LDDR - Niveau 1 : TE 13 (Solutions)

Exercice 1 $f(x) = \dfrac{2e^x - k}{e^x + 1}$ $k \in \mathbb{R}$

1) $f'(x) = \dfrac{2e^x(e^x+1) - (2e^x - k)e^x}{(e^x+1)^2} =$

$= \dfrac{2e^{2x} + 2e^x - 2e^{2x} + ke^x}{(e^x+1)^2} = \dfrac{e^x(2+k)}{(e^x+1)^2} = 0 \Leftrightarrow$

$\Leftrightarrow k = -2$

Donc si $k \neq -2$ f n'a pas de PTH

2) $k = 1$ $f(x) = \dfrac{2e^x - 1}{e^x + 1}$

- $e^x + 1 > 0$ $\forall x \in \mathbb{R}$ $\Rightarrow D_f = \mathbb{R}$
- $f(x) = 0 \Leftrightarrow 2e^x - 1 = 0 \Leftrightarrow e^x = \frac{1}{2} \Rightarrow x = -\ln 2 \approx -0.7$

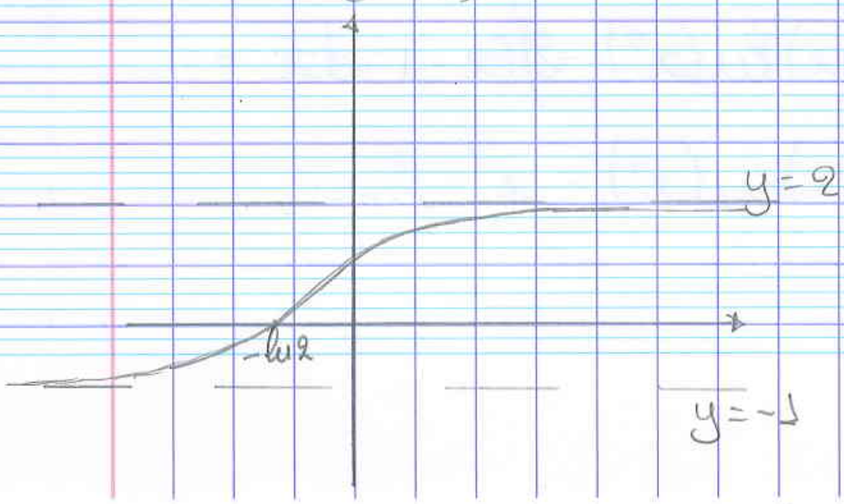
x	$-\infty$		$-\ln 2$		$+\infty$
f		$-$	0	$+$	

- AV $\nexists$ AH: $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \dfrac{e^x(2 - \cancel{1/e^x})}{e^x(1 + \cancel{1/e^x})} = 2$

$\lim\limits_{x\to-\infty} f(x) = \dfrac{2\cdot 0 - 1}{0 + 1} = -1$

$y = 2$ en $+\infty$ $y = -1$ $(-\infty)$

- $f'(x) = \dfrac{3e^x}{(e^x+1)^2} = 0$ impossible $f'(x) > 0$ $\forall x \in \mathbb{R}$



Exercice 2 $f(x) = \frac{\ln(x^2-1)}{x+2}$

1) $x+2 \neq 0 \Leftrightarrow x \neq -2$

$x^2-1>0$

		-1		1	
	+	0	-	0	+

$D_f =]-\infty, -2[\cup]-2; -1[\cup]1, +\infty[$

2) $f(x)=0 \Leftrightarrow \ln(x^2-1)=0 \Leftrightarrow x^2-1=1 \Leftrightarrow x^2=2 \Leftrightarrow x=\pm\sqrt{2}$

	$-\infty$		-2		$-\sqrt{2}$		-1		1		$\sqrt{2}$		$+\infty$
f		-	‖	+	0	-		//////		-	0	+	

Exercice 3 1) $f(x)=2x^2-1+\frac{3}{x}$ $\quad F(x)=\frac{2}{3}x^3-x-\frac{3}{x^2}$

2) $f(x)=\frac{x^2-x}{x+1}$

$$\begin{array}{r|l} x^2-x & x+1 \\ \cline{2-2} -x^2-x & x-2 \\ \hline -2x & \\ 2x+2 & \\ \hline 2 & \end{array}$$

$F(x)=\int\left(x-2+\frac{2}{x+1}\right)dx=\frac{1}{2}x^2-2x+2\ln|x+1|$

3) $f(x)=\cos\left(\frac{x+1}{2}\right)$ $\quad F(x)=2\sin\left(\frac{x+1}{2}\right)$

