LDDR – Niveau 2: TE 20 Eqq.Diff – Taylor-Integral

3MG Level 2	CALCULUS	TEST#4 A
2019/02/15		3MG01

With "formulaire" Name: 90'

Exercise 1.

1) Find all the possible values $k \in \mathbb{R}$ such that $y = f(x) = e^{kx}$ is the solution of the equation $y'' - 6y = 0$.
2) Determine the solution of $y' = x^3 y$ that satisfy $f(0) = -2$.
3) Solve $y' - \frac{2}{x} y = 1 - x^2 \cdot \ln(x)$

Exercise 2.

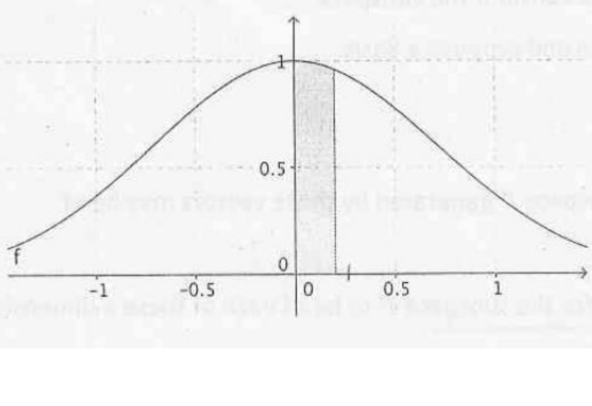
1) Determine by computations the 5 first terms of Taylor's expansion of $f(x) = \ln(1+x)$ around $a = 0$ (MacLaurin).
2) Determine the value of $A = \lim_{x \to 0} \frac{x \cdot \ln(1+x)}{1 - \cos(x)}$ by using Taylor's series (*formulaire* allowed)

Exercise 3.

The surface $A = \int_0^{0.2} f(x)dx$ is represented, for $f(x) = e^{-x^2}$, a function that has no antiderivative !

From Taylor's expension for e^x that is $e^x = \sum_{k \geq 0} \frac{1}{k!} \cdot x^k = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$ you can determine the one of $f(x)$: "just" replace x by $-x^2$.

Use the three first (non zero) terms of that expansion to determine an approximation for the value of A, rounded to three digits.



Exercise 4.

a) Show that $f: \begin{matrix} V_2 \to V_2 \\ \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 2y-x \\ 2x \end{pmatrix} \end{matrix}$ is a linear application.

b) Give the expression of the linear application $g: \begin{matrix} V_2 \to V_2 \\ \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \end{matrix}$ that corresponds to a rotation by +90° degrees around the origin.

c) Give an application from V_3 to V_2 that is not a linear application and justify why : $\begin{matrix} V_3 \to V_2 \\ \begin{pmatrix} \\ \\ \end{pmatrix} \mapsto \begin{pmatrix} \\ \end{pmatrix} \end{matrix}$

Exercise 5.

a) What is the dimension of the vector space M_{2x2} ?

We consider the matrices $A = \begin{pmatrix} 2 & 0 \\ 1 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$.

Determine
1) $tr(A)$ the trace of A
2) $det(B)$ the determinant of B
3) $A \cdot B$
4) ${}^tA$ the transpose of A

5) Give a matrix $C \in M_{2x2}$ such that the subspace of M_{2x2} generated by the linear combinations of A, B, C has dimension 2. That subspace can be written $\{M \in M_{2x2} | M = \alpha A + \beta B + \gamma C, \quad \alpha, \beta, \gamma \in \mathbb{R}\}$

b) In the vector space M_{2x2} of the « 2 by 2 » matrices, we consider the **subset** $\{M \in M_{2x2} | trace(M) \text{ is even}\}$. Determine whether it is a **subspace** of M_{2x2} or not.

c) In the vector space M_{2x2} of the « 2 by 2 » matrices, we consider the **subspace** $\{M \in M_{2x2} | M \text{ is symetric}\}$. Determine its dimension and propose a basis.

Exercise 6.

In V_3 we consider three vectors v_1, v_2 and v_3. The subspace F generated by these vectors may be of dimension 0,1,2 or 3.

Determine what condition(s) the vectors must satisfy for the subspace F to be of each of these 4 dimensions. In case a computation is needed to define the dimension indicate it.

$\dim(F) = 0$ if...

$\dim(F) = 1$ if...

$\dim(F) = 2$ if...

$\dim(F) = 3$ if...