
M_{AB}	$(...;...)$	$(3;3.5)$	$(-6;2)$	$(...;...)$	$(...;...)$	$(1;-5)$
$\vec{AB}$	$\begin{pmatrix} ... \\ ... \end{pmatrix}$	$\begin{pmatrix} ... \\ ... \end{pmatrix}$	$\begin{pmatrix} ... \\ ... \end{pmatrix}$	$\begin{pmatrix} -1.5 \\ 11 \end{pmatrix}$	$\begin{pmatrix} -6 \\ -2 \end{pmatrix}$	$\begin{pmatrix} 6 \\ -4 \end{pmatrix}$

1. 13 : A parallelogram with vertices $ABCD$ is given by the following information : $A(-3;2)$, center $M(-1;0)$, $\vec{AB} \,/\!/ \begin{pmatrix} 1 \\ -3 \end{pmatrix}$, $\vec{BM} \,/\!/ \vec{e_2}$.

1) Calculate the coordinates of the vertices B, C and D.

2) Check your answers thanks to a drawing.

1. 14 : Given a triangle by $A(3;2)$, $B(-1;4)$ and $C(0;-2)$. We consider a homothecy of centre $P(-2;1)$ and dilation factor $k = -2$. Calculate the coordinates of the images A', B' and C' and verify with a drawing.

1. 15 : We consider the homothecy with ratio $k = -5$ that is such that the image of $(-4;7)$ is $(2;19)$. Determine by computations the coordinates of the center of that transformation.

1. 16 : We consider the points $A(-3;7)$, $B(2;4)$ and $C(-5;1)$.

1) Determine the center of gravity of the triangle ABC.

2) Determine the coordinates of D such that the center of gravity of ABD is $(5;2)$.

1. 17 : We consider the line formed by the points $P(-2+3\lambda; 5-4\lambda)$, with $\lambda \in \mathbb{R}$, a parameter.
Calculate the coordinates of :

1) Point A, obtained with $\lambda = 0$

2) Point B, whose ordinate is 0

3) Point C, obtained with $\lambda = 1$

4) Point D, whose ordinate is the double of its abscissa

5) Point E, whose abscissa is 0

6) Point F, whose ordinate is 7

1. 18 : Find parametric and Cartesian equations for the following lines :

1) l_1 through $A(-2;3)$ and $B(8;5)$.

2) l_2 through $A(-4;1)$ and parallel to Ox

3) l_3 through O and parallel to the vector $\vec{d} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

1. 19 : Complete the following table, row by row :

Parametric equations	Cartesian equation	A point	A direction vector
$\begin{cases} x = 5 - 2\lambda \\ y = -1 + 3\lambda \end{cases}$			
	$x + 4y - 10 = 0$		
		$A(-7;1)$	$\begin{pmatrix} 6 \\ 5 \end{pmatrix}$
		$A(0;9)$	$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

1.20 : Given two lines $l_1 : \begin{cases} x = 1 - \lambda \\ y = 2 + 2\lambda \end{cases}$ and $l_2 : \begin{cases} x = 3 + \mu \\ y = 6 + 2\mu \end{cases}$.

1) Draw these lines after having determined a point and a direction vector for each of them.
2) By looking at your drawing, determine the coordinates of their intersection point.
3) How can we know that these two lines are secant without drawing them ?
4) Calculate the coordinates of the intersection point.
5) Write the Cartesian equations of these two lines.

1.21 : Given the triangle ABC with $A(-5;2)$, $B(2;7)$ and $C(3;-4)$.

1) Determine a Cartesian equation and parametric equations of the line through A and A', the midpoint of BC. That line is called the median through A and we denote it by m_A.
2) Does the point $D\,(10;1)$ belong to the median?

1.22 : We consider the lines $l_1 : -x + 2y + 3 = 0$ and $l_2 : 3x - 4y - 12 = 0$.

1) Calculate the intersections of the lines with the axes.
2) Give two direction vectors for each line
3) Determine parametric equations for these two lines.
4) What is the relative position of these two lines ? Justify.
5) Precisely draw these two lines.
6) Compute the coordinates of their intersection point. Check on your drawing.

1.23 : Compute the coordinates of the intersection point of the following lines

1) $l_1 : \begin{cases} x = 7 + 3\lambda \\ y = -1 + 2\lambda \end{cases}$ and $l_2 : \begin{cases} x = -4 - \mu \\ y = 5 + 7\mu \end{cases}$
2) $l_1 : \begin{cases} x = 7 + 3\lambda \\ y = -1 + 2\lambda \end{cases}$ and $l_2 : x + 8y - 5 = 0$
3) $l_1 : 3x - 2y + 6 = 0$ and $l_2 : x + 8y - 5 = 0$

1.24 : The square $ABCD$ is given by :

$$D(-7;2) \qquad C \in l_1 : 3x + y + 2 = 0 \qquad C \in l_2 : \begin{cases} x = 7 + 3\lambda \\ y = -2 - 2\lambda \end{cases}$$

Construct the square(s) and calculate the coordinates of the vertices A, B and C.

