LDDR - Niveau 1 : TE 15 (Solutions)

Exercice 1 $f(x) = \frac{10\ln x}{x^2}$ $\qquad D_f =]0,+\infty[$

a) $I(x_0, y_0)$: $f(x) = 0 \Leftrightarrow \ln x = 0 \Rightarrow x = 1$ $\quad I(1;0)$

$f'(x) = \frac{10\frac{1}{\cancel{x}}\cdot x^{\cancel{2}} - 10\ln x \cdot 2x}{x^4} = \frac{10x - 20x\ln x}{x^4} =$

$= \frac{10\cancel{x}(1-\ln x)}{x^{\cancel{4}3}} \Rightarrow f'(x) = \frac{10(1-\ln x)}{x^3} = 0$

$\Rightarrow \ln x = 1 \Rightarrow x = e \quad S\left(e;\frac{10}{e^2}\right)$

$y_S = f(e) = \frac{10\ln e}{e^2} = \frac{10}{e^2}$

b) $F(x) = \frac{-10(\ln(x)+1)}{x}$

$F'(x) = \frac{-10\cdot\frac{1}{x}\cdot x + 10(\ln x+1)}{x^2} =$

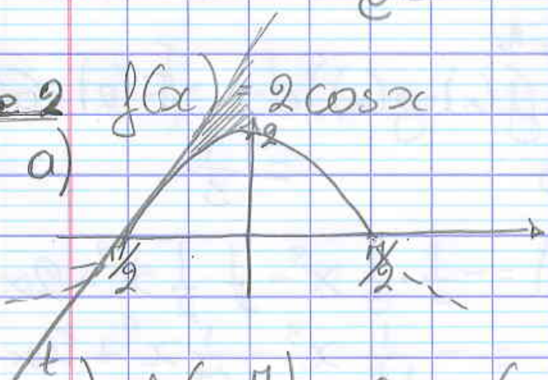
$= \frac{-10+10\ln x+10}{x^2} = \frac{10\ln x}{x^2} = f(x)$

c) $A = \int_1^e f(x)\,dx = F(e) - F(1) = \frac{10}{e^2}$

$F(e) = \frac{10\overset{1}{\ln e}}{e^2} = \frac{10}{e^2} \qquad F(1) = 0$

Exercice 2 $f(x) = 2\cos x$

a)



b) $f'(x) = -2\sin x = 2 \Rightarrow$
$\sin x = -1 \Rightarrow x = -\pi/2$

c) $f\left(-\frac{\pi}{2}\right) = 2\cos\left(-\frac{\pi}{2}\right) = 0 \qquad f'\left(-\frac{\pi}{2}\right) = 2$

$0 = 2\cdot\left(-\frac{\pi}{2}\right) + h \Rightarrow h = \pi$

$t:\ y = 2x + \pi$

$$A = \int_{-\pi/2}^{0} (t(x) - f(x))\,dx = T(0) - F(0) - \left(T\left(-\frac{\pi}{2}\right) - F\left(-\frac{\pi}{2}\right)\right)$$
$$= \frac{1}{4}(\pi^2 - 8) \approx 0{,}46740$$

$T(x) = x^2 + \pi x$
$F(x) = 2\sin x$

Exercice 3 $A(\overset{x_1}{-4};0)$ $B(0;2)$ $C(\overset{x_2}{2};0)$

P_{ABC}: $y = a(x+4)(x-2) \leftarrow B$
$2 = a \cdot 4(-2) \Rightarrow a = -\frac{1}{4}$
$f(x) = -\frac{1}{4}(x+4)(x-2) = -\frac{1}{4}x^2 - \frac{1}{2}x + 2$

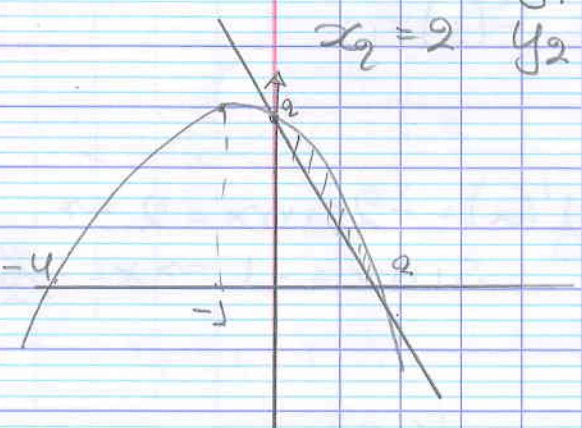
d_{BC}: $m = \frac{2-0}{0-2} = -1$ $\quad 2 = -1 \cdot 0 + h \Rightarrow$
$h = 2$

$g(x) = -x + 2$

sommet: $x_s = \frac{-b}{2a} = \frac{\frac{1}{2}}{2(-\frac{1}{4})} = -1$ $\quad y_s = 9/4$

$f(x) = g(x) \Leftrightarrow -\frac{1}{4}x^2 - \frac{1}{2}x + 2 = -x + 2 \Leftrightarrow$

$\Leftrightarrow -x^2 - 2x + 8 = -4x + 8 \Leftrightarrow$
$\Leftrightarrow x^2 - 2x = 0 \Leftrightarrow x(x-2) = 0$ $\quad x=0$, $x=2$
$x_1 = 0$ $\quad y_1 = 2$ $\quad (0,2)$
$x_2 = 2$ $\quad y_2 = 0$ $\quad (2,0)$



$$A = \int_0^2 (f(x) - g(x))\,dx = [F(2) - G(2)] - (F(0) - G(0))$$
$$= \frac{1}{3}$$

$F(x) = -\frac{1}{4}\cdot\frac{1}{3}x^3 - \frac{1}{2}\cdot\frac{1}{2}x^2 + 2x =$
$= -\frac{1}{12}x^3 - \frac{1}{4}x^2 + 2x$

$G(x) = -\frac{1}{2}x^2 + 2x$