1.16 Exercises

1. 1 : Whenever possible, compute the products $A \cdot B$ and $B \cdot A$.

1) $A = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

2) $A = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 0 & 2 \end{pmatrix}$

3) $A = \begin{pmatrix} 0 & -1 & 5 \\ 2 & -2 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 4 \\ 0 & -1 \\ 3 & 2 \end{pmatrix}$

4) $A = \begin{pmatrix} 1 & -3 & 4 \\ -2 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & -4 & -3 \\ 5 & -1 & 0 \\ 2 & 0 & 0 \end{pmatrix}$

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1. 2 : Let $(\vec{e_1}, \vec{e_2})$ be the standard basis of V_2. Consider the vector $\vec{v} = 2\vec{e_1} - 3\vec{e_2}$.

1) Find the components of $\vec{v}$ with respect to the basis $(\vec{e_1}, \vec{e_2})$, $(\vec{e_2}, \vec{e_1})$ and $(2\vec{e_1}, 6\vec{e_2})$.

2) Show that $(\vec{v_1}, \vec{v_2})$ with $\vec{v_1} = \vec{e_1} - \vec{e_2}$ and $\vec{v_2} = \vec{e_1} + \vec{e_2}$ is a basis of V_2.

3) Give the components of $\vec{u} = \vec{e_1} - 5\vec{e_2}$ with respect to the basis $(\vec{v_1}, \vec{v_2})$.

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1. 3 : We collect the vectors in V_3 whose components x, y, z satisfy the following conditions. Are the subsets obtained subspaces ? If so indicate the dimension of the subspaces obtained and give a basis.

1) $z = 0$

2) $z = 1$

3) $x^2 + y^2 + z^2 = 1$

4) $x = y = 2z$

5) $x + y + 2z = 0$

6) $x + y + 2z = 1$

7) $\begin{cases} x + y - 2z = 0 \\ x - y + 2z = 0 \end{cases}$

8) $\begin{cases} x + y - 2z = 0 \\ 2x + y - z = 0 \\ x + 2y - 5z = 0 \end{cases}$

9) $\begin{cases} x + y - 2z = 0 \\ 2x + y - z = 0 \\ x + 2y - z = 0 \end{cases}$

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1. 4 : In V_3 we consider the vectors $\vec{v_1} = 5\vec{e_1} - 2\vec{e_2} + \vec{e_3}$ and $\vec{v_2} = 3\vec{e_1} + \vec{e_2} - 5\vec{e_3}$.

1) Explain why the set $(\vec{v_1}, \vec{v_2}, \vec{v_1} \times \vec{v_2})$ is a basis of V_3.

2) Express $\vec{e_1}$ in this basis.

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1.5 : In V_3, with the usual (orthonormal direct) basis, let's determine the dimension of the subspaces generated by these vectors. Check that the subspaces 3) and 4) are identical.

1) $\vec{v_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \ \vec{v_2} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \ \vec{v_3} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

2) $\vec{v_1} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \ \vec{v_2} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}, \ \vec{v_3} = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix}$

3) $\vec{v_1} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \ \vec{v_2} = \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix}$

4) $\vec{v_1} = \begin{pmatrix} 4 \\ 6 \\ 7 \end{pmatrix}, \ \vec{v_2} = \begin{pmatrix} 4 \\ -6 \\ 1 \end{pmatrix}$

1.6 : In V_3 we consider the subspace W of all linear combinations of the vectors $\vec{v_1} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}, \ \vec{v_2} = \begin{pmatrix} -1 \\ -3 \\ -4 \end{pmatrix}$ and $\vec{v_3} = \begin{pmatrix} 1 \\ 2 \\ 2.5 \end{pmatrix}$ given with respect to the standard basis.

1) Show that $\vec{v_3}$ is a linear combination of $\vec{v_1}$ and $\vec{v_2}$.

2) Show that $(\vec{v_1}, \vec{v_2})$ is a basis of W.

3) Find the equation of the plane W.

1.7 : Find a basis of the following subspaces of V_3 :

1) The plane $3x - 2y + 5z = 0$.

2) All vectors of the form $\vec{v_1} = \begin{pmatrix} c \\ a - c \\ c \end{pmatrix}$.

1.8 : Let V be the vector space of polynomials of degree ≤ 2.

1) Find the components of $p = 3x^2 - 4x + 5$ with respect to the basis $(1, x, x^2)$.

2) Show that the set of polynomials $p_1 = x^2 + 2x$, $p_2 = x + 2$ and $p_3 = x^2 + 3$ is a basis of V.