1.25 : Given the line $l_m : 4x - my + 2 = 0$. For which values of m (4 different questions):

1) does the line l_m pass through the point $A(2;-3)$?
2) is the line parallel to the y-axis ?
3) does the line have $\vec{t} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ as a direction vector ?
4) is it perpendicular to the line through $B(5;4)$ and $C(7;-1)$?

1. 26 : The triangle ABC is given by its vertex $A(3;1)$, its center of gravity $G(2;3)$ and by$C'(4;4)$, the midpoint of the segment AB. Determine the coordinates of the vertices B and C. An illustration of the situation may help.

1. 27 : The triangle ABC is given by its vertex $A(1;1)$ and its center of gravity $G\left(\frac{7}{3};\frac{1}{3}\right)$. You're also told that $\vec{BC}$ is parallel to $\vec{t}=\begin{pmatrix}-1\\4\end{pmatrix}$ and that the line through A and B is $l_{AB}: 5x+3y-8=0$.
Determine the coordinates of the vertices B and$C$ with a drawing and then by computation.

1. 28 :

1) Show that if the quadrilateral $ABCD$ is such that $\vec{AB}=\vec{DC}$, then it is a parallelogram.
2) Let $ABCD$ be any quadrilateral. Let's name $IJKL$ the midpoints of the sides. Proove "Varignon's theorem" : $IJKL$ is always a parallelogram (whatever the location of $ABCD$).

1. 29 : Determine the coordinates of the vertices of the triangle ABC given that $l_{AB}: 3x-5y+1=0$, $l_{AC}: x-9y-29=0$, $C\,(11;?)$ and $\vec{BC}\,/\!/\begin{pmatrix}2\\-1\end{pmatrix}$.

1. 30 : Given the vectors $\vec{a}=\begin{pmatrix}2\\-3\end{pmatrix}$, $\vec{b}=\begin{pmatrix}5\\7\end{pmatrix}$, $\vec{c}=\begin{pmatrix}-4\\3\end{pmatrix}$ and $\vec{d}=\begin{pmatrix}-5\\-2\end{pmatrix}$.

1) Calculate their norm.
2) Find the unit vector $\vec{u}$ which has same direction as $\vec{c}$.
3) Find a vector $\vec{e}$ orthogonal to $\vec{a}$ and that has the same length as $\vec{a}$.
4) The vector $\vec{f}=\begin{pmatrix}k\\-5\end{pmatrix}$ is perpendicular to $\vec{c}$. What is the value of k?
5) Calculate the scalar products $\vec{a}\bullet\vec{b}$, $\vec{b}\bullet\vec{c}$ and $\vec{b}\bullet\vec{d}$.
6) What vectors among $\vec{a}$,$\vec{c}$ and $\vec{d}$form an obtuse angle with $\vec{b}$?

1. 31 : Determine the type of the triangle ABC with $A\,(2;8)$, $B\,(-4;3)$ and $C\,(4;6)$. Then determine its perimeter and area.

1. 32 : Given the vectors $\vec{a}=\begin{pmatrix}6\\-8\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}7\\2\end{pmatrix}$

1) Draw $\vec{b'}$ the orthogonal projection of $\vec{b}$ on $\vec{a}$.
2) Calculate the norm of $\vec{b'}$.
3) Calculate the area of the triangle OAB, using at least two different methods.
4) Determine the components of $\vec{b'}$.

1. 33 :

1) Given $A(-1;-2)$ and $B(7;4)$ the vertices of the isosceles triangle ABC with base AB. Determine the coordinates of C so that the area of the triangle is 75. Give all the possible answers.

2) Given $A(-1;-2)$ and $B(7;4)$ Find the coordinates of C so that ABC is a right triangle with area 20. Give all the possible answers.

3) Given $A(-1;-2)$ and $B(7;4)$ Find the coordinates of C so that ABC is an isosceles triangle with area 30. Give all the possible answers.

1. 34 :

1) Calculate the angle between the vectors $\vec{a} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

2) Calculate the acute angle between the lines $a : 3x - 4y + 12 = 0$ and $b : 12x + 5y - 15 = 0$.

3) Calculate the angles of the triangle with vertices $A(2;8)$, $B(-4;3)$ and $C(4;6)$.

1. 35 : Find the Cartesian equations of the following lines:

1) a through $A(-2;3)$, direction vector $\vec{d} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$.

2) b through $B(3;-1)$, normal vector $\vec{n} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

3) c through $C(-6;0)$, perpendicular to the line a.

4) d through $D(5;2)$, parallel to the line b.

