LDDR – Niveau 2: TE 19 Calcul Integral

3MG Level 2	CALCULUS	TEST#3A
2019/01/11		3MG01

With formulaire
Only "basic" answers can be used

Name:

90'

Exercise 1.

1) Determine f such that $F(x) = \sin^2(x)$ is an antidervative of f.
   How many solutions are there ?

2) We consider $A = \int f(x) \cdot e^{-x^2} dx$. Give a function f such that A can be determined
   with the *integration by substitution method*, and determine A.

3) Determine $\int \frac{4x^2+3}{x^2+2x+1} dx$

4) Determine an antiderivative of $f(x) = \frac{\ln(2x)}{x}$ by using the *substitution method*.

5) Determine $I = \int_0^2 \frac{1}{\sqrt[4]{2-x}} dx$

Exercise 2.

Show that $\int_0^2 \ln^2(x) \, dx = \lim_{a \to 0} \int_a^2 \ln^2(x) \, dx$ is defined and give its value.

Exercise 3.

The graph of the function $g(x) = 2e^{-x}$ is represented.

1) We consider the right triangle OPP_0 with P a
   point on the graph in the first quadrant.
   Determine the coordinates of P such that the
   area of the triangle is the smallest.

2) The triangle's area $A(x)$ is considered for x
   varing beween 0 and 3. What's the average
   value of the function $A(x)$ on that interval ?
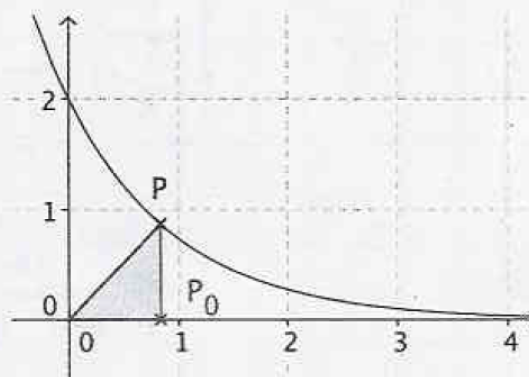
Exercise 4.

We denote by I_n the value of the integral $I_n = \int_0^1 t^n \cdot \sqrt{1-t}\, dt$

Use the *integration by parts method* to show that $I_{n-1} = (1 + \frac{3}{2n}) \cdot I_n$

Hint : You'll need $(1-t)^{\frac{3}{2}} = (1-t) \cdot \sqrt{1-t} = \sqrt{1-t} - t \cdot \sqrt{1-t}$