LDDR – Niveau 2: TE 22 Algèbre Linéaire-Nombres Complexes

3MG01	LINEAR ALGEBRA	TEST 6
2019.05.03	COMPLEX NUMBERS	90'

With « formulaire »

Name :

EXERCISE 1 50% of the points

In the set of the vectors of the 3-dimensional space a linear transformation f is given by its matrix

$$F = \frac{1}{9}\begin{pmatrix} 8 & -2 & 2 \\ -2 & 5 & 4 \\ 2 & 4 & 5 \end{pmatrix}$$

with respect to the orthonormal standard basis $(\overrightarrow{u_1}; \overrightarrow{u_2}; \overrightarrow{u_3})$.

a) For which value of m is the vector $\overrightarrow{v} = \begin{pmatrix} 2 \\ 1 \\ m \end{pmatrix}$ an eigenvector of f ? Determine the corresponding eigenvalue.

b) Knowing that $\det(F - \lambda I) = -\lambda^3 + 2\lambda^2 - \lambda$, find the eigenvalues and the eigenvectors of f.

c) Find a geometric description of the transformation f.

We consider the vectors $\overrightarrow{a} = \frac{1}{3}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\overrightarrow{b} = \frac{1}{3}\begin{pmatrix} -2 \\ 2 \\ n \end{pmatrix}$.

d) Find the value of n and a vector $\overrightarrow{c}$ such that $(\overrightarrow{a}; \overrightarrow{b}; \overrightarrow{c})$ is an orthonormal basis.

e) Find the matrix of f with respect to the basis $(\overrightarrow{a}; \overrightarrow{b}; \overrightarrow{c})$.

In the set of the vectors of the 3-dimensional space, let g be the linear transformation whose matrix with respect to the standard basis $(\overrightarrow{u_1}; \overrightarrow{u_2}; \overrightarrow{u_3})$ is

$$G = \frac{1}{9}\begin{pmatrix} 1 & 8 & 4 \\ -4 & 4 & -7 \\ -8 & -1 & 4 \end{pmatrix}$$

f) The transformation g is a rotation about the line through the origin and parallel to the vector $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$. Calculate the angle of this rotation.

g) The transformation g is orthogonal. What's the main characteristic of such a transformation ?

EXERCICE 2 50% of the points

Part1

The function $f(z) = i\bar{z} + 3z + 1 - i$ is associated to a transformation of the complex plane.
That transformation can also be seen as an affinity $\varphi: \overrightarrow{OP'} = M\,\overrightarrow{OP} + \vec{t}$ in the plane.
Determine the matrix M and the vector $\vec{t}$.

Part2

The function $f : \mathbb{C} \longrightarrow \mathbb{C}$ is defined by $f(z) = z^2 - 9z + 22 + 4i$.

a) The complex numbers $z_1 = f(i)$ and $z_2 = f(2 + 3i)$ are represented by the points P_1 and P_2 in the complex plane. Calculate the angle at the origin in the triangle OP_1P_2.

b) Find the fixed points of the function f.

c) Let $f(x + iy) = u + iv$ (where x, y, u and v are real numbers).
Express u and v as functions of x and y.

d) Demonstrate that the image under f of the line with equation $y = x - 5$ is a parabola. Determine the equation of that parabola.

Consider the function $g : \mathbb{C} \longrightarrow \mathbb{C}$ defined by $g(z) = f(z + i) - z^2$.

e) Precisely describe the geometric transformation associated to g in the complex plane, without using any translation.