1. 36 :

1) Find a direction vector and two points of $l_1 : 3x - 4y - 12 = 0$

2) Find a normal vector and one point of $l_2 : 5x + 3y + 9 = 0$

3) Find a normal vector of l_3 the line through $A(-2;5)$ and $B(4;1)$

1. 37 : Given the line $a : 3x - 4y - 17 = 0$.

1) Determine the Cartesian equation of the line b that is perpendicular to a and that passes through $B(-3;6)$.

2) Calculate the coordinates of the point $I = a \cap b$.

3) Calculate the coordinates of the point C, symmetrical of B about the line a.

1. 38 : We consider the triangle with vertices $A(6;0)$, $B(0;4)$ and $C(-2;0)$.

1) Find the Cartesian equation of the perpendicular bisectors m_{AB}, m_{AC} and m_{BC}.

2) Calculate the coordinates of M the intersection of the perpendicular bisectors.

3) Determine the radius r of the circumcircle of the triangle ABC. Sketch the situation (unit : 2 squares).

1.39 : We consider the line l given by its equation $4x - 3y - 24 = 0$.

1) Calculate the distance from l to the points $O\,(0;0)\,, B\,(11;-10)$ and $C\,(9;4)$.

2) Find the Cartesian equations of the lines e and f that are at distance 2 from the line l. Sketch the situation.

1.40 : Given $a : 4x + 3y - 12 = 0$ and $b : 7x - y - 46 = 0$. Calculate the coordinates of the points of b that are at distance 5 from the line a. Solve the problem with a drawing first.

1.41 : We consider the line $a : 4x + 3y - 24 = 0$ and the line b that is parallel to a and passes through $B\,(0;13)$.

1) Calculate the distance between a and b.

2) Find the Cartesian equation of the line c formed by the points equidistant from a and b.

3) Find the Cartesian equation of the line l whose distance to a is twice its distance to b. Also use a drawing.

1.42 : Find the Cartesian equation of the bisectors of the lines $a : 3x - 4y + 12 = 0$ and $b : 12x + 5y - 15 = 0$.

1.43 : Given the vertices $A(-10;-8)$, $B(6;4)$ and $C(11;-8)$ of a triangle.

1) Calculate its area.

2) Find the Cartesian equation of the internal bisectors of the triangles ABC. Determine the coordinates of the centre and the radius of the incircle.

3) Calculate the coordinates of the contact point of the incircle and the side AB of the triangle. Sketch the situation.

1.44 :

1) Find the equation of the circle centered at $M\,(5;3)$ that has a radius equal to 5.

2) Determine the intersection points of the circle and the x-axis.

1.45 : Do the following equations describe circles ? If yes, give the coordinates of their centre and radius.

1) $x^2 + y^2 - 14x - 2y - 126 = 0$

2) $x^2 + y^2 + 10x + 14y + 123 = 0$

3) $x^2 + y^2 + 8x - 16y + 80 = 0$

4) $3x^2 + 3y^2 + 7x - 10 = 0$

5) $2x^2 + 3y^2 + 7x - 10 = 0$

6) $x^2 - y^2 - 14x - 2y - 126 = 0$

1.46 :

1) Write down the equation of the circle Γ with center $(-7;4)$ and radius 13.

2) The points $A(a_1;9)$ and $B(-2;b_2)$ belong to the circle Γ. Calculate a_1 (>0) and b_2 (<0).

3) Determine the equation of the perpendicular bisector m of the line segment AB.

4) Verify that the centre C lies on the line m.

1.47 : Determine the equation of the circle that passes through $(-3;3)$, $(-1;-3)$ and $(5;3)$.

1.48 : Given the circle $\Gamma : (x-5)^2 + (y+3)^2 = 25$.

1) Determine the relative position, with respect to the circle, of $P_1\,(7;-6)$, $P_2\,(8;1)$ and $P_3\,(1;-6)$.

2) Determine the equations of the tangent to Γ parallel to $\vec{d} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Name them t_1 and t_2. Find the coordinates of the contact points T_1 and T_2.

3) Determine the relative position of $l : -x + y + 7 = 0$ and Γ. Calculate the possible intersections.

1.49 : Determine the equation of the tangent to the circle $\Gamma : x^2 + y^2 + 10x + 2y + 13 = 0$ at the point $T\,(-3;2)$.

1.50 : Determine the intersection between the line $l : x + y - 4 = 0$ and the circle $\Gamma : (x-1)^2 + (y+3)^2 = 20$.

1.51 : Determine the equation of the circle centered at $M\,(-2;3)$ and tangent to the line $l : x + 2y = 0$.

1.52 : Find the equations of the lines tangent to the circle $\Gamma : (x-2)^2 + (y+5)^2 - 17 = 0$ that are parallel to the line $l : x - 4y + 10 = 0$.

