LDDR - Niveau 2 : TE 14 Solutions

Exercice 1 : a) $f(x) = \frac{1}{x^4} = x^{-4} \Rightarrow F(x) = -\frac{1}{3}x^{-3} = -\frac{1}{3x^3}$

$g(x) = -\cos(2x) \Rightarrow G(x) = -\frac{1}{2}\sin(2x)$

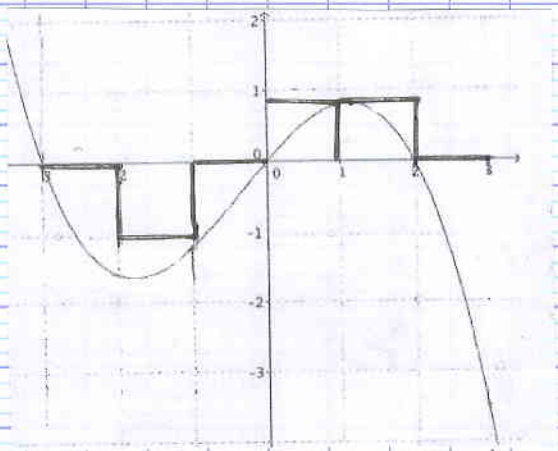
$h(x) = \frac{3}{\sqrt{12x-7}} = 3(12x-7)^{-1/2}$

$H(x) = 3 \cdot \frac{1}{-\frac{1}{2}+1} \cdot \frac{1}{12}(12x-7)^{\frac{1}{2}} = \frac{1}{\frac{1}{2}} \cdot \frac{1}{4}\sqrt{12x-7} =$

$\Rightarrow H(x) = \frac{1}{2}\sqrt{12x-7}$

b) $\int_0^4 e^{-x}\,dx = -e^{-x}\Big|_0^4 = -e^{-4} - (-e^0) = e^{-4} + 1$

c)



d) $\bar{f} = 5 = \frac{1}{b-0}\int_0^b f(x)\,dx = \frac{1}{b}\int_0^b x^3\,dx \Rightarrow$

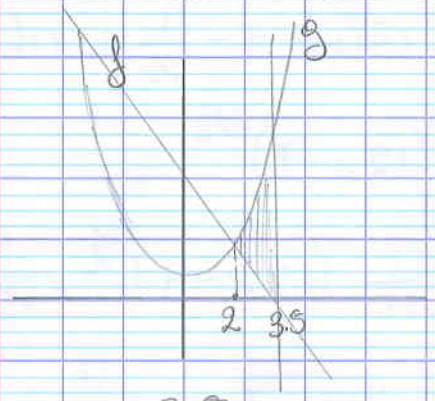
$\Rightarrow 5 = \frac{1}{b}\cdot\frac{1}{4}x^4\Big|_0^b = \frac{1}{4b}b^4 = 5 \Rightarrow \frac{b^3}{4} = 5 \Rightarrow$

$\Rightarrow b^3 = 20 \Rightarrow b = \sqrt[3]{20}$

c) $f(x) = -3x + 26 \quad g(x) = 6x^2 - 3x + 2$

$f(x) = g(x) \Rightarrow 6x^2 - 3x + 2 = -3x + 26 \Leftrightarrow$

$\Leftrightarrow 6x^2 = 24 \Rightarrow x^2 = 4 \Leftrightarrow x = \pm 2$



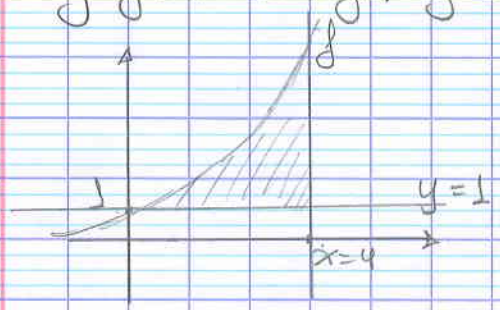
$F(x) = -\frac{3}{2}x^2 + 26x$

$G(x) = 2x^3 - \frac{3}{2}x^2 + 2x$

$$A = \int_2^{3.5} (g(x) - f(x))\,dx = [G(3.5) - F(3.5)] - [G(2) - F(2)] =$$

$$= 33.75$$

g) $y = f(x) = 2^x \quad g(x) = y = 1 \quad x = 4$



$f(x) = 1 \Rightarrow 2^x = 1 \Rightarrow x = 0$

$$V = \pi \int_0^4 (f^2(x) - g^2(x))\,dx = \pi \int_0^4 (2^{2x} - 1)\,dx =$$

$$= \pi \int_0^4 (4^x - 1)\,dx = \pi \left[\frac{4^x}{\ln 4} - x\right]_0^4 = \pi \left[\frac{4^4}{\ln 4} - 4 + \frac{1}{\ln 4}\right]$$

$$\approx \pi \cdot 179.94$$

Exercice 2.

