2.14 Exercises

2. 1 Complete the following tables (leave your answer as a multiple of π for the angles in radians).

degrees	90°	135°	30°	18°	22.5°	720°	270°	1°	$x°$
radians									

radians	$\frac{\pi}{3}$	$\frac{\pi}{20}$	$\frac{3\pi}{8}$	$\frac{2\pi}{3}$	$-\frac{\pi}{2}$	6π	1	2.5	x
degrees									

2. 2 Given a number α. Place precisely, with the help of a protractor, the point P_α which represents α on the trigonometric circle. Measure on your drawing the values of $\cos(\alpha), \sin(\alpha)$ and $\tan(\alpha)$ and finally, complete the table below.

α	$\cos(\alpha)$	$\sin(\alpha)$	$\tan(\alpha)$
230°			
1.9			
14.2π			

2. 3 In each case, find the value(s) of α so that the equality is true.

1) $\cos(\alpha°) = 0$
2) $\sin(\alpha°) = -1$
3) $\cos(\alpha) = -1$
4) $\sin(\alpha°) = 0$
5) $\tan(\alpha°) = -1$
6) $\tan(\alpha°) = 0$

2. 4 Answer the following :

1) Find the angles between 0° and 360° which are equivalent, on the trigonometric circle, to 2578°, 5555°, −1111° and −9876°.
2) Find the angles between 0 and 2π which are equivalent, on the trigonometric circle, to $\frac{97\pi}{6}$, $\frac{111\pi}{8}$, $\frac{-304\pi}{3}$ and $-\frac{123\pi}{4}$.

2. 5 Using a diagram, find the exact values of...

1) $\cos(\alpha)$, $\sin(\alpha)$ and $\tan(\alpha)$ with $\alpha = \frac{\pi}{6}$ rad
2) $\cos(\alpha)$, $\sin(\alpha)$ and $\tan(\alpha)$ with $\alpha = 45°$
3) $\cos(\alpha)$, $\sin(\alpha)$ and $\tan(\alpha)$ with $\alpha = \frac{\pi}{3}$ rad

2. 6 Given a circle with radius 3.2m. We consider, on that circle, an arc corresponding to an angle of 117°. Calculate the length of that arc.

2. 7 Toronto and Quito are on the same meridian. Given that the radius of the earth is about 6370km and that the difference between their latitude is 43°, calculate the distance between these two towns.

2. 8 Without calculator, find the exact value of :

1) $\sin(120°)$
2) $\cos(135°)$
3) $\tan(150°)$
4) $\cos(210°)$
5) $\tan(300°)$
6) $\sin(330°)$
7) $\cos(225°)$

2. 9 Show that :

1) $1 + \tan^2(\alpha) = \dfrac{1}{\cos^2(\alpha)}$
2) $\dfrac{\sin^2(x)}{1 - \cos(x)} = 1 + \cos(x)$

2. 10 In each case, find the exact value of $\sin(t)$, $\cos(t)$ and $\tan(t)$ without looking for the value of the angle t.

1) $\cos(t) = \frac{3}{5}$ with $t \in Q_{IV}$
2) $\sin(t) = -0.28$ with $t \in Q_{III}$
3) $\tan(t) = 2$ with $t \in Q_I$

2. 11 Simplify :

1) $\dfrac{\tan(x)}{1 + \tan^2(x)}$
2) $\dfrac{\sin(x)\cos(x)}{1 - \sin^2(x)}$
3) $\dfrac{1 - (\sin(x) - \cos(x))^2}{\sin(x)}$

2. 12 Using the trigonometric circle :

1) Express $\sin(\pi + \alpha)$, $\cos\left(\frac{\pi}{2} + \alpha\right)$ and $\tan(\pi - \alpha)$ by using $\cos(\alpha)$, $\sin(\alpha)$, or $\tan(\alpha)$.
2) Complete the following equalities :
   - ❑ $-\cos(\alpha) = \cos(...)$
   - ❑ $-\sin(\alpha) = \sin(...)$
   - ❑ $\cos(\alpha) = \sin(...)$
   - ❑ $\sin(\alpha) = \cos(...)$
   - ❑ $-\cos(\alpha) = \sin(...)$
   - ❑ $-\sin(\alpha) = \cos(...)$

2. 13 Sketch, in the same set of axes but with two different colors, the graphs of $f(x) = \cos(x)$ and $g(x) = \sin(x)$. Then, color a period of each function.