3) Find the components of $p = 3x^2 - 4x + 5$ with respect to the basis (p_1, p_2, p_3).

1. 9 : Let V be the vector space of polynomials of degree ≤ 2.

1) Show that the set W of polynomials which take the value 0 for $x = a$ (fixed) is a subspace of V.
2) Find a basis of W.
Hint : a polynomial p such that $p(x) = 0$ for $x = a$ can be factorized by $(x - a)$.

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1. 10 : In the vector space M_{2x2}, we consider $A = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -1 \\ 0 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 7 & -1 \\ 1 & 3 \end{pmatrix}$.

1) What is the dimension of M_{2x2} ? Justify your answer by proposing a basis.
2) What can you deduce about the subspace generated by A, B and C ?
3) Check that A, B and C are linearly dependant.
4) Propose two matrices D and E such that (A, B, D, E) is a basis of M_{2x2}.

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1. 11 : In each case determine whether the application is linear or not. If possible, describe it geometrically.

1) $f : V_2 \to V_2,\ \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ y \end{pmatrix}$
2) $f : V_2 \to V_2,\ \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 0 \\ y \end{pmatrix}$
3) $f : V_2 \to V_2,\ \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} y \\ x \end{pmatrix}$
4) $f : V_2 \to V_2,\ \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x^2 \\ y+1 \end{pmatrix}$
5) $f : V_2 \to V_2,\ \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x \\ x \end{pmatrix}$
6) $f : V_2 \to V_2,\ \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} -y \\ x \end{pmatrix}$
7) $f : V_1 \to V_2,\ x \mapsto \begin{pmatrix} x \\ x^2 \end{pmatrix}$
8) $f : V_2 \to V_1,\ \begin{pmatrix} x \\ y \end{pmatrix} \mapsto x + 3y$
9) $f : V_3 \to V_2,\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} x + a \\ y - 2z \end{pmatrix}$,
where a is a fixed number.

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1. 12 : Two linear transformations f and g from V_2 to V_2 are given by the images of the vectors $\vec{a} = \vec{e_1} + \vec{e_2}$ and $\vec{b} = -\vec{e_1} + 2\vec{e_2}$: $f(\vec{a}) = -\vec{e_1} + 6\vec{e_2}$, $f(\vec{b}) = -5\vec{e_1} + 6\vec{e_2}$, $g(\vec{a}) = 2\vec{e_1} - 6\vec{e_2}$ and $g(\vec{b}) = 7\vec{e_1} - 21\vec{e_2}$.

1) Determine the vectors $f(\vec{e_1})$, $f(\vec{e_2})$, $g(\vec{e_1})$, $g(\vec{e_2})$, $f(\vec{v})$, $g(f(\vec{v}))$, with $\vec{v} = 2\vec{e_1} - \vec{e_2}$.
2) In the standard basis of V_2, express the matrices F of f, G of g and M of $g * f$.
3) Use M to verify the components of $g(f(\vec{v}))$.

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1. 13 : Find the matrix, with respect to the standard basis, of each of the following linear transformation of V_2 and use this matrix to calculate the image of the vector $\vec{v} = 2\vec{e_1} - 3\vec{e_2}$.

1) Homothecy with factor k.
2) Rotation by $+30°$.
3) Projection on the line $3x - 2y = 0$.
4) Orthogonal projection on the axis $a : x - 3y = 0$.
5) Reflection in the axis $a : x - 3y = 0$.
6) Projection on the axis a, in the direction $\vec{d} = -\vec{e_1} + \vec{e_2}$.
7) Perspective affinity with axis a, such that the image of point $R\,(2;2)$ is $R'\,(1;-1)$. Also determine its factor.

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1. 14 : Let V be the set containing the real functions of the form $f(x) = e^{-x}\,(a\cos(2x) + b\sin(2x))$, with a and $b \in \mathbb{R}$.

1) Show that V is a vector space.
2) Explain why the differentiation is a linear operation on V.
   Determine its matrix according to the basis (f_1, f_2), with $f_1(x) = e^{-x}\cos(2x)$ and $f_2(x) = e^{-x}\sin(2x)$.

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1. 15 : Let E be the vector space of the polynomials of degree ≤ 2. Is the transformation $f : E \to \mathbb{R},\ p \mapsto p(a)$ (where a is a fixed number) linear ?

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1. 16 : We consider E the vector space of the polynomials with degree ≤ 2, and the application T defined by $T : E \to F$, $T(p) = p'$, its derivative.

1) Describe F.
2) Propose a basis for E and for F. Determine the matrix of T.