1.53 : Given the points $P\,(-2;7)$, $Q\,(2;3)$ and $R\,(4;5)$. Prove that the triangle PQR is a right triangle. Determine the equation of the circle through P, Q and R.

1.54 : Determine the centre and the radius of the incircle of the triangle formed by the lines $l_1 : x + 2 = 0$, $l_2 : y - 3 = 0$ and $l_3 : 5x + 12y - 60 = 0$.

1.55 : Given the points $A(-3;3)$ and $B(4;0)$. Determine the coordinates of the points $P_1, P_2 \in l : y = x$ so that the triangle ABP is an isosceles triangle at B. Then, calculate the area of the quadrilateral AP_1BP_2.

1.56 : We consider the circle $x^2 + y^2 - 10x + 16 = 0$ and the lines $y = mx$ $(m \in \mathbb{R})$. For which values of m are the circle and the line secant ? And tangent ?

1.57 : Determine the equation of the circles centered on the line $l : 3x + 7y - 39 = 0$ and tangent to the lines $a : 3x - 4y + 12 = 0$ and $b : x = 0$.

1.58 : In each case, determine the points of intersection of the two circles below :

1) $\Gamma_1 : x^2 + y^2 = 25$ and $\Gamma_2 : (x+1)^2 + (y-1)^2 = 29$

2) $\Gamma_1 : (x+4)^2 + (y+5)^2 = 194$ and $\Gamma_2 : (x-3)^2 + (y-2)^2 = 40$

1.59 : Determine the equation of the lines that are tangent to the circle $\Gamma : (x-4)^2 + (y+1)^2 = 5$ and that pass through $C(9;4)$.

1.24 Solutions

1.1 : -

1.2 : $\vec{c} = -0.8\vec{a} - 2\vec{b}$, $\vec{d} = 1.5\vec{a} + 0.8\vec{b}$, $\vec{e} = -1.3\vec{a} + 1.5\vec{b}$

1.3 : $y = -2.4$

1.4 : -

1.5 :

1) $\vec{a} \parallel \begin{pmatrix} -6 \\ -8 \end{pmatrix}$ $\quad \vec{b} \parallel \begin{pmatrix} 7 \\ -17.5 \end{pmatrix}$ $\quad \vec{c} \parallel \begin{pmatrix} \frac{11}{3} \\ -11 \end{pmatrix}$

2) $2\vec{a} - 3\vec{b} = \begin{pmatrix} 12 \\ -7 \end{pmatrix}$ $\quad \frac{1}{3}\vec{a} + \frac{3}{2}\vec{c} = \begin{pmatrix} -2 \\ \frac{31}{3} \end{pmatrix}$ $\quad -4\left(\vec{a} - \vec{b}\right) + 3\left(-\vec{b} + \vec{c}\right) = \begin{pmatrix} -20 \\ 7 \end{pmatrix}$

3) $\vec{c} = \frac{2}{23}\vec{a} + \frac{26}{23}\vec{b}$

4) $\vec{b} = -\frac{1}{13}\vec{a} + \frac{23}{26}\vec{c}$

1.6 :

1) $\det(\vec{a}, \vec{b}) = -13 \neq 0 \implies \vec{a} \nparallel \vec{b}.$

2) $\vec{c} = -\frac{29}{13}\vec{a} + \frac{20}{13}\vec{b}$

3) $m = -10.5$

4) $n = 1.5$

1.7 : $\vec{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \vec{b} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

1.8 : $C = F = (-5; -3), D = H = (5; 3), E = G(1; 5)$.
The line segments AC, BD and OE are the medians of the triangle CDE. They intersect at the centre of gravity of the triangle.

1.9 :

1) $A(7; 5) \quad B(-4; 1) \longrightarrow \vec{AB} = \begin{pmatrix} -11 \\ -4 \end{pmatrix}$

$A\left(5; -\frac{5}{3}\right) \quad B\left(2; \frac{1}{3}\right) \longrightarrow \vec{AB} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$A(8.75; -2) \quad B(0; 3) \longrightarrow \vec{AB} = \begin{pmatrix} -8.75 \\ 5 \end{pmatrix} \parallel \begin{pmatrix} 7 \\ -4 \end{pmatrix}$

$A\left(\frac{1}{2}; 3\right) \quad B(0.5; -1) \longrightarrow \vec{AB} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \parallel \vec{e_2}$

2) $M(0.5; -2.5)$

3) $E(-4; -1)$

4) $\vec{OP'} = 2 \cdot \vec{OM} - \vec{OP}$ so $P'(2m_1 - x; 2m_2 - y)$.

