

## LDDR Niveau 1 TE 9 solutions.

Exercice 1

$$1) \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^{\frac{n+1}{2}} = \lim_{n \rightarrow +\infty} \left(\left(1 + \frac{1}{n}\right)^n\right)^{\frac{1}{2}} \left(1 + \frac{1}{n}\right) =$$

$$= e^{\frac{1}{2}} \cdot 1 = \sqrt{e}$$

$$2) \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{3n}\right)^{3n} = e^{\frac{3}{2}}$$

Exercice 2

$$1) e^{x-2} = 3e \Leftrightarrow \frac{e^{x-2}}{e} = 3 \Leftrightarrow e^{x-2-1} = 3 \Leftrightarrow$$

$$e^{x-3} = 3 \Leftrightarrow x-3 = \ln 3 \Leftrightarrow x = 3 + \ln 3$$

$$x > 0 \quad 2) \ln(x) - 2 = \frac{3}{\ln x} \Leftrightarrow \ln^2 x - 2 \ln x - 3 = 0.$$

$\ln x \neq 0 \Leftrightarrow$

$x \neq 1$

$$t = \ln x \quad t^2 - 2t - 3 = 0$$

$$\Delta = 4 + 12 = 16 \quad t_{1,2} = \frac{2 \pm 4}{2} \quad t_1 = 3 \quad t_2 = -1$$

$$\ln x = 3 \Leftrightarrow x = e^3 \quad \checkmark$$

$$\ln x = -1 \Leftrightarrow x = e^{-1} \quad \checkmark$$

$$3) \frac{2e^x - 5}{3e^x} = \frac{1}{2} \Leftrightarrow 4e^x - 10 = 3e^x \Leftrightarrow e^x = 10$$

$$\Leftrightarrow x = \ln 10$$

$$4) \ln\left(\frac{x}{x+2}\right) = -1 \Leftrightarrow \frac{x}{x+2} = e^{-1} \Leftrightarrow x = \frac{x+2}{e} \Leftrightarrow$$

$$\frac{x}{x+2} > 0$$

$$\begin{array}{c|ccc} & + & - & + \\ \hline x & + & - & + \\ x+2 & + & - & + \end{array}$$

$$\Leftrightarrow ex = x+2 \Leftrightarrow ex - x = 2 \Leftrightarrow (e-1)x = 2$$

$$\Leftrightarrow x = \frac{2}{e-1} \approx 1.16 > 0 \quad \checkmark$$

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Exercise 3 1)  $f(x) = \frac{xc}{\ln(x+3)}$

$\cdot x+3 > 0 \Leftrightarrow x > -3$   
 $\cdot \ln(x+3) \neq 0 \Leftrightarrow x+3 \neq 1$   
 $\Leftrightarrow x \neq -2$

$\cdot D_f = [-3; -2] \cup [-2; +\infty[$

$\cdot \text{AV} \lim_{x \rightarrow -3} f(x) = \frac{-3}{-\infty} = 0 \quad \text{AV: } x = -3$

$\begin{array}{c|ccc} & -3 & -2 & +\infty \\ \hline \ln(x+3) & - & 0 & + \end{array}$

$\lim_{x \rightarrow -2} f(x) = \frac{-2}{\ln 1} = \frac{-2}{0} = \begin{cases} +\infty & x > -2 \\ -\infty & x < -2 \end{cases}$

$\text{AV } x = -2$

$\cdot \text{AH} \lim_{x \rightarrow +\infty} f(x) = \frac{+\infty}{+\infty} = +\infty \Rightarrow \text{f}$

2)  $f(x) = \frac{\ln(x^2)}{e^{x-2}}$

$\cdot D_f = \mathbb{R} \setminus \{\ln 2, 0\}$

$\cdot x^2 > 0 \Rightarrow x \neq 0$

$\cdot e^{x-2} \neq 0 \Leftrightarrow e^{x-2} \neq 1 \Leftrightarrow x \neq \ln 2$

$\begin{array}{c|ccc} & -\infty & \ln 2 & +\infty \\ \hline x^2 & - & 0 & + \end{array}$