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1. 17 : Answer the following :

1) Determine the matrix S_α of the reflection in the axis that forms an angle α with the x-axis in V_2.
2) Indicate the matrix R_φ of the rotation by an angle φ.
3) Determine $S_\alpha \cdot S_\alpha$ by first guessing and then computing.
4) Check that $S_\alpha \cdot S_\beta = R_{2(\alpha-\beta)}$.

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1. 18 : Find the matrix, with respect to the standard basis, of each of the following linear transformation of $\mathbb{R}^3$ and use this matrix to calculate the image of the vector $\vec{v} = 2\vec{e_1} - 3\vec{e_2} + 5\vec{e_3}$. Determine the eigenvalues, propose eigenvectors and give the eigenmatrix.

1) Reflection in the plane $x = y$.
2) Reflection in the y-axis.
3) Rotation of $+120°$ around the oriented axis given by $\vec{a} = \vec{e_1} + \vec{e_2} + \vec{e_3}$.
4) Orthogonal projection onto the line through the origin and parallel to $\vec{a} = \vec{e_1} + \vec{e_2} + \vec{e_3}$.
5) Orthogonal projection onto the yz-plane.
6) Vertical projection onto the plane $x + y - z = 0$.
7) Orthogonal projection onto the plane $x + y - z = 0$.
8) Reflection in the plane $x + y - z = 0$.
9) Rotation of $90°$ about the x-axis and so that the vector $\vec{e_3}$ is mapped to $\vec{e_2}$.

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1. 19 : Find the matrix of $f : V_3 \to V_2$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} x+2y \\ z-2y \end{pmatrix}$ with respect the standard basis and then use this matrix to calculate the image of the vector $\vec{v} = 3\vec{e_1} - 4\vec{e_2} - \vec{e_3}$.

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1. 20 : In each case describe geometrically the operator on V_n given by its matrix with respect to the standard basis.

1) $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ 4) $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 7) $G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

2) $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ 5) $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 8) $H = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

3) $C = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ 6) $F = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 9) $I = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$

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1. 21 : We consider V_3, with the standard basis $(\vec{e_1}, \vec{e_2}, \vec{e_3})$.

1) Determine P the matrix of the orthogonal projection onto the plane $x - y - z = 0$.

We call I the matrix of the transformation $i : \vec{v} \mapsto \vec{v}$ (identity).

2) Determine I, $Q = I - P$, P^2 and PQ.
3) Find the vectors that are constant under Q.
4) Find the vectors whose image under Q is the zero vector.
5) Deduce a geometrical interpretation of the transformation given by the matrix Q.

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1. 22 : A linear application is given by the symmetric matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$. Verify that there is at least one eigenvalue.

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1. 23 : Show that the characteristic polynomial of M (type 2×2) is

$$p(\lambda) = \lambda^2 - tr(M) \cdot \lambda + \det(M)$$

Generalization
$Tr(M) = \sum \lambda_i = \lambda_1 + \lambda_2 + ... + \lambda_n$ and
$\det(M) = \prod \lambda_i = \lambda_1 \cdot \lambda_2 \cdot ... \cdot \lambda_n$

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1. 24 : Describe the linear transformations associated to the matrices $F = \begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix}$, $G = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$ and $H = \begin{pmatrix} 1 & 4 & 5 \\ 0 & 0.75 & 6 \\ 0 & 0 & 0.5 \end{pmatrix}$.

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1. 25 : When possible, determine the inverse matrix of

1) $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$ 3) $C = \begin{pmatrix} 4 & 12 \\ 3 & 9 \end{pmatrix}$ 5) $E = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

2) $B = \begin{pmatrix} 4 & -1 \\ 0 & 1 \end{pmatrix}$ 4) $D = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ 6) $F = \begin{pmatrix} -3 & 1 & 6 \\ 2 & -6 & -10 \\ 0 & 2 & 4 \end{pmatrix}$

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1. 26 : Give the interpretation of the transformation f, with matrix $F = \begin{pmatrix} -5/13 & 12/13 \\ 12/13 & 5/13 \end{pmatrix}$. Find the matrix of the transformation g such that $g * f$ is the reflection in the axis $y = x$. What's the geometrical interpretation of g ?

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1. 27 : Answer the following :

1) Compute the inverse matrix of $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 2 & 4 \\ 1 & 1 & 1 \end{pmatrix}$.

2) Determine m so that $B = \begin{pmatrix} 3 & 0 & 2m \\ m & 11 & -m \\ m-1 & 0 & 8 \end{pmatrix}$ is singular.