4) $f(x)=(2x-1)\ln(x^2)$

$u'=2x-1 \Rightarrow u=x^2-x$

$v=\ln(x^2) \Rightarrow v'=\frac{2x}{x^2}=\frac{2}{x}$

$\int(2x-1)\ln(x^2)dx=(x^2-x)\ln(x^2)-\int(x^2-x)\cdot\frac{2}{x}dx$

$=(x^2-x)\ln(x^2)-2\int(x-1)dx=$

$=(x^2-x)\ln(x^2)-x^2+2x$

5) $f(x) = e^{-2x}(x-x^2)$ $\quad u' = e^{-2x} \Rightarrow u = -\frac{1}{2}e^{-2x}$

$v = -x^2+x \Rightarrow v' = -2x+1$

$$\int f(x)\,dx = -\frac{1}{2}e^{-2x}(-x^2+x) + \frac{1}{2}\int(-2x+1)e^{-2x}\,dx$$

$u' = e^{-2x} \Rightarrow u = -\frac{1}{2}e^{-2x}$

$v = -2x+1 \Rightarrow v' = -2$

$$= -\frac{1}{2}e^{-2x}(-x^2+x) + \frac{1}{2}\left[-\frac{1}{2}e^{-2x}(-2x+1) + \frac{2}{2}\int e^{-2x}\,dx\right]$$

$$= -\frac{1}{2}e^{-2x}(-x^2+x) - \frac{1}{4}e^{-2x}(-2x+1) - \frac{1}{4}e^{-2x} =$$

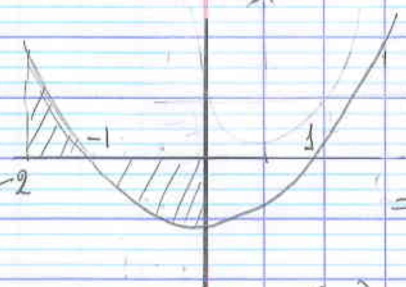
$$= -\frac{1}{2}e^{-2x}\left(-x^2+x+\frac{1}{2}(-2x+1)+\frac{1}{2}\right) =$$

$$= -\frac{1}{2}e^{-2x}\left(-x^2+\cancel{x}-\cancel{x}+\frac{1}{2}+\frac{1}{2}\right) =$$

$$= -\frac{1}{2}e^{-2x}(-x^2+1)$$

<u>Exercice 4</u>

1) $f(x) = x^2-1$ $\quad a=-2 \quad b=0$



$$A = \int_{-2}^{-1} f(x)\,dx - \int_{-1}^{0} f(x)\,dx =$$

$$= F(-1) - F(-2) - F(0) + F(-1) = \frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2$$

$F(x) = \frac{1}{3}x^3 - x$

2) $f(x) = \sqrt[3]{7x-6} > 0$ $\quad a=1 \quad b=2$

$$A = \int_{1}^{2} f(x)\,dx = F(2) - F(1) = \frac{45}{28}$$

$$F(x) = \int (7x-6)^{1/3}\,dx = \frac{3}{4}\cdot\frac{1}{7}(7x-6)^{4/3} = \frac{3}{28}\sqrt[3]{(7x-6)^4}$$

3) $f(x) = x\cos x$ $\quad a = 0 \quad b = \frac{3\pi}{2}$

$f(x) = 0 \Leftrightarrow x = 0 \quad \cos x = 0 \Rightarrow x = \pi/2$

$x = 3\pi/2$

x	0		$\pi/2$		$3\pi/2$	$+\infty$
x	0	$+$		$+$		$+$
$\cos x$		$+$	0	$-$	0	$+$
f	0	$+$	0	$-$	0	$+$

$$A = \int_0^{\pi/2} f(x)\,dx - \int_{\pi/2}^{3\pi/2} f(x)\,dx =$$

$$= F\left(\frac{\pi}{2}\right) - F(0) - F\left(\frac{3\pi}{2}\right) + F\left(\frac{\pi}{2}\right) = \frac{1}{2}(\pi - 2) - (-2\pi)$$

$$= \frac{5\pi}{2} - 1$$

$$F(x) = \int x\cos x\,dx$$

$u' = \cos x \Rightarrow u = \sin x$

$v = x \Rightarrow v' = 1$

$$= x\sin x - \int \sin x\,dx =$$

$$= x\sin x + \cos x$$