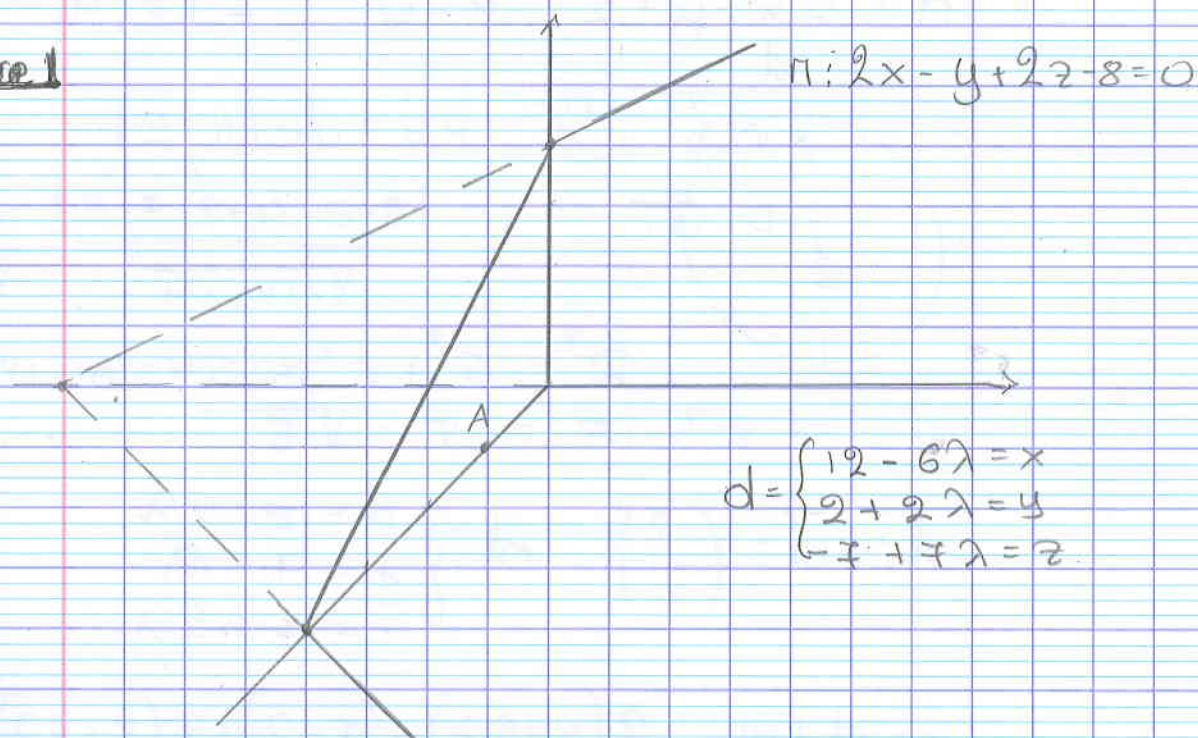


LDDR - Niveau 1 TE 8 (Solutions)

Exercise 1



1) $\Pi_x: y = z = 0 \quad \Pi_x(4; 0; 0)$
 $\Pi_y: x = z = 0 \quad \Pi_y(0; -8; 0)$
 $\Pi_z: x = y = 0 \quad \Pi_z(0; 0; 4)$

2) $A(3; 2; 2) \quad B(3; 6; 0) \quad D(5; 10; 4)$

$\vec{AB} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} \quad \vec{AD} = \begin{pmatrix} 2 \\ 8 \\ 2 \end{pmatrix} \quad \vec{AB} \cdot \vec{AD} = 32 - 4 = 28$

$\|\vec{AB}\| = \sqrt{16 + 4} = \sqrt{20} \quad \|\vec{AD}\| = \sqrt{4 + 64 + 4} = \sqrt{72}$

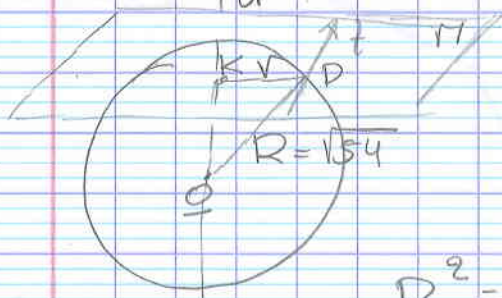
$\cos \varphi = \frac{\vec{AB} \cdot \vec{AD}}{\|\vec{AB}\| \cdot \|\vec{AD}\|} = \frac{28}{\sqrt{20} \cdot \sqrt{72}} \Rightarrow \varphi = 42.45^\circ$

3) $O(8; 4; 7) \quad r = \|\vec{AO}\| = \sqrt{25 + 4 + 25} = \sqrt{54}$
 $\vec{AO} = \begin{pmatrix} 5 \\ 2 \\ 7 \end{pmatrix}$