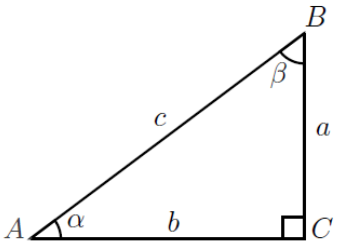
2. 14 Analyse the function $f(x) = \tan(x)$ (domain, zeroes, parity, periodicity, table of signes and graph).

2. 15 Given that $\tan(t) = \frac{5}{4}$ with t in Q_{III}, find the exact value of $\sin(t)$ and $\cos(t)$ without looking for the value of the angle t.

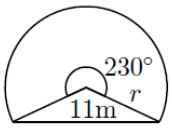
2. 16 Determine the exact value of $\cos(-945°)$ and $\sin\left(\frac{-10\pi}{3}\right)$.

2. 17 The triangle ABC is represented opposite. In each case, find the unknown angles and sides.

1) $a = 7.8\text{cm}$, $\alpha = 27°$
2) $a = 6.3\text{cm}$, $c = 9.2\text{cm}$
3) $c = 4.8\text{cm}$, $\beta = 10°$
4) $a = 13.4\text{cm}$, $b = 20\text{cm}$
5) $a = 5\text{cm}$, Area $= 6\text{cm}^2$



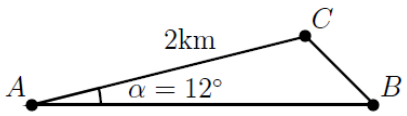
2. 18 The archway of a road tunnel is an arc of circle whose angle at the center mesure 230°. Calculate the radius r of this arc given that the width of the road is 11m. Then, calculate the maximum height of the archway.



2. 19 Answer the following :

1) Calculate the height of a tower whose shadow on the ground measures 36 meters when the sun is $37,5°$ over the horizon ?
2) The average speed of a cable car is 7m/s and the run lasts 4 minutes and 20 seconds. The start is at the altitude of 1315 meters and the cables make a 19° angle with the ground. What is the altitude of the arrival of the cable car ?
3) The perimeter of a regular pentagon is 50 centimeters. Calculate the radius of the incircle and the one of the circumcircle of the pentagon.

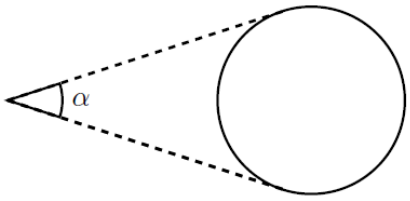
2. 20 In the fog, a man wants to walk from a point A to a point B. He starts its walk at point A but its trajectory makes an angle of 12° with the right way. After 2km he arrives at point C and modifies its course. He reaches the point B after 0,8 km. What is the distance between A and B ?



2. 21 In a circle with radius 10cm, we consider a sector corresponding to a chord of 6cm. Calculate the perimeter and the area of that sector.

2. 22 A man watches a circular tower whose circumference is 50m. He looks at the tower with an angle α of 18°.

1) What is the shortest distance from the man to the tower ?
2) What would the angle be if that distance was 100m ?



2. 23 The area of an isosceles triangle is 120m^2, the angle at its vertex is 54°. Calculate its basis, its height and the length of its sides.

2. 24 Find, correct to one decimal place, all the positive solutions of the trigonometric equations above.

1) $\sin(\theta^\circ) = 0, 1$
2) $\sin(\theta^\circ) = -0.84$
3) $\sin(\theta^\circ + 15^\circ) = 0.951$
4) $\cos(\theta^\circ) = 0.8$
5) $\cos(\theta^\circ) = -0.84$
6) $\cos(\theta^\circ - 20^\circ) = \sqrt{\frac{2}{3}}$
7) $\tan(\theta^\circ) = 4$
8) $\tan(\theta^\circ) = -0.32$
9) $\tan(\theta^\circ + 100^\circ) = 0.11$

2. 25 Determine all the solutions of the following equations in the interval $0^\circ < \theta^\circ \leq 360^\circ$.

1) $\cos(2\theta^\circ) = \frac{1}{3}$
2) $\tan(3\theta^\circ) = 2$
3) $\sin(2\theta^\circ) = -0.6$
4) $\cos(4\theta^\circ + 30^\circ) = -\frac{1}{4}$
5) $\tan(2\theta^\circ - 90^\circ) = 0.4$
6) $\sin(3\theta^\circ - 45^\circ) = -0.42$

2. 26 Find a number between 10 and 11 whose sine is equal to $-0, 88$.