$\text{AV: } \lim_{x \rightarrow \ln 2} f(x) = \frac{\ln(\ln 2^2)}{0} = \begin{cases} +\infty & x > \ln 2 \\ -\infty & x < \ln 2 \end{cases}$

$\text{AV } x = \ln 2$

$\lim_{x \rightarrow 0} f(x) = \frac{-\infty}{-1} = +\infty \quad \text{AV } x = 0$

$\cdot \text{AH} \lim_{x \rightarrow +\infty} f(x) = \frac{+\infty}{+\infty} = 0 \quad \text{AH: } y = 0$

$\lim_{x \rightarrow -\infty} f(x) = \frac{+\infty}{-2} = -\infty \quad \text{AH: } y$

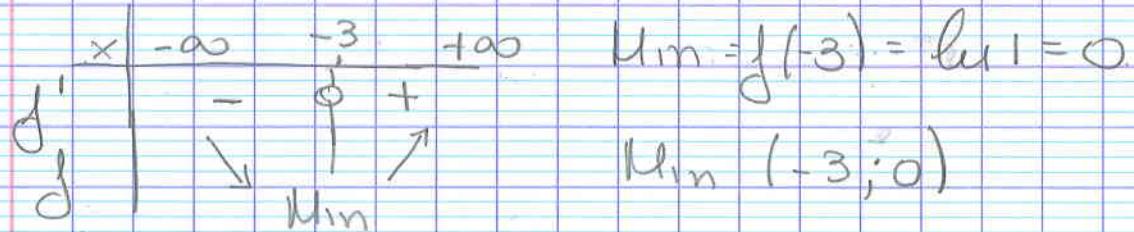
Exercise 4

1)  $f(x) = \ln(x^2 + 6x + 10)$

$x^2 + 6x + 10 > 0 \quad \forall x \in \mathbb{R} \rightarrow D_f = \mathbb{R}$

$\Delta = 36 - 100 < 0$

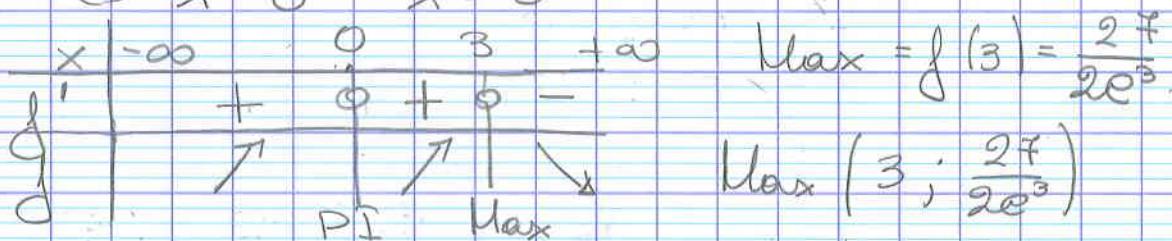
$f'(x) = \frac{2x+6}{x^2+6x+10} = 0 \Leftrightarrow 2x = -6 \Leftrightarrow x = -3$



2)  $f(x) = \frac{x^3}{2e^x} \quad D_f = \mathbb{R}$

$f'(x) = \frac{3x^2 \cdot 2e^x - x^3 \cdot 2e^x}{4e^{2x}} = \frac{2e^x x^2 (3 - x)}{4e^{2x}} = 0$

$\Leftrightarrow x = 0 \quad x = 3$



Exercise 5  $f(x) = (ax+b)e^{-x} \quad E(3; \frac{2}{e^3})$

$f'(x) = ae^{-x} - (ax+b)e^{-x} \equiv e^{-x}(-ax-b+a)$

$f'(3) = 0 \Leftrightarrow e^{-3}(-3a - b + a) = 0 \Leftrightarrow$

$-2a - b = 0 \Leftrightarrow 2a + b = 0 \quad ①$

$f(3) = \frac{2}{e^3} \Leftrightarrow e^{-3}(-2a - b + a) = \frac{2}{e^3} \Leftrightarrow$

$\Leftrightarrow -a - b = 2 \Leftrightarrow a + b = -2 \quad ②$

$$\begin{cases} 2a + b = 0 \\ a + b = -2 \end{cases}$$

$\overline{a = 2 \text{ et } b = -4}$