3) For the value(s) of m obtained at 2), give the geometrical interpretation of the transformation associated to B ?

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1. 28 : Answer the following :

1) Prove that $(AB)^{-1} = B^{-1}A^{-1}$.

2) Prove that $\det({}^tA) = \det(A)$ (for dimensions 2 and 3).

3) Prove that the product of the eigenvalues of a matrix is equal to the determinant of that matrix.

4) Deduce from 3), that when the determinant of a matrix is zero then the transformation associated is (or contains) a projection.

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1. 29 : Determine the eigenvalues and eigenvectors of the transformation f described, in a standard basis of $\mathbb{R}^3$, by the matrix $F = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$. Then, propose an eigenbasis and the associated matrix F_p.

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1. 30 : A linear application f has matrix $F = \begin{pmatrix} 3 & -3 \\ 2 & 10 \end{pmatrix}$, in the standard basis $(\vec{e_1}, \vec{e_2})$.

1) Determine the eigenvalues and eigenvectors of F.

2) Choose an eigenbasis $(\vec{p_1}, \vec{p_2})$ and express the matrix of f in that basis (name it F_p).

3) Find the images of $\vec{a} = 2\vec{e_1} + \vec{e_2}$ and $\vec{b} = \frac{1}{2}\vec{p_1} + \frac{1}{3}\vec{p_2}$ under f :
First : By working in $(\vec{e_1}, \vec{e_2})$ with the matrix F
Then : By working in $(\vec{p_1}, \vec{p_2})$ with the matrix F_p

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1. 31 : In the standard basis, we consider the application f with matrix $A = \begin{pmatrix} 0 & 2 \\ 2 & 3 \end{pmatrix}$.

1) Compute A^{-1}, A^2, A^3.

2) Determine the eigenvalues and eigenvectors of f, f^{-1}, $f * f$ and $f * f * f$.

3) Choose an eigenbasis $(\vec{p_1}, \vec{p_2})$ and give the matrix B of f in that new basis.

4) Compute B^{-1}, B^2, B^3 and B^n.

5) Determine the matrix P of the application p such that $(\vec{p_1}, \vec{p_2})$ has image $(\vec{e_1}, \vec{e_2})$.

6) Compute P^{-1}.

7) Use the eigenbasis of f and the matrices P and P^{-1} in order to determine A^n.
Check your previous formula for $n = -1$, $n = 1$, $n = 2$ and $n = 3$.

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1. 32 : We consider the application f whose matrix in the standard basis is
$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 9 & -1 \\ -1 & -1 & 4 \end{pmatrix}.$$

1) Verify that it has three different eigenvalues, given that one of them is an integer.

2) Diagonalize A by expressing it in an eigenbasis. You don't have to find that eigenbasis.

3) Let's consider the application $g = f * f - 12f + 25i$, where i is the identity. Find the eigenvalues of g by working in a *good* basis.

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1. 33 : The application f is defined by its matrix $F = \begin{pmatrix} -7/5 & 0 & 4/5 \\ 0 & -1 & 0 \\ 4/5 & 0 & -13/5 \end{pmatrix}$.

Determine the matrix of f^{-1} and the one of $(f * f)^{-1}$. Find $\vec{v}$ such that $f(\vec{v}) = \vec{e_1} + 5\vec{e_2} - 22\vec{e_3}$.

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1. 34 : Diagonalize the matrices $A = \begin{pmatrix} 2 & 4 \\ 3 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 & 4 \\ 3 & -4 & 12 \\ 1 & -2 & 5 \end{pmatrix}$.

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1. 35 : In V_3 with a standard basis, we consider a transformation given by its matrix $A = \begin{pmatrix} 1 & -4 & 8 \\ 8 & 4 & 1 \\ -4 & 7 & 4 \end{pmatrix}$

1) Verify that $B = \frac{1}{9}A$ is an orthogonal matrix. Deduce the matrices B^{-1} and A^{-1}.

2) Determine the eigenvalues and eigenvectors of B. Deduce the ones of A.

3) Give the geometric transformation of B and A.

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1. 36 : In the standard basis $(\vec{e_1}, \vec{e_2}, \vec{e_3})$, we consider the transformation f from V_3 in V_3 given by :

$$f(\vec{e_1}) = \vec{e_2} + \vec{e_3},\ f(\vec{e_2}) = \vec{e_1} + \vec{e_3} \text{ and } f(\vec{e_3}) = \vec{e_1} + \vec{e_2}$$

1) Show that its possible to find an orthornormal basis of f formed of eigenvectors : $(\vec{p_1}, \vec{p_2}, \vec{p_3})$.