2. 27 Solve the following :

1) $2\sin(x) + \cos(x) = 0$
2) $4\cos^2(x) - 4\cos(x) - 3 = 0$
3) $2\sin^2(x) - 3\sin(x) + 1 = 0$
4) $\sqrt{3}\sin(2x) - \cos(2x) = 0$
5) $2\cos^2(x) + \sin(x) - 1 = 0$
6) $\sqrt{3}\cos(x) - \sin(x) = 1$

2.15 Solutions

2.1

degrees	90°	135°	30°	18°	22.5°	720°	270°	1°	$x°$
radians	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	$\frac{\pi}{6}$	$\frac{\pi}{10}$	$\frac{\pi}{8}$	4π	$\frac{3\pi}{2}$	$\frac{\pi}{180}$	$\frac{\pi x}{180}$

radians	$\frac{\pi}{3}$	$\frac{\pi}{20}$	$\frac{3\pi}{8}$	$\frac{2\pi}{3}$	$-\frac{\pi}{2}$	6π	1	2.5	x
degrees	60°	9°	67.5°	120°	−90°	1080°	57.3°	143.2°	$\frac{180x°}{\pi}$

2.2

α	$\cos(\alpha)$	$\sin(\alpha)$	$\tan(\alpha)$
230°	$\simeq -0.64$	$\simeq -0.77$	$\simeq 1.19$
1.9	$\simeq -0.32$	$\simeq 0.95$	$\simeq -2.93$
14.2π	$\simeq 0.81$	$\simeq 0.59$	$\simeq 0.73$

2.3

1) $\alpha = 90° + k \cdot 180°,\ k \in \mathbb{Z}$
2) $\alpha = 270° + k \cdot 360°,\ k \in \mathbb{Z}$
3) $\alpha = \pi + k \cdot 2\pi,\ k \in \mathbb{Z}$
4) $\alpha = k \cdot 180°,\ k \in \mathbb{Z}$
5) $\alpha = 135° + k \cdot 180°,\ k \in \mathbb{Z}$
6) $\alpha = k \cdot 360°,\ k \in \mathbb{Z}$

2.4 1) 58° 155° 329° 204° 2) $\frac{\pi}{6}$ $\frac{15\pi}{8}$ $\frac{2\pi}{3}$ $\frac{5\pi}{4}$

2.5

α	$\cos(\alpha)$	$\sin(\alpha)$	$\tan(\alpha)$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$

2.6 $l \simeq 6,53\text{m}$

2.7 $d \simeq 4780,6\text{km}$

2.8

1) $\sin(120°) = \frac{\sqrt{3}}{2}$
2) $\cos(135°) = -\frac{\sqrt{2}}{2}$
3) $\tan(150°) = -\frac{\sqrt{3}}{3}$
4) $\cos(210°) = -\frac{\sqrt{3}}{2}$
5) $\tan(300°) = -\sqrt{3}$
6) $\sin(330°) = -\frac{1}{2}$
7) $\cos(225°) = -\frac{\sqrt{2}}{2}$

2.9 —

2.10 1) $\sin(t) = -\frac{4}{5}$ $\tan(t) = -\frac{4}{3}$

2) $\cos(t) = -\frac{24}{25}$ $\tan(t) = \frac{7}{24}$

3) $\sin(t) = -\frac{2\sqrt{5}}{5}$ $\cos(t) = -\frac{\sqrt{5}}{5}$

2.11 1) $\sin(x)\cos(x)$ 2) $\frac{1)}{\tan(x)}$ 3) $2\cos(x)$

2.12 1)
$$\sin(\pi+\alpha) = -\sin(\alpha)$$
$$\cos(\frac{\pi}{2}+\alpha) = -\sin(\alpha)$$
$$\tan(\pi-\alpha) = -\tan(\alpha)$$

2)
$$-\cos(\alpha) = \cos(\pi-\alpha) = \cos(\pi+\alpha)$$
$$-\sin(\alpha) = \sin(-\alpha) = \sin(\pi+\alpha)$$
$$\cos(\alpha) = \sin(\frac{\pi}{2}-\alpha) = \sin(\frac{\pi}{2}+\alpha)$$
$$\sin(\alpha) = \cos(\frac{\pi}{2}-\alpha) = \sin(-\frac{\pi}{2}+\alpha)$$
$$-\cos(\alpha) = \sin(-\frac{\pi}{2}-\alpha) = \sin(-\frac{\pi}{2}+\alpha)$$
$$-\sin(\alpha) = \cos(\frac{\pi}{2}+\alpha) = \cos(-\frac{\pi}{2}-\alpha)$$