5) $F(14; -4)$

1.10 : $B(7;-4), C(2;0), D(-5;4), E(0;0)$

1.11 : $D(-1.5;9.5), M(-0.25;3.25)$

1.12 :

A	$(4;9)$	$(0.5;-2)$	$(-12;-3)$	$(2;7)$	$(0.5;19)$	$(-2;-3)$
B	$(-2;5)$	$(5.5;9)$	$(0;7)$	$(0.5;18)$	$(-5.5;17)$	$(4;-7)$
M_{AB}	$(1;7)$	$(3;3.5)$	$(-6;2)$	$(1.25;12.5)$	$(-2.5;18)$	$(1;-5)$
$\vec{AB}$	$\begin{pmatrix}-6\\-4\end{pmatrix}$	$\begin{pmatrix}5\\11\end{pmatrix}$	$\begin{pmatrix}12\\10\end{pmatrix}$	$\begin{pmatrix}-1.5\\11\end{pmatrix}$	$\begin{pmatrix}-6\\-2\end{pmatrix}$	$\begin{pmatrix}6\\-4\end{pmatrix}$

1.13 : $B(-1;-4), C(1;-2), D(-1;4)$

1.14 : $A'(-12;-1), B'(-4;5), C'(-6;7)$

1.15 : $Center(-3;9)$

1.16 : $G_{ABC}(-2;4)$ and $D(16;-5)$.

1.17 : $A(-2;5), B(\frac{7}{4};0), C(1;1), D(0.7;1.4, E(0;\frac{7}{3}), F(-3.5;7)$

1.18 :

$l_1 : \begin{cases} x = -2 + 10\lambda \\ y = 3 + 2\lambda \end{cases}$ or $-2x + 10y - 34 = 0$

$l_2 : \begin{cases} x = -4 + \lambda \\ y = 1 \end{cases}$ or $y - 1 = 0$

$l_3 : \begin{cases} x = 2\lambda \\ y = -5\lambda \end{cases}$ or $5x + 2y = 0$

1.19 :

Parametric equations	Cartesian equation	A point	A direction vector
$\begin{cases} x = 5 - 2\lambda \\ y = -1 + 3\lambda \end{cases}$	$3x + 2y - 13 = 0$	$(5;-1)$	$\begin{pmatrix}-2\\3\end{pmatrix}$
$\begin{cases} x = 10 - 4\lambda \\ y = \lambda \end{cases}$	$x + 4y - 10 = 0$	$(10;0)$	$\begin{pmatrix}-4\\1\end{pmatrix}$
$\begin{cases} x = -7 + 6\lambda \\ y = 1 + 5\lambda \end{cases}$	$5x - 6y + 41 = 0$	$A(-7;1)$	$\begin{pmatrix}6\\5\end{pmatrix}$
$\begin{cases} x = -2\lambda \\ y = 9 + \lambda \end{cases}$	$x + 2y - 18 = 0$	$A(0;9)$	$\begin{pmatrix}-2\\1\end{pmatrix}$

1.20 : $I(1;2)$. Secant as the direction vectors aren't parallel. $\vec{l_1} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\vec{l_2} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
$l_1 : 2x + y - 4 = 0$, $l_2 : 2x - y = 0$.

1.21 : $A'(2.5;1.5), \vec{AA'} = \begin{pmatrix} 7.5 \\ -0.5 \end{pmatrix} \parallel \begin{pmatrix} 15 \\ -1 \end{pmatrix}$.
$x + 15y - 25 = 0$ $\left\{ \begin{array}{l} x = -5 + 15\lambda \\ y = 2 - \lambda \end{array} \right.$. D belongs to the median.

1.22 :

1) $l_1 : (0;-1.5),(3;0)$. $l_2 : (0;-3),(4;0)$
2) $l_1 : \begin{pmatrix} 2 \\ 1 \end{pmatrix} \parallel \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ $l_2 : \begin{pmatrix} 4 \\ 3 \end{pmatrix} \parallel \begin{pmatrix} -4 \\ -3 \end{pmatrix}$
3) $l_1 : \left\{ \begin{array}{l} x = 3 + 2\lambda \\ y = \lambda \end{array} \right.$ and $l_2 : \left\{ \begin{array}{l} x = 4\lambda \\ y = -3 + 3\lambda \end{array} \right.$
4) Secant as the direction vectors aren't parallel to each other.
6) $I(6;1.5)$

1.23 : $I_1(-\frac{52}{23}; -\frac{165}{23})(\lambda = -\frac{71}{23}, \mu = -\frac{40}{23})$ $I_2(\frac{151}{19}; -\frac{7}{19})(\lambda = \frac{6}{19})$ $I_3(-\frac{19}{13}; \frac{21}{26})$.

1.24 : First answer :$A(-5;-3), B(0;-1)$. Second answer : $A(-9;7), B(-4;9)$

1.25 : $m = -\frac{10}{3}, m = 0, m = \frac{4}{3}, m = 10$.

1.26 : $A'(1.5;4), C(-2;1), B(5;7)$.

1.27 : $B(4;-4), C(2;4)$.

