LDDR – Niveau 2: TE 21 Algebre Lineaire

3MG01 LINEAR ALGEBRA **TEST 5 - B**

2019.03.19 90'

With « formulaire » Name :

EXERCISE 1 10'

1) Determine the values of k such that the matrix $M = \begin{pmatrix} 6 & k \\ k & -1 \end{pmatrix}$ has two different eigenvalues.
2) The vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ is a λ −eigenvector of M. Determine the values of λ and of k.

EXERCICE 2 [17 pts] 20'

In V_2 with basis $(\overrightarrow{e_1}, \overrightarrow{e_2})$, we consider the vector $\vec{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

1) The application r is the **rotation** by 180°. Determine its matrix R.
2) The application s is the **symmetry** about the axis parallel to $\vec{a}$. Determine its matrix S.
3) The application p is the orthogonal **projection** on $\vec{a}$. Determine its matrix P.
4) Give an eigenbasis of s and the eigenmatrix S' associated.
5) Give an eigenbasis of p and the eigenmatrix P' associated.
6) Without « row by column multiplications », determine

   b) $P^2 =$ b) $S^3 =$ c) $SP =$ d) $RP =$ e) $S + I =$

7) Give a precise and as simple as possible description of the transformation m with matrix $M = RS$.

EXERCICE 3 [6 pts] 10'

We consider f the vertical projection on the plane $\pi: x + 2y - 3z = 0$.

3) Determine the **second column** of the matrix of f, expressed in the standard basis.
4) Determine the eigenvalues and eigenspaces of f, and give the eigenmatrix.

EXERCISE 4 [7 pts] 20'

A linear application f from V_3 to V_3 is given by its matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 3 & -2 \end{pmatrix}$ (in the standard basis)

1) Determine the image of the vector $\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ under f.

2) Determine the vector whose image under f is $\vec{b} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

3) The invariant* vectors of f form a subspace of V_3.

   Determine its dimension and give a basis of that subspace.

**: a vector is invariant if it is equal to its image.*

EXERCICE 5 20'

We consider g a linear transformation with matrix $G = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (in the standard basis).

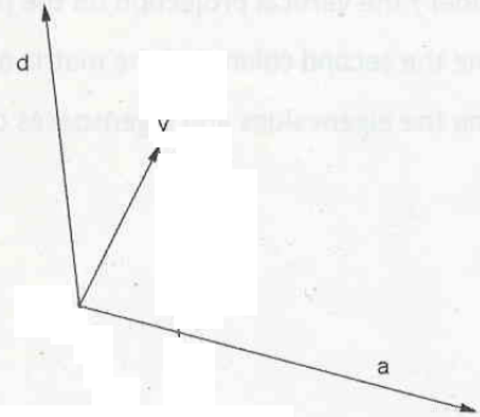
1) Determine the matrix of the composition $g * g$.
2) Check the result $p(\lambda) = \det(G - \lambda I) = -\lambda^3 + 3\lambda^2$
3) Determine the eigenvalues and eigenspaces of g and give a geometrical description of g.

EXERCICE 6

1) Draw the image of $\vec{v}$ under f : a perspective affinity

   with axis $\vec{a}$, direction $\vec{d}$ and factor -2.

2) Determine the matrix of f in the basis $(\vec{d}, \vec{a})$



Exercise 4.

a) Show that $f: \begin{matrix} V_2 \to V_2 \\ \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 2y-x \\ 2x \end{pmatrix} \end{matrix}$ is a linear application.

b) Give the expression of the linear application $g: \begin{matrix} V_2 \to V_2 \\ \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \end{matrix}$ that corresponds to a rotation by +90° degrees around the origin.

c) Give an application from V_3 to V_2 that is not a linear application and justify why : $\begin{matrix} V_3 \to V_2 \\ \begin{pmatrix} \ \end{pmatrix} \mapsto \begin{pmatrix} \ \end{pmatrix} \end{matrix}$

Exercise 5.

a) What is the dimension of the vector space M_{2x2} ?

We consider the matrices $A = \begin{pmatrix} 2 & 0 \\ 1 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$.

Determine
1) $tr(A)$ the trace of A
2) $det(B)$ the determinant of B
3) $A \cdot B$
4) ${}^tA$ the transpose of A

5) Give a matrix $C \in M_{2x2}$ such that the subspace of M_{2x2} generated by the linear combinations of A, B, C has dimension 2. That subspace can be written $\{M \in M_{2x2} | M = \alpha A + \beta B + \gamma C, \quad \alpha, \beta, \gamma \in \mathbb{R}\}$

b) In the vector space M_{2x2} of the « 2 by 2 » matrices, we consider the **subset** $\{M \in M_{2x2} | trace(M) \text{ is even}\}$. Determine whether it is a **subspace** of M_{2x2} or not.

c) In the vector space M_{2x2} of the « 2 by 2 » matrices, we consider the **subspace** $\{M \in M_{2x2} | M \text{ is symetric}\}$. Determine its dimension and propose a basis.

Exercise 6.

In V_3 we consider three vectors v_1, v_2 and v_3. The subspace F generated by these vectors may be of dimension 0,1,2 or 3.

Determine what condition(s) the vectors must satisfy for the subspace F to be of each of these 4 dimensions. In case a computation is needed to define the dimension indicate it.

$\dim(F) = 0$ if...

$\dim(F) = 1$ if...

$\dim(F) = 2$ if...

$\dim(F) = 3$ if...