2) Explain then why the transformation t which maps the basis $(\vec{e_1}, \vec{e_2}, \vec{e_3})$ to $(\vec{p_1}, \vec{p_2}, \vec{p_3})$ is a rotation. Without doing the computations, explain how to find the matrix of t; its axis of rotation and the angle of that rotation.

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1. 37 : We consider $\vec{u}$ a fixed unit vector of V_3 and the the four following transformations :

$$i : \vec{v} \to \vec{v},\ f : \vec{v} \to \vec{u} \times \vec{v},\ g : \vec{v} \to (\vec{u} \bullet \vec{v})\vec{u} \text{ and } h = i - g$$

1) Verify that f, g, h and i are linear applications.

2) Relatively to a standard basis $(\vec{u_1}, \vec{u_2}, \vec{u_3})$, the vector $\vec{u}$ is given by $\vec{u} = a\vec{u_1} + b\vec{u_2} + c\vec{u_3}$, with $a^2 + b^2 + c^2 = 1$.
Determine, in that basis, the matrices F, G and H of f, g and h.

3) Verify that $f * f = -h$.

4) Find the image of $\vec{u}$ and of the vectors orthogonal to $\vec{u}$ through the applications g and h.

5) Give a geometrical interpretation of g and of h.
Deduce the equalities $g * g = g,\ h * h = h$ and $h * g = 0$.

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1. 38 : **First part**

In V_2 an operator is given by its matrix $M = \begin{pmatrix} 6 & k \\ k & -1 \end{pmatrix}$ with respect an orthonormal basis, k being a constant.

1) Show that, for any k, the operator has two different eigenvalues.

2) Find k so that the vector $\vec{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ is an eigenvector. Give the corresponding eigenvalue. For this value of k, check that any vector orthogonal to $\vec{a}$ is also an eigenvector and give the corresponding eigenvalue.

Second part

In the space V_3 a linear transformation T is given by its matrix
$F = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$ with respect to the standard basis.

1) Find the matrices F^2 and F^3. Is it possible to find immediately the matrix F^n ?

2) Check that the eigenvalues of T are -1, 0 and 1.

3) Find a basis $(\vec{p_1}, \vec{p_2}, \vec{p_3})$ consisting in eigenvectors of T.

4) What are the images under $T \circ T$ of each of the vectors $\vec{p_1}$, $\vec{p_2}$ and $\vec{p_3}$? Deduce, without any further calculations, the eigenvalues of $T \circ T$.

5) Describe geometrically the transformation $T \circ T$.

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1. 39 : An affine transformation is given by its associated matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$ and by $O'(-4;4)$ the image of the origin.

1) Determine the invariant points.
2) Choose a system $(\Omega, \vec{p_1}, \vec{p_2})$, where Ω is invariant and $(\vec{p_1}, \vec{p_2})$ forms an eigen-basis of A.
3) Give the formula of the affinity in that system.
4) Give a geometrical interpretation.

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1. 40 : Determine the formula of the following affine transformations of the plane.

1) Reflection in the line $x - y + 1 = 0$.
2) Perspective affinity with axis $y = x + 1$ such that the image of the origin is $O'(2;-1)$.
3) Translation with translation vector $\vec{t} = -2\vec{u_1} + 3\vec{u_2}$.
4) Homothecy with centre $\Omega(\alpha;\beta)$ and factor λ.
5) Rotation through 120° that maps $A(-2;1)$ to $A'(1;0)$. Also find the centre.

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1. 41 : An affine transformation f is given by three points $A(1;2)$, $B(3;0)$, $C(5;4)$ and their images $A'(-10;-1)$, $B'(12;13)$, $C'(-2;3)$.
Find the formula of f. Find the fixed points of f. Find the invariant directions of f.

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1. 42 : An affine transformation φ is given by $\begin{cases} x' = 0.8x - 0.6y + 1 \\ y' = 0.6x + 0.8y - 3 \end{cases}$

1) Find the Cartesian equation of the curve Γ whose image is the parabola with equation $y = x^2$.
2) Find the expression of the inverse affinity.
3) Find the equation of the curve Γ', image through φ of the circle with radius 1 centered at $(1;3)$.
4) Show that φ is a rotation. Calculate its angle and its center.

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1. 43 : What's the centre of the rotation obtained by composing a rotation through 180° around $(1;-1)$ and a rotation by $-90°$ around $(3;1)$? Is the result the same if the order of the composition is changed ?

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