1.28 :

1) $\vec{OB} - \vec{OA} = \vec{OC} - \vec{OD} \implies \vec{OB} - \vec{OC} = \vec{OA} - \vec{OD} \implies \vec{CB} = \vec{DA}$
So it's a parallelogram.
2) $\vec{OI} = \frac{1}{2}(\vec{OA} + \vec{OB})$ $\vec{OJ} = \frac{1}{2}(\vec{OB} + \vec{OC})$ $\vec{OK} = \frac{1}{2}(\vec{OC} + \vec{OD})$ $\vec{OL} = \frac{1}{2}(\vec{OA} + \vec{OD})$
$\vec{IJ} = \frac{1}{2}(\vec{OB} + \vec{OC}) - \frac{1}{2}(\vec{OA} + \vec{OB}) = \frac{1}{2}(\vec{OC} - \vec{OA}) = \frac{1}{2}\vec{AC}$
$\vec{LK} = \frac{1}{2}(\vec{OC} + \vec{OD}) - \frac{1}{2}(\vec{OA} + \vec{OD}) = \frac{1}{2}(\vec{OC} - \vec{OA}) = \frac{1}{2}\vec{AC}$.
$\vec{IJ} = \vec{LK}$ so from the result 1) : it's a parallelogram.

1.29 : $A(-7;-4), C(11;-2), B(3;2)$

1.30 :

1) $\|\vec{a}\| = \sqrt{13}$ $\left\|\vec{b}\right\| = \sqrt{74}$ $\|\vec{c}\| = 5$ $\left\|\vec{d}\right\| = \sqrt{29}$
2) $\vec{u} = \begin{pmatrix} -4/5 \\ 3/5 \end{pmatrix}$
3) $\vec{e} = \pm \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

4) $k = -\frac{15}{4}$

5) $\vec{a} \bullet \vec{b} = -11$ $\vec{b} \bullet \vec{c} = 1$ $\vec{b} \bullet \vec{d} = -39$

6) $\vec{a}$ $\vec{d}$

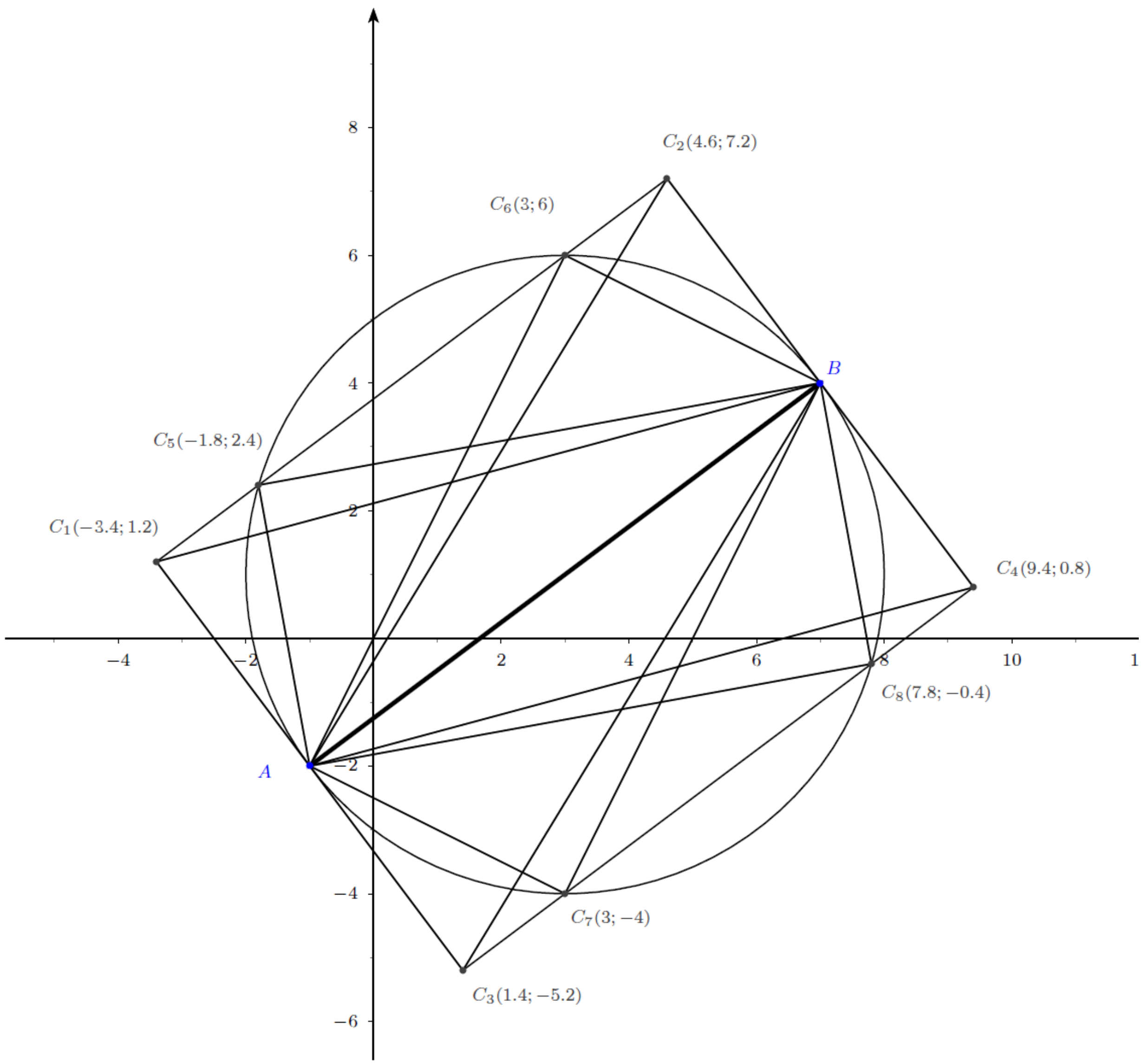
1. 31 : Obtuse $p = \sqrt{61} + \sqrt{8} + \sqrt{73} \cong 19.18$

