LDDR - Niveau 1 : TE 14 (Solutions)

Exercice 1. $f(x) = (e^x-3)(e^x+1)$

a) $D_f = \mathbb{R}$

$\cap O_x \quad f(x)=0 \Leftrightarrow e^x=3 \Leftrightarrow x=\ln 3 \qquad (\ln 3;0)$

$e^x+1=0 \Leftrightarrow e^x=-1$ impossible

$\cap O_y \quad x=0 \quad y=(1-3)(1+1)=-4 \quad (0;-4)$

$\lim\limits_{x\to+\infty} f(x) = +\infty \qquad \lim\limits_{x\to-\infty} f(x) = -3\cdot 1 = -3$

AH $y=-3$

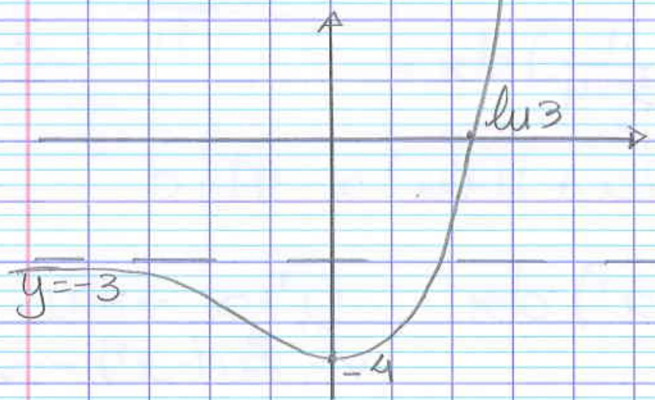
b) $f'(x) = e^x(e^x+1)+(e^x-3)e^x = e^{2x}+e^x+e^{2x}-3e^x \Rightarrow$

$f'(x) = 2e^{2x}-2e^x = e^x(2e^x-2) = 2e^x(e^x-1)=0$

$\Leftrightarrow e^x=1 \Rightarrow x=0$

x	$-\infty$		0		$+\infty$
f'		$-$	0	$+$	
f		$\searrow$	Min	$\nearrow$	

Min $= f(0) = -6$



Exercice 2

a) $\int x e^2 dx = \frac{e^2}{2} x^2 + c$

b) $\int \frac{1}{\sqrt{x^3}} dx = \int x^{-3/2} dx = \frac{1}{-\frac{3}{2}+1} x^{-\frac{3}{2}+1} = -2x^{-1/2} + c =$

$= -\frac{2}{\sqrt{x}} + c$

c) $\int (x^2+1) e^{-x} dx =$ $\quad u' = e^{-x} \Rightarrow u = -e^{-x}$

$\quad v = x^2+1 \Rightarrow v' = 2x$

$= -e^{-x}(x^2+1) + 2\int x e^{-x} dx$ $\quad u' = e^{-x} \Rightarrow u = -e^{-x}$

$\quad v = x \Rightarrow v' = 1$

$= -e^{-x}(x^2+1) + 2\left[-x e^{-x} + \int e^{-x} dx\right] =$

$= -e^{-x}(x^2+1) - 2x e^{-x} - 2e^{-x} =$

$= -e^{-x}(x^2+1+2x+2) = -e^{-x}(x^2+2x+3) + c$

d) $\int \frac{x^2-2x+3}{x-4} dx =$

$$\begin{array}{r|l} x^2-2x+3 & x-4 \\ \hline -x^2+4x & x+2 \\ 2x+3 & \\ -2x+8 & \\ \hline 11 & \end{array}$$

$= \int \left(x+2 + \frac{11}{x-4}\right) dx =$

$= \frac{1}{2}x^2 + 2x + 11 \ln|x-4| + c$

e) $\int x \ln(-x^2) dx$ $\quad u' = x \Rightarrow u = \frac{1}{2}x^2$

$\quad v = \ln(-x^2) \Rightarrow v' = \frac{-2\cancel{x}}{-x^{\cancel{2}}} = \frac{2}{x}$

$= \frac{1}{2}x^2 \ln(-x^2) - \frac{2}{2}\int \frac{x^2}{x} dx =$

$= \frac{1}{2}x^2 \ln(-x^2) - \int x\, dx = \frac{1}{2}x^2 \ln(-x) - \frac{1}{2}x^2 + c$

f) $\int 2x \cos(x^2+1) dx$ $\quad t = x^2+1 \Rightarrow dt = 2x\, dx$

$= \cancel{2}\int \cancel{x} \cos t \cdot \frac{1}{\cancel{2x}} dt = \int \cos t\, dt = \sin t$

$= \sin(x^2+1) + c$