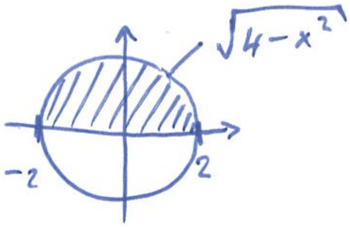
Corrigé TE N°2 – 3Mg – Niv 1

(4) Ex. 1 :

2 a) $$\int_{-2}^{2}\sqrt{4-x^2}\,dx = \frac{\pi\cdot 2^2}{2} = 2\pi$$



2 b) $$\int_{1}^{8} f(x)\,dx = \underbrace{\underbrace{2\cdot 3}_{6} + \frac{\cancel{2}\cdot 4}{\cancel{2}}}_{10} + \underbrace{\underbrace{3\cdot 7}_{21}}_{21} + \underbrace{\frac{\cancel{2}\cdot 1}{\cancel{2}} + \underbrace{2\cdot 6}_{12}}_{13} = 44$$

(3) Ex. 2 :

2

$$f(x) = 4\cdot\left(-\cos(2x)\cdot\frac{1}{2}\right) + c = -2\cos(2x) + c$$

1

$$f\left(\frac{\pi}{2}\right) = -2\cdot\underbrace{\cos\left(2\cdot\frac{\pi}{2}\right)}_{-1} + c = 1 \Rightarrow c = -1 \Rightarrow \boxed{f(x) = -2\cos(2x) - 1}$$

(4) Ex. 3 :

2

$$F(x) = (Ax^2 + Bx + C)e^{-2x} \Rightarrow F'(x) = (2Ax + B)e^{-2x} - 2\cdot(Ax^2 + Bx + C)e^{-2x}$$

$$= \left(\underbrace{-2A}_{4}x^2 + (2A - 2B)x + \underbrace{B - 2C}_{=1}\right)e^{-2x}$$

$$\Rightarrow \begin{cases} -2A = 4 \Rightarrow A = -2 \\ 2A - 2B = 0 \Rightarrow B = -2 \\ B - 2C = 1 \Rightarrow C = -\frac{3}{2} \end{cases} \Rightarrow \int \ldots\,dx = -2\left(x^2 + x + \frac{3}{4}\right)e^{-2x} + C$$

1 1

$\boxed{17}$ Ex. 4 :

1 1/2

(2.5) a) $$\int x(4-x^2)^3\,dx = -\frac{1}{2}\int t^3\,dt = -\frac{1}{8}t^4 + c = -\frac{1}{8}(4-x^2)^4 + c,\ c\in\mathbb{R}$$

$t = 4 - x^2$

$dt = -2x\,dx$

$dx = \dfrac{dt}{-2x}$

1

(3) b) $$\int \ln(x)(\ln(x)-2)\,dx \underset{\uparrow}{=} (\ln(x)-2)\cdot x(\ln(x)-1) - \int \ln(x)-1\,dx \quad 1/2$$

$$u=\ln(x)-2 \qquad u'=\frac{1}{x}$$
$$1 \qquad v'=\ln(x) \qquad v=x(\ln(x)-1)$$

$$= x(\ln^2(x)-3\ln(x)+2) - x(\ln(x)-1) + x + C \quad 1$$

$$= x(\ln^2(x)-4\ln(x)+4) + C = \underline{x(\ln(x)-2)^2 + C}, \quad C\in\mathbb{R}$$

1/2 (Bonus de 1/2)

(3.5) c)
$$\begin{array}{r|l} 2x^2-5 & 2x-2 \\ -(2x^2-2x) & x+1 \\ \hline 2x-5 & \\ -(2x-2) & \\ \hline -3 & \end{array} \quad 1$$

$$\Rightarrow \int \frac{2x^2-5}{2x-2}\,dx = \int x+1-\frac{3}{2x-2}\,dx \quad 1$$

$$= \underline{\frac{1}{2}x^2 + x - \frac{3}{2}\ln|2x-2| + C}, \quad C\in\mathbb{R}$$

1.5

(2.5) d) $$\int \frac{2}{\sqrt{3x-5}}\,dx = 2\int (3x-5)^{-\frac{1}{2}}\,dx = 4(3x-5)^{\frac{1}{2}}\cdot\frac{1}{3} + C = \underline{\frac{4}{3}\sqrt{3x-5} + C}, \quad C\in\mathbb{R}$$

2 1/2

(1.5) e) $$\int x(x^3-3)\,dx = \int x^4-3x\,dx = \frac{1}{5}x^5 - \frac{3}{2}x^2 = x^2\left(\frac{1}{5}x^3-\frac{3}{2}\right) + C, \quad C\in\mathbb{R}$$

1.5 (Bonus de 1/2)

(4) f) $$\int \cos(x)e^{-x}\,dx \underset{\uparrow}{=} -\cos(x)e^{-x} - \int \sin(x)e^{-x} \underset{\uparrow}{=} \dots \quad 1/2$$

$$1 \quad u=\cos(x) \quad u'=-\sin(x) \qquad 1 \quad u=\sin(x) \quad u'=\cos(x)$$
$$v'=e^{-x} \quad v=-e^{-x} \qquad v'=e^{-x} \quad v=-e^{-x}$$

$$\dots = -\cos(x)e^{-x} - \left(-\sin(x)e^{-x} + \int \cos(x)e^{-x}\,dx\right) \quad 1/2$$

$$\Rightarrow \int \cos(x)e^{-x}\,dx = \underline{\frac{1}{2}e^{-x}(-\cos(x)+\sin(x)) + C}, \quad C\in\mathbb{R}$$

1

8 **Ex. 3 :**

a) Par défaut : $\frac{2}{3}\cdot(0^3+1)+\frac{2}{3}\left(\left(\frac{2}{3}\right)^3+1\right)+\frac{2}{3}\left(\left(2\cdot\frac{2}{3}\right)^3+1\right) = \overbrace{\frac{2}{3}\cdot 3}^{2} + \overbrace{\left(\frac{2}{3}\right)^4\cdot(1+8)}^{\frac{16}{9}} = \frac{34}{9}$ 2

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Par excès : $\frac{2}{3}\cdot\left(\left(\frac{2}{3}\right)^3+1\right)+\frac{2}{3}\left(\left(2\cdot\frac{2}{3}\right)^3+1\right)+\frac{2}{3}\left(\left(3\cdot\frac{2}{3}\right)^3+1\right) = \underbrace{\frac{2}{3}\cdot 3}_{2} + \underbrace{\left(\frac{2}{3}\right)^4\cdot(1+8+27)}_{\frac{64}{9}} = \frac{82}{9}$ 2

$$\Rightarrow \underbrace{\frac{34}{9}}_{3,\overline{7}} < \int_0^2 f(x)\,dx < \underbrace{\frac{82}{9}}_{9,\overline{1}}$$

b) Par excès : $\int_0^2 f(x)\,dx = \lim_{n\to+\infty}\left(\frac{2}{\cancel{n}}\cdot\cancel{n} + \left(\frac{2}{n}\right)^4\cdot\left(\frac{n(n+1)}{2}\right)^2\right)$ 2

4

$$= 2 + \lim_{n\to+\infty}\frac{2^{\cancel{4}\,2}}{n^{\cancel{4}\,2}}\cdot\frac{\cancel{n^2}\cdot(n+1)^2}{\cancel{2^2}} = 2 + \underbrace{\lim_{n\to+\infty}\frac{4(n+1)^2}{n^2}}_{=4} = \underline{\underline{6}}$$

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