1. 32 : 2) $\left\| \vec{b'} \right\| = \frac{13}{5}$ 3) $Area = 34$ 4) $\vec{b'} = \begin{pmatrix} 1.56 \\ -2.08 \end{pmatrix}$

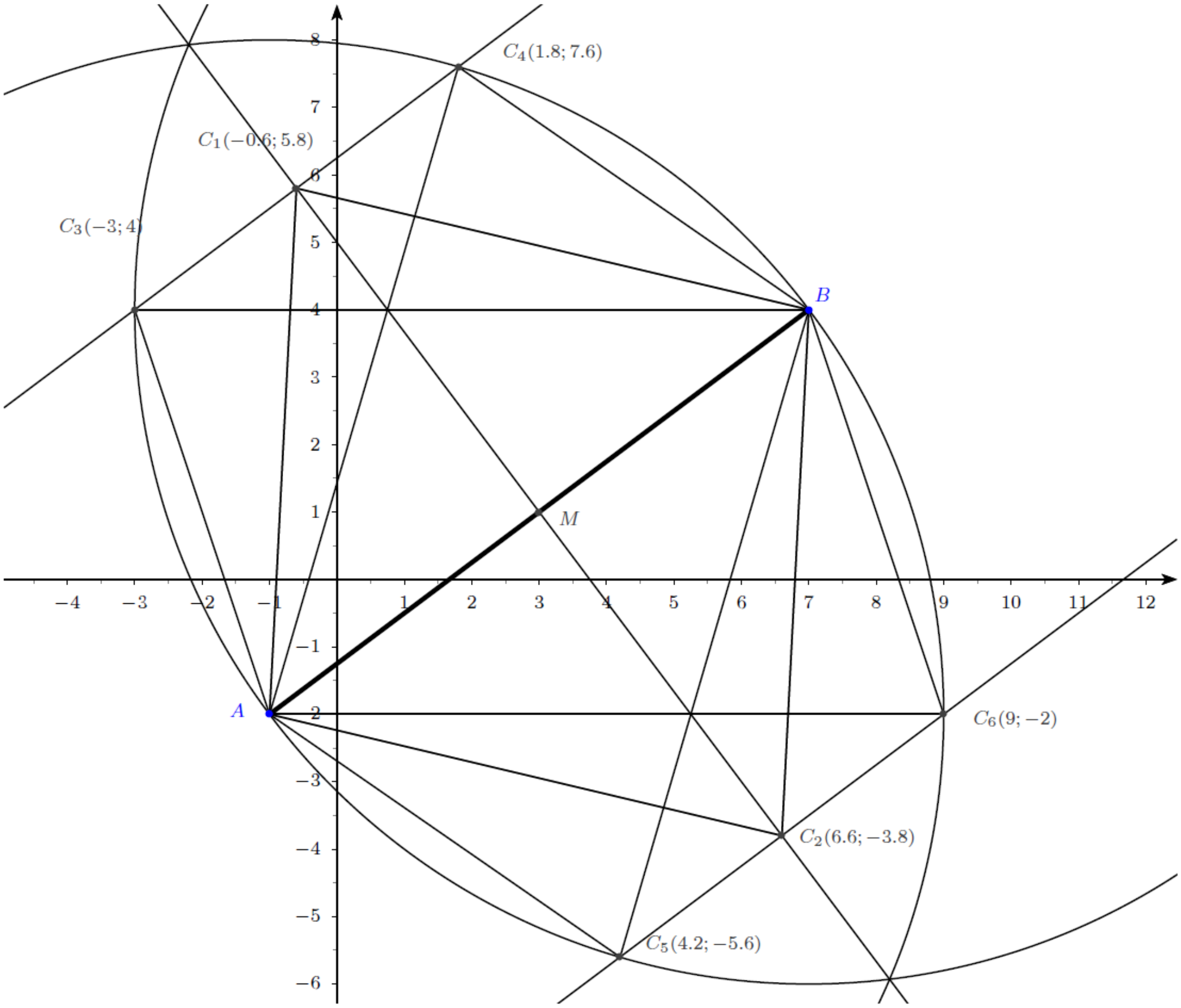
1. 33 :

1) $C_1(-6; 13)$ $C_2(12; -11)$

2) There are 8 answers !