$S: (x-8)^2 + (y-4)^2 + (z-7)^2 = 54$

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$$\Pi: 2x - y + 2z - 8 = 0 \quad o(8; 4; 7)$$



$$KO = \text{dis}(\Pi; o) =$$

$$= \frac{|2 \cdot 8 - 4 + 2 \cdot 7 - 8|}{\sqrt{4 + 1 + 4}} = \frac{18}{\sqrt{9}} = 6$$

$$R^2 = 6^2 + r^2 \Leftrightarrow 54 = 36 + r^2 \Rightarrow$$

$$\Rightarrow r^2 = 18 \Rightarrow r = \sqrt{18}$$

$$ok: \vec{d} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \quad ol: \begin{cases} x = 8 + 2\lambda \\ y = 7 - \lambda \\ z = 7 + 2\lambda \end{cases}$$

$$K = d \cap \Pi \quad 2(8 + 2\lambda) - (7 - \lambda) + 2(7 + 2\lambda) - 8 = 0$$

$$\Leftrightarrow 16 + 4\lambda - 7 + \lambda + 14 + 4\lambda - 8 = 0 \Leftrightarrow$$

$$\Leftrightarrow 9\lambda = -15 \Leftrightarrow \lambda = -\frac{5}{3}$$

$$K \left(8 - \frac{10}{3}; 7 + \frac{5}{3}; 7 - \frac{10}{3} \right) = \left(\frac{14}{3}; \frac{26}{3}; \frac{11}{3} \right)$$

$$s) D(5; 10; 4)$$

$$S: (5-8)^2 + (10-4)^2 + (4-7)^2 = 9 + 36 + 9 = 54 \checkmark$$

$$\Pi: 2 \cdot 5 - 10 + 2 \cdot 4 - 8 = 0 \checkmark$$

$$D \in S \cap \Pi \Rightarrow D \in \text{cerce}$$

$$\vec{DO} = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix} \quad \vec{d} = \vec{n}_\Pi = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

$$\vec{t} = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix} \wedge \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -12 + 3 \\ 6 - 6 \\ -3 + 12 \end{pmatrix} = \begin{pmatrix} -9 \\ 0 \\ 9 \end{pmatrix} \parallel \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$t: \begin{cases} x = 5 - \lambda \\ y = 10 \\ z = 4 + \lambda \end{cases}$$

Exercise 2 1) $f(x) = \frac{x}{\sin(2x)}$

$$f'(x) = \frac{\sin(2x) - x \cdot 2 \cos(2x)}{\sin^2(2x)} = \frac{\sin(2x) - 2x \cos(2x)}{\sin^2(2x)}$$

2) $f(x) = 7 \cdot \sqrt[4]{1-8x} = 7(1-8x)^{1/4}$

$$f'(x) = 7 \cdot \frac{1}{4} (1-8x)^{-3/4} \cdot (-8) = -14 \frac{1}{\sqrt[4]{(1-8x)^3}}$$

Exercise 3 1) $\frac{(49r^3s^2)^2}{(7rs)^3} = \frac{49^2 r^6 s^4}{7^3 r^3 s^3} = \frac{7^4 r^6 s^4}{7^3 r^3 s^3} =$

$$= 7 r^3 s$$

2) $\frac{\sqrt{\frac{1}{a^3}}}{\sqrt[3]{a}} = \frac{a^{-3/2}}{a^{1/3}} = \frac{1}{a^{3/2} \cdot a^{1/3}} = \frac{1}{a^{11/6}} = \frac{1}{\sqrt[6]{a^{11}}}$

Exercise 4 1) $\log_{16}(x) = -\frac{3}{2} \Leftrightarrow x = 16^{-3/2} > 0$

$$\begin{matrix} x > 0 \\ x \neq 1 \end{matrix}$$

2) $\log_x \sqrt{27} = 3 \Leftrightarrow x^3 = \sqrt{27} = 3^{3/2} \Leftrightarrow x = \sqrt{3} \checkmark$

3) $4 \cdot 16^{x-1} = \frac{1}{8^x} \Rightarrow 4 \cdot 2^{4x-4} = 2^{-3x} \Leftrightarrow$

$$\Leftrightarrow 2^2 \cdot 2^{4x-4} = 2^{-3x} \Leftrightarrow 2^{4x-2} = 2^{-3x} \Leftrightarrow$$

$$4x-2 = -3x \Leftrightarrow 7x = 2 \Leftrightarrow x = \frac{2}{7}$$

$$\begin{cases} x > -1/2 \\ x > -1 \end{cases}$$

4) $\log(2x+1) + \log(x+1) = 1 \Leftrightarrow$
 $\Leftrightarrow \log[(2x+1)(x+1)] = 1 \Rightarrow$

$$(2x+1)(x+1) = 10 \Leftrightarrow 2x^2 + 2x + x + 1 - 10 = 0$$

$$\Leftrightarrow 2x^2 + 3x - 9 = 0$$

$$\Delta = 81 \quad x_{1,2} = \frac{-3 \pm 9}{4} \begin{cases} -3 \times \\ \frac{3}{2} \checkmark \end{cases}$$

$$\boxed{x = \frac{3}{2}}$$