LJP: TE 18 Solutions

EXERCICE 1

$$\int_0^{2m} e^{3x}dx = \frac{2}{3} \Leftrightarrow \frac{1}{3}e^{3x}\Big|_0^{2m} = \frac{2}{3} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{3}e^{6m} - \frac{1}{3}e^0 = \frac{2}{3} \Leftrightarrow \frac{1}{3}e^{6m} - \frac{1}{3} = \frac{2}{3} \Leftrightarrow$$

$$\Leftrightarrow e^{6m} - 1 = 2 \Leftrightarrow e^{6m} = 3 \Leftrightarrow 6m = \ln 3 \Leftrightarrow$$

$$\Leftrightarrow m = \frac{1}{6}\ln 3$$

EXERCICE 2

a) $$\int (4-5x^3)^2 dx = \int (16 - 40x^3 + 25x^6)dx =$$

$$= 16x - \frac{40}{4}x^4 + \frac{25}{7}x^7 + C = 16x - 10x^4 + \frac{25}{7}x^7 + C$$

b) $$\int \frac{-2x+3}{-3x^2+9x-8}dx$$

$t = -3x^2+9x-8 \Rightarrow$

$\mapsto dt = (-6x+9)dx \Rightarrow$

$dt = 3(-2x+3)dx \Rightarrow dx = \frac{1}{3(-2x+3)}dt$

$$= \int \frac{(-2x+3)}{t}\frac{dt}{3(-2x+3)} =$$

$$= \frac{1}{3}\int \frac{dt}{t} = \frac{1}{3}\ln t = \frac{1}{3}\ln(3x^2+9x-8) + C.$$

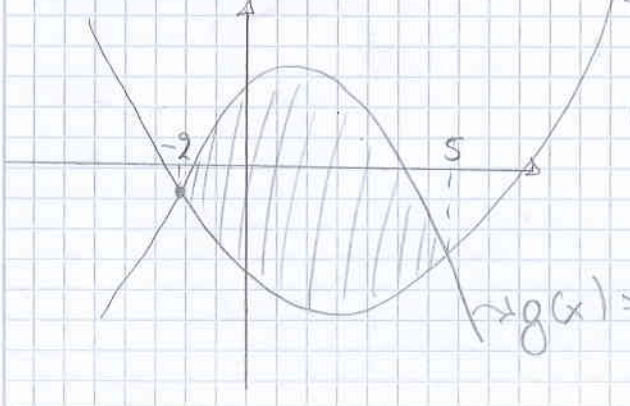
c) $$\int \frac{x^2-3x+12}{x-5}dx =$$

$$x^2-3x+12 \;\big|\; x-5$$
$$-x^2+5x \quad\quad x+2$$
$$2x+12$$
$$-2x+10$$
$$12$$

$$= \int \left(x+2+\frac{12}{x-5}\right)dx =$$

$$= \frac{1}{2}x^2 + 2x + 12\ln|x-5| + C.$$

EXERCICE 3



$\mapsto f(x) = x^2 - 6x - 20$

$f(x) = g(x) \Leftrightarrow x^2 - 6x - 20 = -2x^2 + 3x + 10$

$\Leftrightarrow 3x^2 - 9x - 30 = 0 \Leftrightarrow x^2 - 3x - 10 = 0$

$x_1 = 5 \quad x_2 = -2$

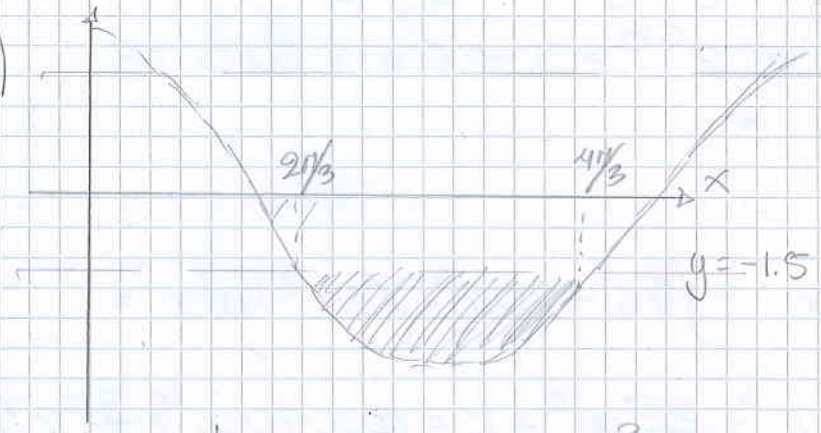
$\mapsto g(x) = -2x^2 + 3x + 10$

$F(x) = \frac{1}{3}x^3 - 3x^2 - 20x$

$G(x) = -\frac{2}{3}x^3 + \frac{3}{2}x^2 + 10x$

$$A = \int_{-2}^{5} (g(x) - f(x))\,dx = (G(5) - F(5)) - (G(2) - F(2)) \approx 171.5$$

b)



$f(x) = -1.5 \Leftrightarrow 3\cos x = -\frac{3}{2} \Leftrightarrow \cos x = -\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \quad x = \frac{2\pi}{3} \quad x = \frac{4\pi}{3}$

$$A = \int_{2\pi/3}^{4\pi/3} \left(-\frac{3}{2} - 3\cos x\right)dx = -\left[-\frac{3}{2}x - 3\sin x\right]_{2\pi/3}^{4\pi/3} =$$

$$= \frac{3}{2}x + 3\sin x \Big|_{2\pi/3}^{4\pi/3} \approx 2.0546$$