3) There are 6 answers !



1.34 : 159.23° / 75.75° / $\alpha \cong 95.19°$, $\beta \cong 19.25°$, $\gamma \cong 65.56°$

1.35 :

1) $a : 5x + 2y + 4 = 0$
2) $b : -4x + 3y + 15 = 0$
3) $c : -2x + 5y - 12 = 0$
4) $d : -4x + 3y + 14 = 0$

1.36 : 1) $\vec{l_1} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, $(4;0)$, $(0;-3)$ 2) $\vec{n} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, $(0;-3)$ 3) $\vec{n} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

1.37 :

1) $b : 4x + 3y - 6 = 0$
2) $I(3;-2)$
3) $C(9;-10)$

1.38 :

1) $m_{AB} : 3x - 2y - 5 = 0$ $m_{AC} : x = 2$ $m_{BC} : x + 2y - 3 = 0$
2) $M(2;0.5)$ 3) $r = \frac{\sqrt{65}}{2}$

1.39 : $\text{dist}(d;O) = 4.8$ $\text{dist}(d;B) = 10$ $\text{dist}(d;C) = 0$ $e : 4x - 3y - 14 = 0$ $f : 4x - 3y - 34 = 0$

1.40 : $P_1(7;3)$ $P_2(5;-11)$

1.41 :

1) $\text{dist}(a;b) = 3$
2) $c : 8x + 6y - 63 = 0$
3) $d : 4x + 3y - 34 = 0$

1.42 : $c : 21x + 77y - 231 = 0$ $d : 99x - 27y + 81 = 0$

1.43 :

1) $A = 126$
2) $b_A : x - 3y - 14 = 0$ $b_B : 11x - 3y - 54 = 0$ $b_C : 2x + 3y + 2 = 0$
   $I\left(4; -\frac{10}{3}\right)$ $r = \frac{14}{3}$
3) $C\left(\frac{6}{5}; \frac{2}{5}\right)$

1.44 :

1) $(x-5)^2 + (y-3)^2 = 25$
2) $P_1(1;0)$ $P_2(9;0)$

1.45 :

1) Yes $C(7;1)$ $r = \sqrt{176}$
2) No $r^2 < 0$
3) No $r = 0$, but it's the point $(-4;8)$
4) Yes $C\left(-\frac{7}{6};0\right)$ $r = \frac{13}{6}$
5) No (coeff of $x^2 \neq$ coeff of y^2)
6) No (coeff of $x^2 \neq$ coeff of y^2)

1.46 : $(x+7)^2 + (y-4)^2 = 169$ $a_1 = 5$ $b_2 = -8$ $m : 7x + 17y - 19 = 0$

1.47 : $(x-1)^2 + (y-1)^2 = 20$

1.48 :

1) P_1 inside P_2 on P_3 on
2) $t_1 : 4x - 3y - 4 = 0$ $t_2 : 4x - 3y - 54 = 0$, $T_1(1;0)$ $T_2(9;-6)$
3) Secant $I_1(1;-6)$ $I_2(8;1)$

1.49 : $t : 2x + 3y = 0$

1.50 : $(3;1)$ and $(5;-1)$

1.51 : $(x+2)^2 + (y-3)^2 = 3.2$

1.52 : $x - 4y - 5 = 0$ and $x - 4y - 39 = 0$

1.53 : $x^2 + (y-5)^2 = 10$

1.54 : $(x+\frac{13}{15})^2 + (y-\frac{62}{15})^2 = \frac{289}{225}$

1.55 : $P_1(7;7)$ and $P_2(-3;-3)$. Area=50

1.56 : Secant for $m \in]-0.75; 0.75[$ and tangent for $m = \pm 0.75$

1.57 : $(x+36)^2 + (y-21)^2 = 1296$ and $(x-\frac{18}{17})^2 + (y-\frac{87}{17})^2 = \frac{324}{289}$

1.58 : 1) $(4;3)$ and $(-3;-4)$ 2) $(1;8)$ and $(9;0)$

1.59 : $t_1 : -x + 2y + 1 = 0 (T_1(3;1))$ and $t_2 : -2x + y + 14 = 0 (T_2(6;-2))$