a) $0 \le p \le 0.8$ car $0 \le p+0.2 \le 1 \Rightarrow p \le 0.8$
$0 \le p+0.15 \le 1 \Rightarrow p \le 0.85$ $\Big\} p \le 0.8$

b) $A \cap P = \emptyset \Rightarrow p = 0$

c) $P(A) = 0.15 + p \quad P(B) = p + 0.2$
$P(A) \cdot P(B) = p \Rightarrow (0.15+p)(p+0.2) = p \Leftrightarrow$
$\Leftrightarrow 0.15p + 0.2p + p^2 + 0.03 = p \Leftrightarrow$
$\Leftrightarrow p^2 - 0.65p + 0.03 = 0$
$\Delta = 0.3025 \quad p_{1,2} = \dfrac{0.65 \pm 0.55}{2}$ $\Big/ \begin{matrix} p_1 = 0.6 \\ p_2 = 0.05 \end{matrix}$

d) $p = 0.3 \quad p(\bar{B}|\bar{A}) = \dfrac{P(\bar{B} \cap \bar{A})}{P(\bar{A})}$
$P(B) = 0.2 + 0.3 = 0.5$
$P(A) = 0.15 + 0.3 \Rightarrow P(A) = 0.45 \Rightarrow P(\bar{A}) = 0.55$
$P(\bar{B} \cap \bar{A}) = P(\overline{A \cup B}) = 1 - P(A \cup B) =$
$= 1 - P(A) - P(B) + P(A \cap B) =$
$= 1 - 0.45 + 0.5 + 0.3 = 0.35$

$P(\bar{B}|\bar{A}) = \dfrac{0.35}{0.55} \approx 0.636$

Exercice 3. A : 1 B 3 W B : 1 B 5 W

a) B : noir $\bar{B}$: pas noir

$P_A(\bar{B}) = \frac{3}{4} \qquad P_B(\bar{B}) = \frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36}$

$P_{AB}(\bar{B}) = \frac{3}{4} \cdot \frac{25}{36} = \frac{75}{144}$

b) $P(B \text{ wins}) = P_A(B) \cdot P_B(BB) + P_A(\bar{B}) P(BB) + P_A(\bar{B}) P_B(BB)$

$= \frac{1}{4} \cdot \frac{1}{6} \cdot \frac{1}{6} + \frac{3}{4} \cdot \frac{1}{6} \cdot \frac{1}{6} + \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{1}{6} \cdot 2 = \frac{1}{144} + \frac{3}{144} + \frac{30}{144}$

$= \frac{34}{144}$

c) $P(A \text{ wins}) = 1 - \frac{75}{144} - \frac{34}{144} = \frac{35}{144}$

d) $P(A|\omega) = \frac{P(A \cap \omega)}{P(\omega)} = \frac{P(A)}{P(\omega)} = \frac{35/144}{69/144} = \frac{35}{69}$

$P(\omega) = P(A) + P(B) = \frac{34}{144} + \frac{35}{144} = \frac{69}{144}$

e) $n = 10 \quad p = \frac{35}{144} \quad k = 2$

$P(X = 2) = \binom{10}{2} \left(\frac{35}{144}\right)^2 \left(\frac{109}{144}\right)^8$

f) n?

$P(A \geq 1) > 0.999 \Rightarrow 1 - P(A = 0) > 0.999 \Leftrightarrow$

$\Leftrightarrow 0.001 > \binom{n}{0} \left(\frac{35}{144}\right)^0 \left(\frac{109}{144}\right)^n \Rightarrow$

$\Rightarrow n > \frac{\ln 0.001}{\ln \frac{109}{144}} \rightarrow n > 24.8 \Rightarrow n \geq 25$