LDDR Niveau 1 TE 9 solutions

Exercice 1

1) $\lim\limits_{n\to+\infty}\left(1+\frac{1}{n}\right)^{\frac{n}{2}+1} = \lim\limits_{n\to+\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{1}{2}}\left(1+\frac{1}{n}\right) =$

$= e^{1/2}\cdot 1 = \sqrt{e}$

2) $\lim\limits_{n\to+\infty}\left(1+\frac{1}{3n}\right)^{5n} = e^{5/3}$

Exercice 2

1) $e^{x-2} = 3e \Leftrightarrow \frac{e^{x-2}}{e} = 3 \Leftrightarrow e^{x-2-1} = 3 \Leftrightarrow$

$e^{x-3} = 3 \Leftrightarrow x-3 = \ln 3 \Leftrightarrow x = 3+\ln 3$

$\cdot\ x>0$
$\cdot\ \ln x \neq 0 \Leftrightarrow x \neq 1$

2) $\ln(x) - 2 = \frac{3}{\ln x} \Leftrightarrow \ln^2 x - 2\ln x - 3 = 0$

$t = \ln x \qquad t^2 - 2t - 3 = 0$

$\Delta = 4+12 = 16 \qquad t_{1,2} = \frac{2\pm 4}{2} \begin{cases} t_1 = 3 \\ t_2 = -1 \end{cases}$

$\ln x = 3 \Leftrightarrow x = e^3$ ✓
$\ln x = -1 \Leftrightarrow x = e^{-1}$ ✓

3) $\frac{2e^x - 5}{3e^x} = \frac{1}{2} \Leftrightarrow 4e^x - 10 = 3e^x \Leftrightarrow e^x = 10$
$\Leftrightarrow x = \ln 10$

4) $\ln\left(\frac{x}{x+2}\right) = -1 \Leftrightarrow \frac{x}{x+2} = e^{-1} \Leftrightarrow x = \frac{x+2}{e} \Leftrightarrow$

$\frac{x}{x+2} > 0$

		-2		0	
$\frac{x}{x+2}$	$+$	$\parallel$	$-$	$\parallel$	$+$

$\Leftrightarrow ex = x+2 \Leftrightarrow ex - x = 2 \Leftrightarrow (e-1)x = 2$

$\Leftrightarrow x = \frac{2}{e-1} \simeq 1{,}16 > 0$ ✓

Exercice 3 1) $f(x) = \dfrac{x}{\ln(x+3)}$

$\cdot\ x+3>0 \Leftrightarrow x>-3$

$\cdot\ \ln(x+3) \neq 0 \Leftrightarrow x+3 \neq 1$

$\Leftrightarrow x \neq -2.$

$\cdot\ D_f =]-3;-2[\cup]-2;+\infty[$

x	-3		-2		$+\infty$
$\ln(x+3)$		$-$	0	$+$	

$\cdot$ AV $\lim\limits_{x \to -3} f(x) = \text{"}\dfrac{-3}{-\infty}\text{"} = 0$ AV : $x=-3$

$\lim\limits_{x \to -2} f(x) = \text{"}\dfrac{-2}{\ln 1}\text{"} = \text{"}\dfrac{-2}{0}\text{"} = \begin{cases} x>-2 & -\infty \\ x<-2 & +\infty \end{cases}$

AV $x=-2$

$\cdot$ AH $\lim\limits_{x \to +\infty} f(x) = \text{"}\dfrac{+\infty}{+\infty}\text{"} = +\infty \Rightarrow \nexists$

2) $f(x) = \dfrac{\ln(x^2)}{e^x-2}$ $\cdot\ x^2>0 \Rightarrow x \neq 0$

$\cdot\ e^x-2 \neq 0 \Leftrightarrow e^x \neq 2 \Leftrightarrow x \neq \ln 2$

$\cdot\ D_f = \mathbb{R} \setminus \{\ln 2, 0\}$

x	$-\infty$		$\ln 2$		$+\infty$
e^x-2		$-$	0	$+$	

$\cdot$ AV : $\lim\limits_{x \to \ln 2} f(x) = \text{"}\dfrac{\overset{(-)}{\ln(\ln^2 2)}}{0}\text{"} \begin{cases} x>\ln 2 & -\infty \\ x<\ln 2 & +\infty \end{cases}$

AV $x = \ln 2$

$\lim\limits_{x \to 0} f(x) = \dfrac{-\infty}{-1} = +\infty$ AV $x=0$

$\cdot$ AH $\lim\limits_{x \to +\infty} f(x) = \text{"}\dfrac{+\infty}{+\infty}\text{"} = 0$ AH : $y=0$

$\lim\limits_{x \to -\infty} f(x) = \text{"}\dfrac{+\infty}{-2}\text{"} = -\infty$ AH $\nexists y$

Exercice 4

1) $f(x) = \ln(x^2+6x+10)$

$x^2+6x+10 > 0 \quad \forall x \in \mathbb{R} \Rightarrow D_f = \mathbb{R}$

$\Delta = 36 - 100 < 0$

$$f'(x) = \frac{2x+6}{x^2+6x+10} = 0 \Leftrightarrow 2x = -6 \Leftrightarrow x = -3$$

x	$-\infty$		-3		$+\infty$
f'		$-$	0	$+$	
f		$\searrow$	Min	$\nearrow$	

$\text{Min} = f(-3) = \ln 1 = 0$

$\text{Min}\,(-3\,;0)$

2) $f(x) = \dfrac{x^3}{2e^x} \qquad D_f = \mathbb{R}$

$$f'(x) = \frac{3x^2 \cdot 2e^x - x^3 \cdot 2e^x}{4e^{2x}} = \frac{2e^x x^2(3-x)}{4e^{2x}} = 0$$

$\Leftrightarrow x = 0 \quad x = 3$

x	$-\infty$		0		3		$+\infty$
f'		$+$	0	$+$	0	$-$	
f		$\nearrow$	PI	$\nearrow$	Max	$\searrow$	

$\text{Max} = f(3) = \dfrac{27}{2e^3}$

$\text{Max}\left(3\,;\dfrac{27}{2e^3}\right)$

Exercice 5

$f(x) = (ax+b)e^{-x} \qquad E\left(3\,;\dfrac{2}{e^3}\right)$

$$f'(x) = ae^{-x} - (ax+b)e^{-x} = e^{-x}(-ax-b+a)$$

$$f'(3) = 0 \Leftrightarrow e^{-3}(-3a-b+a) = 0 \Leftrightarrow$$

$$-2a - b = 0 \Leftrightarrow 2a+b = 0 \quad ①$$

$$f(3) = \frac{2}{e^3} \Leftrightarrow e^{-3}(-2a-b+a) = \frac{2}{e^3} \Leftrightarrow$$

$$\Leftrightarrow -a-b = 2 \Leftrightarrow a+b = -2 \quad ②$$

① - ② $\begin{cases} 2a+b=0 \\ a+b=-2 \end{cases}$

$a = 2$ et $b = -4$