2.13 —

2.14 —

2.15 $\cos(t) = -\frac{4\sqrt{41}}{41}$ $\sin(t) = -\frac{5\sqrt{41}}{41}$

2.16 $-\frac{\sqrt{2}}{2}$ $\frac{\sqrt{3}}{2}$

2.17

Questions	α	β	$a_{[\,cm\,]}$	$b_{[\,cm\,]}$	$c_{[\,cm\,]}$
1)	27°	63°	7.8	b	c
2)	43.22°	46.78°	6.3	6.7	9.2
3)	80°	10°	4.73	0.83	4.8
4)	33.82°	56.18°	13.4	20	24.07
5)	64.36°	25.64°	5	2.4	5.55

2.18 $r \simeq 6.07\text{m}$ $h \simeq 2.56\text{m}$

2.19 $h \simeq 27.62\text{m}$

2.20 $d_{AB} \simeq 2.6397\text{km}$

2.21 $p \simeq 26.09\text{cm}$ $A \simeq 30.47\text{cm}^2$

2.22 $d \simeq 42.91\text{m}$ $\alpha \simeq 8.45°$

2.23 $b \simeq 15.64\text{m}$ $c \simeq 17.22\text{m}$ $h \simeq 15.35\text{m}$

2. 24

1) $x_1 = 5.7° + k \cdot 360°,\ k \in \mathbb{Z}$ $x_2 = 174.3° + k \cdot 360°,\ k \in \mathbb{Z}$
2) $x_1 = 302.9° + k \cdot 360°,\ k \in \mathbb{Z}$ $x_2 = 237.1° + k \cdot 360°,\ k \in \mathbb{Z}$
3) $x_1 = 57.0° + k \cdot 360°,\ k \in \mathbb{Z}$ $x_2 = 93.0° + k \cdot 360°,\ k \in \mathbb{Z}$
4) $x_1 = 36.9° + k \cdot 360°,\ k \in \mathbb{Z}$ $x_2 = 323.1° + k \cdot 360°,\ k \in \mathbb{Z}$
5) $x_1 = 147.1° + k \cdot 360°,\ k \in \mathbb{Z}$ $x_2 = 212.9° + k \cdot 360°,\ k \in \mathbb{Z}$
6) $x_1 = 55.3° + k \cdot 360°,\ k \in \mathbb{Z}$ $x_2 = 344.7° + k \cdot 360°,\ k \in \mathbb{Z}$
7) $x_1 = 76° + k \cdot 180°,\ k \in \mathbb{Z}$
8) $x_1 = 162.5° + k \cdot 180°,\ k \in \mathbb{Z}$
9) $x_1 = 86.3° + k \cdot 180°,\ k \in \mathbb{Z}$

2. 25

Questions	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1)	35.26°	144.74°	215.26°	324.74°				
2)	21.14°	81.14°	141.14°	201.14°	261.14°	321.14°		
3)	108.43°	161.57°	288.43°	341.57°				
4)	18.62°	56.38°	108.62°	146.38°	198.62°	236.38°	288.62°	326.38°
5)	55.9°	145.9°	235.9°	325.9°				
6)	6.7°	83.3°	126.7°	203.3°	246.7°	323.3°		

2. 26 $x \simeq 10.5$

2. 27

1) $x = -26.57° + k \cdot 180°,\ k \in \mathbb{Z}$
2) $x_1 = \frac{2\pi}{3} + k \cdot 2\pi,\ k \in \mathbb{Z}$
   $x_2 = \frac{4\pi}{3} + k \cdot 2\pi,\ k \in \mathbb{Z}$
3) $x_1 = \frac{\pi}{2} + k \cdot 2\pi,\ k \in \mathbb{Z}$
   $x_2 = \frac{\pi}{6} + k \cdot 2\pi,\ k \in \mathbb{Z}$
   $x_3 = \frac{5\pi}{6} + k \cdot 2\pi,\ k \in \mathbb{Z}$
4) $x = 15° + k \cdot 90°,\ k \in \mathbb{Z}$
5) $x_1 = 210° + k \cdot 360°,\ k \in \mathbb{Z}$
   $x_2 = 330° + k \cdot 360°,\ k \in \mathbb{Z}$
   $x_3 = 90° + k \cdot 360°,\ k \in \mathbb{Z}$
6) $x_1 = \frac{3\pi}{2} + k \cdot 2\pi,\ k \in \mathbb{Z}$
   $x_2 = \frac{\pi}{6} + k \cdot 2\pi,\ k \in \mathbb{Z}$